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Application of the Energy Dot Product (EDP) in Recognizing the Energy Flow Synchronization

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In the article construction and application of the EDP is introduced. This construction is based on the transformation of the traditional phase space into the energy space. After this transformation the state vector and its derivative determine the energy flow plane. The angle between these vectors shows energy flow direction. Application of such a construction in recognizing the energy flow synchronization is shown.

Keywords: energy space, EDP, nonlinear dynamics

1. Introduction

An energy flow as the final effect of the external and internal interactions is very important aspect of the system dynamics. Observations of the energy state of the system, or its part gives possibility of the determining and explaining the reasons of the system behaviour. It allows to control the system dynamics, what is especially important in the case of the chaotic systems [4, 9, 32]. Such a control can cause for instance oscillation reduction [7–10, 32], giving possibilities of the system application.

An energy flow modelling still arouses interests in the scientific world [6, 7, 11–15, 29–34]. Different methods are applied to solve problems connected with energy flow: Statistical Energy Analysis [7, 11, 12], Finite Element Method [7], modes theory [13]. But these methods do not allow for a special kind of a geometrical view on energy changes, which could develop our intuitional knowledge on energy flow phenomenon. This intuition is very important especially in modelling the systems, where we still can not measure energy flow, such as the brain dynamics [25–27], or applied in medicine diagnostics bioenergy, observed thanks to GDV method based on Kirlian effect [6, 7, 8].

In this article an application of the EDP in detecting synchronization, nowadays one of the most investigated phenomenon [16–25] was shown. Some aspects of motion, energy flow, synchronization are considered.

2. Introduction to the EDP analysis

The EDP is the traditional dot product function introduced into energy space. The energy space [2, 3, 33, 34] is constructed in a way, that the norm $|\vec{x}|$ of the dynamic state vector is the function of the energy E, that is accumulated in the system. After this transformation vector $\vec{x}$ and its derivative $\frac{d\vec{x}}{dt} = \dot{\vec{x}}$ determine the current energy flow plane. The angle between these vectors shows energy E changes. In a particular case, when the amount of the energy accumulated in the system is constant, the angle between $\vec{x}$ and $\dot{\vec{x}}$ is the right angle. In that case EDP function applied to these vectors:

$$\vec{x} \circ \dot{\vec{x}} = 0 \tag{1}$$

The same construction can be also applied in the energy subspaces, showing the energy flow in some parts of the system. Consider for instance the particular energy flow synchronization, when the same amount of the energy flows out and in some parts of the system. In that case the projection $\vec{x}_p$ of the vector $\vec{x}$ on to the considered subspace, together with the projection $\dot{\vec{x}}_p$ of the vector $\dot{\vec{x}}$, are vectors with the right inclination angle. EDP function applied to these vectors:

$$\vec{x}_p \circ \dot{\vec{x}}_p = 0 \tag{2}$$

3. Energy flow synchronization in the one DOF vibrating system

The most simple energy flow synchronization phenomena can be observed in the one DOF vibrating system. Consider the system with the mathematical model:

$$\frac{d\vec{x}}{dt} = \dot{\vec{x}} = \begin{bmatrix} x_2 \\ q\cos\eta\tau - 2hx_2 - \alpha^2 x_1 \end{bmatrix} \tag{3}$$

where:

$$q = \frac{F}{m}; \quad \eta = \frac{\omega}{\alpha}; \quad 2h = \frac{c}{m}; \quad \alpha = \sqrt{\frac{k}{m}};$$

m – mass of the oscillator,
k – spring stiffness,
c – viscotic damping coefficient,
F – external excitation force amplitude,
ω – frequency of the external excitation force.

The phase vector $\vec{x}$in the standard phase space $\mathbb{R}^2$ is represented by two components $x_1;\ x_2$.

$\vec{x} = [x_1;\ x_2]$

Transform the phase space of the system using the function f: $\mathrm{R}^2 \rightarrow \mathrm{R}^2$:

$$f(\vec{x}) = \begin{bmatrix} \sqrt{\frac{k}{2}} \\ \sqrt{\frac{m}{2}} \end{bmatrix} \vec{x} = \vec{z} \tag{4}$$

After this transformation the mathematical model of the system in the energy space:

$$\frac{d\vec{z}}{dt} = \dot{\vec{z}} = \begin{bmatrix} \alpha z_2 \\ q\sqrt{\frac{m}{2}}\cos\eta\tau - 2hz_2 - \alpha^2 z_1 \end{bmatrix} \tag{5}$$

To simplify the description of the transformation note, that in the energy space there are square roots of the potential (z_1) or kinetic (z_2) energies on each axis. Note also, that function f transforms energy into vector form.

Assume the solution as:

$$\vec{z} = \begin{bmatrix} \sqrt{\frac{k}{2}}A\cos(\eta\tau - \beta) \\ \sqrt{\frac{m}{2}}A\sin(\eta\tau - \beta) \end{bmatrix} \tag{6}$$

then

$$\dot{\vec{z}} = \begin{bmatrix} -\alpha\sqrt{\frac{m}{2}}A\eta\sin(\eta\tau - \beta) \\ q\sqrt{\frac{m}{2}}\cos(\eta\tau) + 2h\sqrt{\frac{m}{2}}A\eta\sin(\eta\tau - \beta) - \alpha\sqrt{\frac{k}{2}}A\cos(\eta\tau - \beta) \end{bmatrix} \tag{7}$$

Consider the EDP function as:

$$\vec{z} \circ \dot{\vec{z}} = z_1\dot{z}_1 + z_2\dot{z}_2$$

Condition

$$\vec{z} \circ \dot{\vec{z}} = 0 \tag{8}$$

means that the norm $|\vec{z}|$ of the energy state vector $\vec{z}$ has the constant value. In such a case an amount of the energy accumulated in the system is constant. In the system there exists continuous energy changes from the kinetic into potential and vice versa. All the excitation energy is dissipated in the viscous damper. Condition (8) is satisfied when:

$$2hA\eta = q \quad \text{and} \quad \beta = \frac{\pi}{2} \tag{9}$$

For such a parameter values the system is in the resonance state. It can be seen in the Fig. 1a and 1b - resonance diagram and EDP diagrams respectively. One can see that for the excitation frequency $\eta = 1$ EDP is equal to zero. The energy flow in such a case can be seen in the Fig. 2. The vector trajectory is a regular circle. It means that amount of the energy accumulated in the system is constant. In the system there exists continuous energy changes from the kinetic into potential and vice versa. All the excitation energy is dissipated in the viscous damper. Such an energy flow synchronization is possible only for the resonance state.

4. Energy flow synchronization in the two DOF vibrating system

Consider the vibrating system consisted of two joined oscillators. The external harmonic force excites the oscillator μ. This oscillator is joined with the second oscillator μ_1. The mathematical model of the system is given by four differential equations of the first order:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \left(F \sin \eta\tau - c x_2 - c_1 \left(x_2 - x_4\right) - \sigma x_1 - \sigma_1 \left(x_1 - x_3\right)\right) \dfrac{1}{\mu} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \left(c_1 \left(x_2 - x_4\right) + \sigma_1 \left(x_1 - x_3\right)\right) \dfrac{1}{\mu_1} \end{cases} \tag{10}$$

where:

$\mu,\ \mu_1$– masses of the oscillators
$\sigma,\ \sigma_1$– stiffness coefficients of the springs
c, c_1 – damping coefficients
F – amplitude of the external excitation force
ω – frequency of the external excitation force

$$\eta = \frac{\omega}{\alpha} \quad \alpha = \sqrt{\frac{\sigma}{\mu}} \tag{11}$$

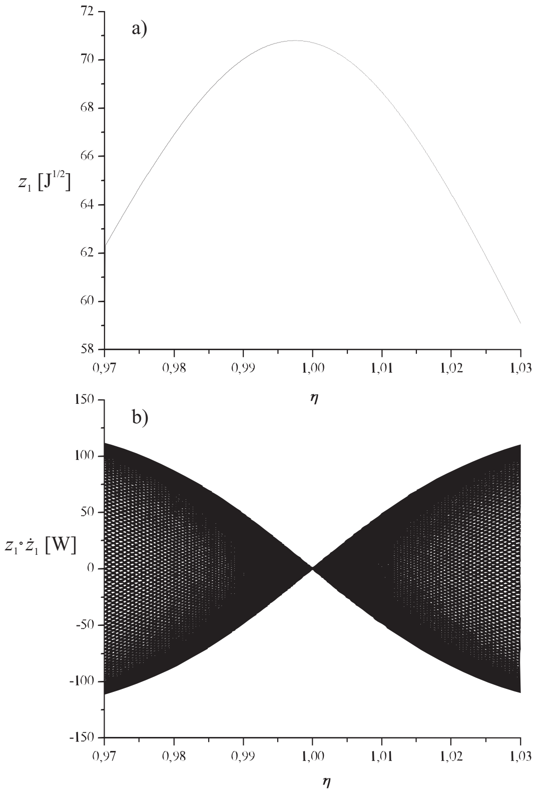

Figure 1 a) Resonance diagram, b) EDP diagram, $\alpha = 1,\ h = 0.05,\ q = 10$

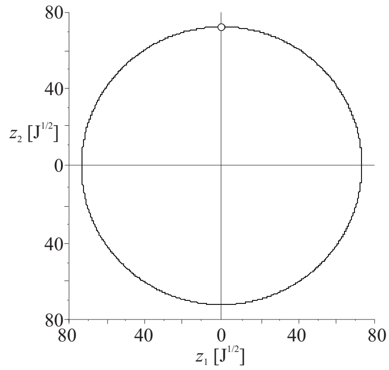

Figure 2 Energy plane, $\alpha = 1,\ h = 0.05,\ q = 10,\ \mu = 1$

The phase vector in the standard phase space $\mathbb{R}^4$ is represented by four components: $x_1;\ x_2;\ x_3;\ x_4$.

Transform the phase space of the system using the given function f: $\mathrm{R}^4 \to \mathrm{R}^4$:

$$f(x_1, x_2, x_3, x_4) = \begin{cases} \sqrt{\frac{\sigma}{2}}x_1 = z_1 \\ \sqrt{\frac{\mu}{2}}x_2 = z_2 \\ \sqrt{\frac{\sigma_1}{2}}(x_3 - x_1) = z_3 \\ \sqrt{\frac{\mu_1}{2}}x_4 = z_1 \end{cases} \tag{12}$$

After this transformation the mathematical model of the system in the energy space is given by the following differential equations:

$$\begin{cases} \dot{z}_1 = \sqrt{\frac{\sigma}{\mu}}z_1 \\ \dot{z}_2 = -\frac{c}{\mu}z_2 - \frac{c_1}{\mu}z_2 + \frac{c_1}{\sqrt{\mu\mu_1}}z_4 - \sqrt{\frac{\sigma}{\mu}}z_1 + \sqrt{\frac{\sigma_1}{\mu}}z_2 + \frac{F}{\mu}\sin\eta\tau \\ \dot{z}_3 = \sqrt{\frac{\sigma_1}{\mu_1}}z_4 - \sqrt{\frac{\sigma_1}{\mu}}z_2 \\ \dot{z}_4 = -\sqrt{\frac{\sigma_1}{\mu_1}}z_3 - \frac{c_1}{\mu_1}z_4 + \frac{c_1}{\sqrt{\mu\mu_1}}z_2 \end{cases} \tag{13}$$

To simplify the description of the transformation note, that in the energy space there are square roots of the potential or kinetic energies on each axis. Note also, that function $f$transforms energy into vector form.

The EDP function for such a system is constructed as follows:

$$\vec{z} \circ \dot{\vec{z}} = z_1\dot{z}_1 + z_2\dot{z}_2 + z_3\dot{z}_3 + z_4\dot{z}_4 \tag{14}$$

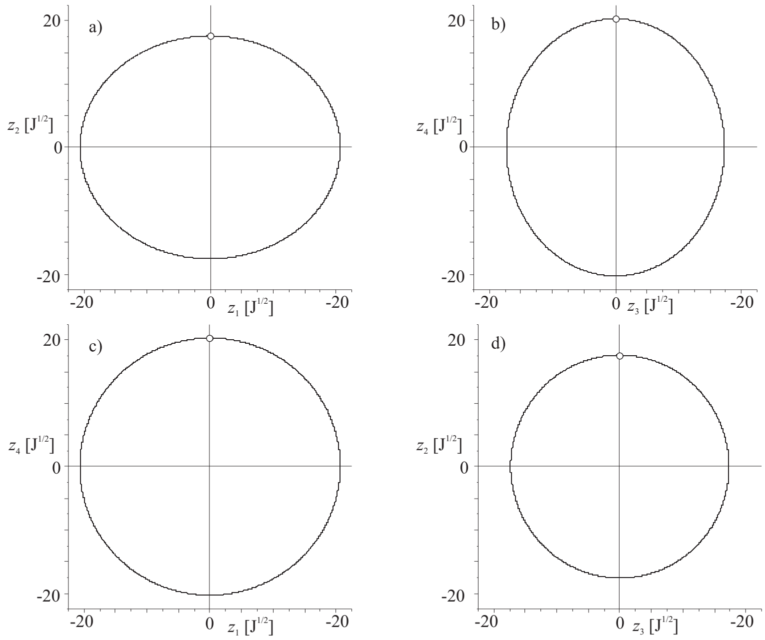

Figure 3 a) Energy plane of the oscillator μ system,b) Energy plane of the oscillator μ_1 system, c), d) Combined energy planes, $\eta = 0.875$, $\mu = 1$ kg, $\mu_1 = 0.07$ kg, $\sigma = 1$ N/m, $\sigma_1 = 0.07$ N/m, $c = 0.1$ Ns/m, $c_1 = 0.0001$, Ns/m, $F = 2.5$ N

4.1. *Energy flow and synchronization*

The special advantages brings consideration of the energy planes. These subspaces are the equivalents of the phase planes of the phase space. In our case there are four interesting planes.

The first one $(z_1 : z_2)$ – the energy plane of the oscillator μ system.

This plane is obtained after transformation of the plane $(x_1 : x_2)$. This transformation is linear. One can find eigeinvectors of the transformation as orthogonal basis vectors [1, 0] and [0, 1], of the space $(x_1 : x_2)$. Eigenvalues of the transformation are equal:

$\lambda_1 = \sqrt{\frac{\sigma}{2}} \quad \lambda_2 = \sqrt{\frac{\mu}{2}}$ respectively.

Thus depending on σ and μ the phase space is just only squeezed or stretched in directions of the eigenvectors.

In Fig. 3a plane $(z_1 : z_2)$ can be seen. The norm of the vector projection on that plane shows the energy accumulated in the oscillator μ system. The value of z_1 shows the potential energy of the spring σ, the value of z_2 – the kinetic energy of the mass μ, and by means of these coordinates these energies can be calculated. One can see the changes from the potential energy into the kinetic one and vice versa. Note that total amount of the energy accumulated in the oscillator μ system is not constant. There exists an energy flow between the oscillators μ and μ_1. This flow will be analysed further.

The second interesting plane is $(z_3 : z_4)$ – the energy plane of the oscillator μ_1

system.

There exist transformations of two kinds which were made on the phase plane $(x_3 \ : \ x_4)$ to obtain this energy plane.

The first one: instead of the state variable x_3 we have the spring σ_1 deflection: $z_1 = x_3 - x_1$.

The second one is similar to the considered transformation of the plane $(x_1 \ : \ x_2)$: transformation is linear and one can find eigeinvectors of the transformation as orthogonal basis vectors [1, 0] and [0, 1]. Eigenvalues of the transformation are equal:

$$\lambda_1 = \sqrt{\frac{\sigma_1}{2}} \quad \lambda_2 = \sqrt{\frac{\mu_1}{2}} \qquad \text{respectively.}$$

Thus in time of this part of transformation, depending on σ_1 and μ_1 the phase space is just only squeezed or stretched in directions of the given eigenvectors.

In Fig. 3b plane $(z_3 \ : \ z_4)$ can be seen. The projection of the vector on that energy plane shows the energy accumulated in the oscillator μ_1 system. The value of z_3 shows the potential energy of the spring σ_1 and the value of z_4 – the kinetic energy of the mass μ_1, and by means of these coordinates the energies can be calculated.

The position of the vector projections on the energy planes $(z_1 \ : \ z_2)$ and $(z_3 \ : \ z_4)$ at the same moment of time is marked by a small circle. See that at the same moment the spring σ and spring σ_1 have the extremes of the potential energy, and the same concerns kinetic energies. Note also that the maximum of the potential energy of the oscillator μ system equals the maximum of the kinetic energy of the oscillator μ_1 system and vice versa.

The energy flow between the main mass μ and the dynamical absorber μ_1 can be seen if you compare Fig. 3c and Fig. 3d, which show the combined energy planes. Note, that these trajectory projections are regular circles. It means, that energy accumulated in that parts of the system is constant. It shows some interesting aspects of the continuous energy flow between oscillator μ and μ_1. All the energy, that flows out the oscillator μ is intercepted by oscillator μ_1, and vice versa. The potential energy of the spring σ transforms into kinetic energy of the oscillator μ, and vice versa (Fig. 3c).It can be also seen in the Fig. 4e, where for the first resonance EDP value is equal to zero. The same situation takes place with the potential energy of the spring σ_1 and the kinetic energy of the oscillator μ (Fig. 3d, 4f). It can be seen in the Fig. 4a, b, e and f, that this energy flow synchronization phenomena takes place just for the excitation frequencies equal to the system resonance frequencies. Moreover from the Fig. 4b one can conclude that for these frequencies the energy of the whole system is constant. Energy flows only between the μ and μ_1 systems and the excitation energy is dissipated in the dampers c and c_1.

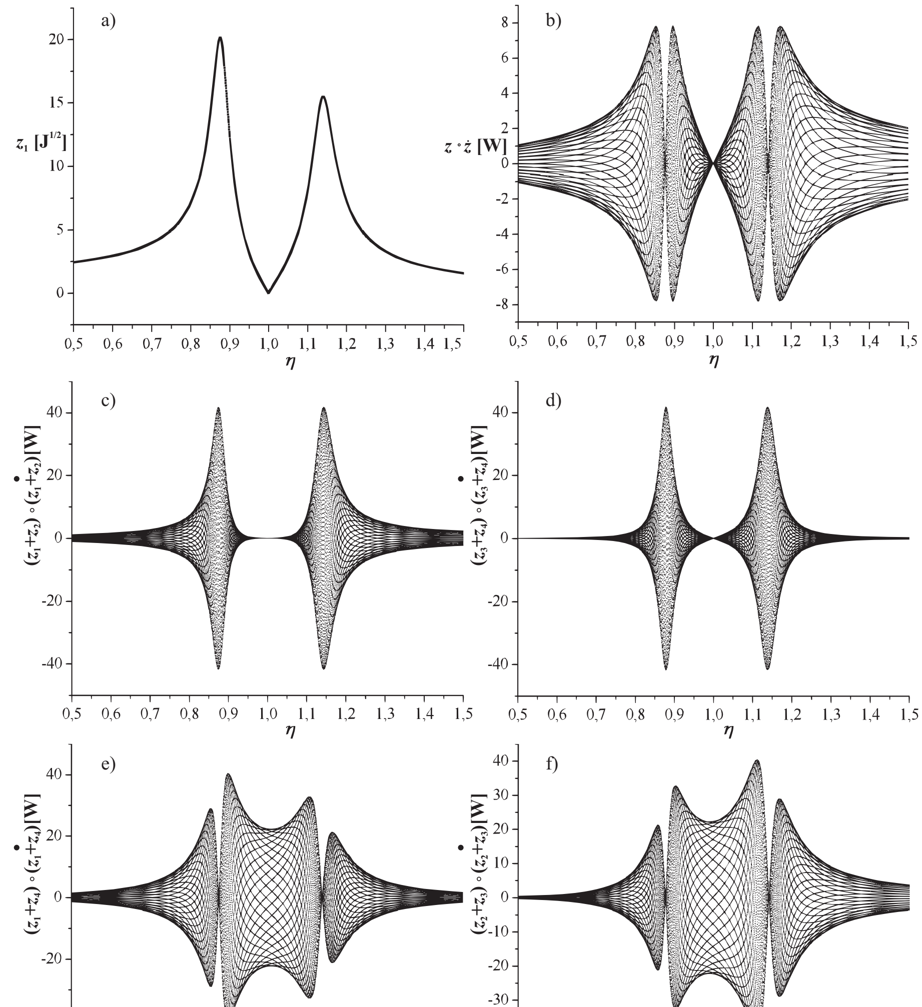

Figure 4 a) Resonance diagram, b, c, d, e, f) EDP diagrams, $\eta = 1$, $\mu = 1$ kg, $\mu_1 = 0.07$ kg, $\sigma = 1$ N/m, $\sigma_1 = 0.07$ N/m, $c = 0.1$Ns/m, $c_1 = 0.0001$ Ns/m, $F = 2.5$ N

5. Energy flow and synchronization in the chaotic system

Consider the two DOF system with nonlinear spring σ. The mathematical model of the system is given by four differential equations of the first order:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \left(F \sin \eta\tau - c x_2 - c_1 \left(x_2 - x_4\right) - \sigma x_1^3 - \sigma_1 \left(x_1 - x_3\right)\right) \dfrac{1}{\mu} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \left(c_1 \left(x_2 - x_4\right) + \sigma_1 \left(x_1 - x_3\right)\right) \dfrac{1}{\mu_1} \end{cases} \tag{15}$$

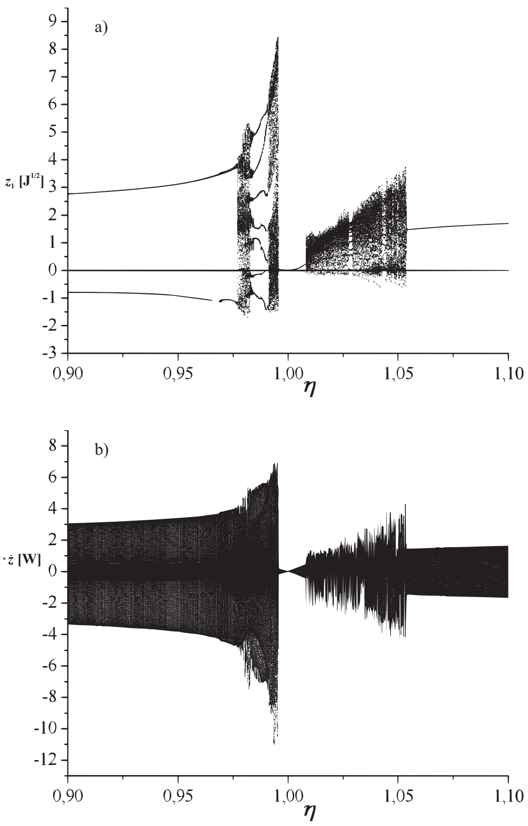

Figure 5 a) Resonance diagram, b) Dot product, $\mu = 1$ kg, $\mu_1 = 0.07$ kg, $\sigma = 1$ N/m, $\sigma_1 = 0.07$ N/m, $c = 0.1$Ns/m, $c_1 = 0.0001$ Ns/m, $F = 2.5$ N

The transformation of the phase space, to obtain the energy space, in direction of x is nonlinear. In the considered case we have to show the transformation of the space, as function $f : R^4 \rightarrow R^4$, that transforms the phase space the following way:

$$f(x_1, x_2, x_3, x_4) = \begin{cases} \sqrt{\frac{\sigma}{2}} x_1^2 sign(x_1) = z_1 \\ \sqrt{\frac{\mu}{2}} x_2 = z_2 \\ \sqrt{\frac{\sigma_1}{2}} (x_3 - x_1) = z_3 \\ \sqrt{\frac{\mu_1}{2}} x_4 = z_1 \end{cases} \tag{16}$$

To simplify the description of the transformation note, that in the energy space there are square roots of the potential or kinetic energies on each axis. Note also, that function f transforms energy into vector form. Similarly to the model given in paragraph **4.** one can obtain the mathematical model of the system after this transformation, but for the nonlinear system it is easier to integrate the mathematical model (15) and afterwards use the function f to calculate vector coordinates in the energy space. Both methods were applied to examine the system dynamics, and the same results were obtained. There exists just only one unsolved problem with the critical point for the transformed mathematical model, thus these results are not presented.

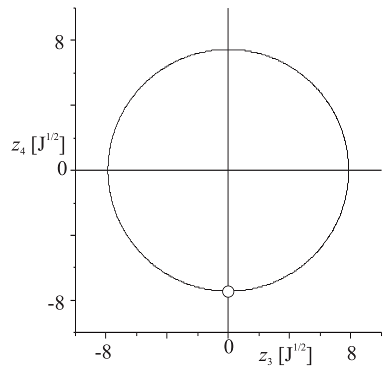

Figure 6 a) Energy plane of the oscillator μ system,b) Energy plane of the oscillator μ_1 system, $\eta = 1$, $\mu = 1$ kg, $\mu_1 = 0.07$ kg, $\sigma = 1$ N/m, $\sigma_1 = 0.07$ N/m, $c = 0.1$Ns/m, $c_1 = 0.0001$ Ns/m, $F = 2.5$ N

Dependence of the system dynamics on the excitation force frequency is shown in the Fig. 5. The resonance and EDP diagrams can be seen in it. One can see (Fig. 5a), that for the $\eta = 1$ oscillations amplitude of the vibrations of the oscillator μ is close to zero. The value of the EDP for $\eta = 1$ is also close to zero, while the energy of the oscillator μ_1 is constant, and not close to zero (Fig. 6). It means, that although this part of the system is forced by external force whole energy accumulated in the oscillator μ system is close to zero. It is the case, when the oscillator μ_1 works as the dynamical damper. For the excitation frequency equal to the system μ_1 free vibrations frequency. All energy of the excitation just only flows through the μ system and is intercepted by μ_1 system. What is interesting, the energy of the μ_1 system is constant, what can be seen in Fig. 6. Similarly to the one DOF system, accumulated energy only changes its form from potential into kinetic energy, and vice versa and the excitation energy is dissipated in the damper and c_1. The energy of the whole system is almost constant, what can be concluded from Fig. 5b.

6. Conclusions

In the article construction and application of the EDP is introduced. An application of the EDP in detecting synchronization, was shown. Some aspects of motion, energy flow, synchronization are considered. Transformation of the traditional phase space into the energy space was shown. It was shown, that after this transformation the state vector and its derivative determine the energy flow plane. The angle between these vectors shows energy flow direction. Application of such a construction in recognizing the energy flow synchronization was shown. It has been proved that this new kind of space allows for concluding about the energy state of a vibrating system. The projection of the vector space on energy subspaces showing the amount of the energy that is accumulated in some parts of the system were considered. It has been shown that using this kind of spaces, all aspects of the kind of motion can be concluded about, like from the phase space and, moreover, the energy state, accumulation, flow and dissipation can be observed. Different types of the energy flow synchronization were shown.

References

[1] **Kapitaniak, T.**: Chaos for engineers, Springer-Verlag Berlin Heidelberg **1998**.

[2] **Dąbrowski, A.**: The construction of the energy space, *Chaos, Solitons and Fractals*, 26, 1277-1292, **2005**.

[3] **Dąbrowski, A.**: The energy space, energy flow and synchronization, *Mechanics and Mechanical Engineering*, 8, (1), 200-218, **2005**.

[4] **T. Kapitaniak**: Controlling Chaos, *Academic Press*, London, **1996**.

[5] **Chen, G. and Dong, X.**: From Chaos to Order, *World scientific*, Singapore, **1998**.

[6] **Maidanik, G. and Becker, K.J.**: Dependence of the induced loss factor on the coupling forms and coupling strengths: energy analysis, *Journal of Sound and Vibration*, 266, 33-48, **2003**.

[7] **Xu, H.D., Lee, H.P. and Lu, C.**: Numerical study on energy transmission for rotating hard disk systems by structural intensity technique, *International Journal of Technical Sciences*, 46, 639-652, **2004**.

[8] **Pennestrì E.**: An application of Chebyshev's min-max criterion to the optimal design of a damped dynamic vibration absorber, *Journal of Sound and Vibration*, 217, 757-765, **1998**.

[9] **Dąbrowski, A. and Kapitaniak, T.**: Using chaos to reduce oscillations, Nonlinear Phenomena in Complex Systems, 4, (2), p. 206-211, **2001**.

[10] **Dąbrowski, A.**: New design of the impact damper, *Mechanics and Mechanical Engineering*, 4, (2), (2000), p. 191-196.

[11] **Kishimoto, Y., Bernstein, D.S. and Hall, S.R.**: Energy flow modeling of interconnected structures: a deterministic foundation for statistical energy analysis, *Journal of Sound and Vibration*, 183, (3), 407-445, **1995**.

[12] **B**.R.Mace: Power flow between two continuous one-dimensional subsystems: a wave solution. Journal of Sound and Vibration 154 (1992), 289-320,

[13] **D.Sado**: Przenoszenie energii w nieliniowo sprzężonych ukadach o dwóch stopniach swobody, Politechnika Warszawska, Warszawa, **1992**.

[14] **Keane, A.J. and Price, W.G.**: A note on the power flowing between two conservatively coupled multi-modal subsystems, *Journal of Sound and Vibration*, 144,, 185-186, **1991**.

[15] **Mace, B.R.**: The statistical energy analysis of two continuous one-dimensional subsystems, *Journal of Sound and Vibration*, 166, 429-461, **1994**.

[16] **Stefański, A.**: Praca Habilitacyjna: Estymacja maksymalnego wykładnika Lapunowa układów dynamicznych w oparciu o zjawisko synchronizacji, Politechnika Łódzka, *Zeszyty Naukowe*, Nr 941, **2004**.

[17] **Lachaux, J.P., Chavez, M. and Lutz, A.**: A simple measure of correlation across time, frequency and space between continuous brain signals, *J. Neuroscience Methods*, 123, 175-188, **2003**.

[18] **Fries, P., Reynolds, J., Rorie, A. and Dismone, R.**: Modulation of oscillatory neuronal synchronization by selective visual attention, *Science*, 291, (5508), 1560-3, **2001**.

[19] **Rodrigues, E., George, N., Lachaux, J.P., Martinerie, J., Renault, B. and Varela, F.**:Perception's shadow: long distance synchronization of human brain activity, *Nature*, 397, 430-3, **1999**.

[20] **Lachaux, J.P., Rodrigues, E., Le Van Quyen, M., Lutz, A., Martinerie, J. and Varela, F.**: Studying single-trials of phase synchronous activity in the brain, *Int. J. Bifurcat. Chaos*, 10(9), 2429-39, **2000**.

[21] **Shu, Y., Zhang, A. and Tang, B.**: Switching among three different kinds of synchronization for delay chaotic systems, *Chaos, Solitons and Fractals*, 23(2), 563-571, **2005**.

[22] **Lindner, B. and Schimansky-Geier, L.**: Coherence and stochastic resonance in a two-state system, *Phys. Rev.*, E 61, 6103–6110, **2000**.

[23] **Pecora L.M., Carroll T.L., Johnson G.A., Mar D.J. and Heagy J.F.**: Fundamentals of synchronization in chaotic systems, concepts and applications, *Chaos*, 7, 520-543, **1997**.

[24] **Blekhman II**, Synchronization in science and technology, *ASME Press*, New York, **1988**.

[25] **Yamaguchi, A., Watanabe, H., Mikami, S. and Wada, M.**: Characterization of biological internal dynamics by the synchronization of coupled chaotic system, *Robotic and Astronomous Systems*, 28, 195-206, **1999**.

[26] **Liley, D.T.J., Alexander, D.M.,Wright, J.J. and Aldous, M.D.**: Alpha rythm emerges from large-scale networks of realistically coupled multicomparamental model cortical neurons, *Comput. Neural Syst.*, 10, 79-92, **1999**.

[27] **Reynolds, D.V. and Ashwin, C.**: A large scale neuroanatomical model of attention implemented as a computer simulation, *Neurocomuting*, 32-33, 967-971, **2000**.

[28] **Bankovskiv, N.G., Korotkov, K.G. and Petrov, N.N.**: Physical image-forming processes in gas-discharge visualisation – the Kirlian effect (review), *Soviet Journal of Communications and Technology*, 31, 29-45, **1986**.

[29] **Korotkov, K.G.**: Human Energy Field: study with GDV Bioelectrography, *Backbone Publishing Company*, USA, **2002**.

[30] **Gourdon, E. and Lamarque, C.H.**: Energy pumping for a larger span of energy, *Journal of Sound and Vibration*, 285, (3), 711-720, **2005**.

[31] **Tsakirtzis, S., Kerschen, G.,,Panagopoulos, P.N., and Vakakis, A.F.**: Multi-frequency nonlinear energy transfer from linear oscillators to mdof essentially nonlinear attachments, *Journal of Sound and Vibration*, 285, (1-2), 483-490, **2005**.

[32] **Dąbrowski, A.and Kapitaniak, T.**: Using chaos to reduce oscillations: experimental results, accepted for publication in *Chaos, Solitons and Fractals*, **2007**.

[33] **Dąbrowski, A.**: Energy-Vector Method in mechanical oscillations, accepted for publication in *Chaos, Solitons and Fractals*, **2007**.

[34] **Dąbrowski, A.**: Application of the Energy space in Chaotic Systems Research, *Mechanics and Mechanical Engineering* 11, (1), 21-36, **2007**.