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Alternative Solutions of the Problem of Load–capacity of Thin–walled Plated Structures

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The paper presents results of the comparative study of structural behaviour of collapse of thin–walled plated structures performed using two different approaches: energy method and equilibrium strip method. Both methods ate based on the rigid–plastic theory of thin–walled structures and apply the concept of the yield line mechanism. Solutions based on both methods lead the upper–bound estimation of the load capacity. Two structural members, namely thin plate and channel section column subjected to compression are under investigation. Plastic mechanisms of failure in both members are discussed and detailed solutions based on two approaches mentioned above are derived. Exemplary numerical calculations are presented in diagrams of loading paths (load–deformation curves). Some concluding remarks are formulated.

Keywords: Thin–walled structures, load–carrying capacity, plastic mechanisms

1. Introduction

Design of a modern thin–walled structure (lightweight and safe) is a complex process, where is necessary to recognize a structural behaviour in the whole range of loading, up to and beyond an ultimate load, which is represented by the load equilibrium path (load–versus deformation diagram). It should be underlined here, that a thin–walled structure load path is significantly different from the analogous load–deformation diagram of a structure built from members of compact cross–section (e.g. truss members). A thin–walled structure is usually characterized by a very high characteristic (predominant) dimension to wall thickness ratio. Thus, its load–carrying capacity is determined by buckling and post–buckling behaviour.

In Fig.1. exemplary load equilibrium paths (load versus deflection) of thin–walled beam or column subject to bending or eccentric compression are presented.

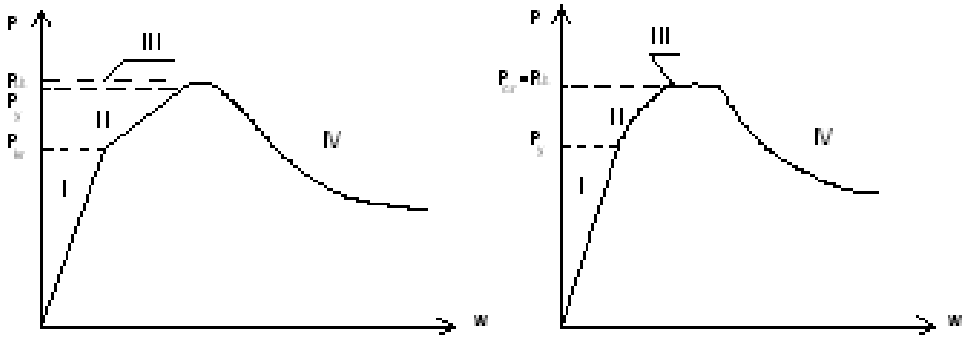

Figure 1 Load paths of thin–walled beam–column, a) – elastic buckling of the weakest plate element, b) – elasto–plastic buckling of the weakest plate element

Four separate phases are observed in the behaviour of a thin–walled structural member in the complete range of loading. The first phase (I) is the pre–buckling one. The second phase (II) is the non–linear, post–buckling behaviour. After exceeding the buckling load, a local buckling and also an interaction of different buckling modes occurs. The second phase can be regarded as an elastic, non–linear post–buckling behaviour. The third phase (III) is initiated by the first yield in one of the plate elements and is the elasto–plastic one, in which both geometrical and physical relations are non-linear. At the highest point of this phase, the load is at its ultimate value and, at the same instant, the last, post–failure phase begins. The propagation of yield areas proceeds and, in consequence, the plastic mechanism of failure is created.

Also four phases are observed in the structural behaviour of a profile for which the plastic buckling takes place: (I) – elastic pre–buckling, (II) – elasto–plastic pre–buckling, (III) – characteristic plateau of plastic buckling, and finally (IV) – failure phase.

Thus, the investigation of structural behaviour under such circumstances is very complex, particularly in the post-buckling elastic and elasto–plastic range. The analysis of this problem involves great mathematical difficulties, which consist in solving systems of non–linear equations taking into account many factors: both geometrical and physical non-linearities, different global and local buckling modes as well as coupled buckling. One can omit some of those problems carrying out buckling and elastic post–buckling analysis only and combining afterwards the obtained pre– and post–buckling path with the post–failure path derived from the collapse analysis conducted using the plastic mechanism approach and performing the yield line mechanism analysis [1,2,12] based on the rigid–plastic theory. This approach leads to the upper–bound estimation of the load–carrying capacity and is widely used to study plated structures that involve plastic collapse mechanism.

The investigation of the structural behaviour at collapse is also necessary since a designer should know in what way the structure fails: either it happens rapidly without earlier signs of catastrophe (brittle structure) or it proceeds slowly with warnings against collapse (ductile structure). Since thin–walled profiles can work as energy absorbers, an amount of the energy dissipated at collapse is very important in that case. Those three factors mentioned above induce the designer's interest in the post–failure behaviour of a structure.

Generallly, a determination of the load–carrying capacity of a thin–walled structure can be achieved by means of the limit analysis based on the lower and upper bound theorems [13,14]. The application of these theorems leads to the lower or upper bound estmation of the load–carrying capacity.

The methods allowing for the lower–bound determination of the load–capacity can be classified into three categories:

To the first category belong analytical–numerical methods consisting in the determination of the buckling load and post–buckling path by means of the solution of the boundary problem, among others using variational methods based on the Rayleigth–Ritz variational principle [16,17] or based on Koiter's asymptotic theory of conservative systems [17]; afterwards, approximated estimation of load carrying capacity can be obtained on the basis of an appropriate threshold (usually – first yield) criterion. The second group form numerical methods (finite element and finite strip methods) [16]. All these methods utilize basically the first yield criterion. This approach is currently used in almost all design codes. However, structural members often display a significant post–elastic capacity. It means that the actual load–carrying capacity is higher than an ultimate load calculated using the method mentioned above. It concerns both static and dynamic loads. Thus, a certain "redundancy" of the load–carrying capacity has to be determined in order to be able to design thin–walled members in more economic way. It can be realized using the upper bound estimation of the structure's load–carrying capacity.

The upper bound estimation of the load–capacity corresponds to the ordinate of the intersection point of the extrapolated elastic post–buckling path and the unloading path (Fig. 2), which is termed *a failure curve.* Thus, evaluation of this curve is of crucial importance in that case and can be realized using the plastic mechanism analysis (yield line analysis).

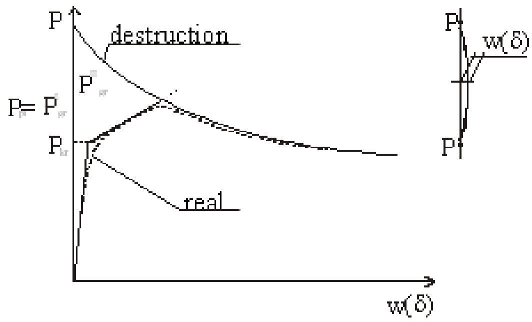

Figure 2 Thin–walled column subject to eccentric compression. Lower and upper bound estimation of the load–capacity

It should be underlined here that in spite of an enormous development in numerical methods (FEM), analytical solutions are still useful, particularly in the initial phase of design process since the results can be obtained in a short time so that several versions of a structure can be analysed. In the context of the simplicity of calculations to be conducted, compilation of post–buckling analysis with the plastic mechanism approach leads to a relatively simple and quick solution of the load–carrying capacity problem of thin–walled plated structures.

The plastic mechanism approach is based on two basic methods [2,3,10], namely the *energy method (work method)* and the *equilibrium strip method*. The aim of the present work is a comparative analysis of results obtained using those two methods, i.e. the energy method versus equilibrium strip method, leading to the solution of the plastic mechanism problem and subsequently – to the upper bound estimation of the load–capacity of the plated structure. Such a comparative study has been already carried out by Flockhart et al. [8], who analysed a thin–walled spot welded box–section beam. He has come to conclusion that for large rotations of the mechanism (global plastic hinge), the energy absorption determined by the energy method is higher up to 30% than that determined by the equilibrium strip method. No further investigation into a comparison of those two approaches has been made so far.

In this paper the results obtained from these two analytical methods are validated by FE numerical calculations. As subjects of the analysis a separate thin plate under uniform compression and a typical thin–walled plated structure, i.e. a channel–section column under compression have been chosen. The analysis is restricted to the isotropic material members.

2. Plastic mechanisms in thin–walled plate structures

A starting point of the plastic mechanism analysis, by means of both methods mentioned above, is determination of the geometrical model of the mechanism. There are many factors, which determine a geometry of the plastic mechanism to be formed in the phase of failure, among them a shape of the member's cross–section, buckling mode, boundary and load conditions and , to a great extent – preliminary geometrical imperfections.

Plates prone to uniform compression and under symmetrical boundary conditions can develop two basic types of plastic mechanisms, namely the flip disc mechanism [3,10] and the mechanism termed in literature as "pitched roof" (Fig. 3). The latter was described by many researchers, among others Kato [4], Korol & Sherbourne [5], as well as Mahendran [6] and Sin [7]. Modifications of this mechanism have been elaborated by Rondal & Maquoi [14] and Kragerup [15]. These mechanisms were discussed in details by Ungureanu et al. [3,11,18]. A modification of the pitched–roof mechanism is a mechanism "pyramid" type shown in Fig. 4. Theoretical models of 3D plastic mechanisms in channel–section columns were originally elaborated by Murray and Khoo [9] – Fig. 5. They are discussed in details in [10], where the Reader finds also references. Among them a true mechanism (three–hinge flange mechanism), which develops due to axial compression and/or bending (flanges deflect laterally towards the free edge) has been developed. This mechanism denoted as CF1 is shown in Fig. 5. In this work the attention is focused on the latter. Two solutions based on the energy (work) method and equilibrium strip method concerning this mechanism are discussed in the present paper.

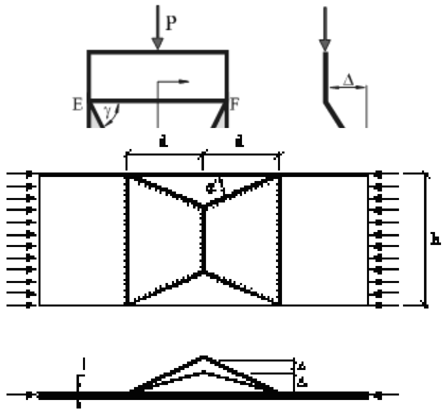

Figure 3 Pitched–roof mechanism

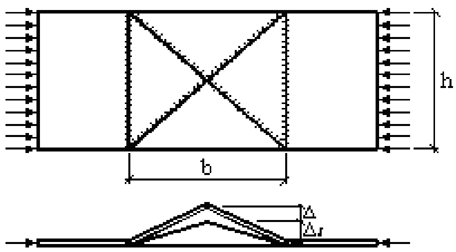

Figure 4 Mechanism type "pyramid" [18]

CF1

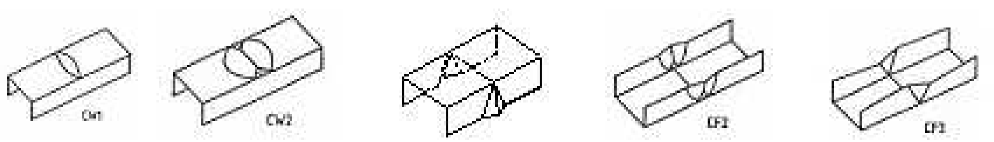

Figure 5 Plastic mechanisms of failure in channel–section column

3. Energy method

Using the energy method we apply the Principle of Virtual Velocities of the general following form:

$$P\dot{\delta} = \int_V \sigma_{ij}\dot{\varepsilon}_{ij}^{p}(\beta,\chi)dV \tag{1}$$

where δ is the global generalized displacement, $\dot{\delta}$ is the rate of change of the global generalized displacement, β is the vector of kinematical parameters of the plastic mechanisms (kinematically admissible displacements), χ is the vector of geometrical parameters of the plastic mechanisms,$\dot{\varepsilon}_{ij}^{p}$ is the strain rate tensor. As a result, a load – deformation relation is obtained, the graphical representation of which is a *failure curve.* As mentioned above, the intersection of the elastic curve and the failure curve can be used to estimate the ultimate strength of a plated member.

The energy method is widely used in analyzing collapse behaviour of thin–walled structures, particularly crushing and progressive buckling of thin–walled columns and beams. It is capable to estimate an energy dissipation directly, which is especially important in the analysis of energy absorbers. Furthermore, it enables to evaluate an energy absorbed not only at so called stationary yield lines, but at traveling and rolling yield lines (plastic corners) as well [1, 2, 10].

3.1. *Thin plate subject to uniform compression pitched roof mechanism*

The analysis of pitched roof mechanism was performed by many researchers mentioned above. Among them only Sin [7] and Koteko et al. [16] (for multi–layered plated structures) applied the energy method. The examined mechanism is in fact a quasi–mechanism [1, 10], since in triangles ACE and BDF (Fig. 3) plastic membrane strain zones are formed. Thus, the energy of plastic deformation consists of the energy of bending plastic deformation and the energy of membrane plastic deformation.

In the case of the plate subject to compression, from the principle of virtual velocities we obtain the following variational relation:

$$\delta W_{ext} = \delta W_b + \delta W_m \tag{2}$$

where δW_{ext} is the variation of work of external forces, δW_b is the variation of the energy of bending plastic deformation, δW_m – variation of the energy of membrane plastic deformation.

The variation of the energy of bending plastic deformation dissipated at yield–lines amounts

$$\delta W_b = \sum_k l_k m_p \delta\beta_k \tag{3}$$

where l_k is a length of the yield–line and β_k is an angle of relative rotation of two walls of the global plastic hinge along that line, m_p is a fully plastic moment in the wall cross–section.

In the pitched roof mechanism the energy of bending deformation takes form (Fig. 3) :

$$W_b = W_{AB+CD+\ldots} + W_{AE+\ldots} = \sum_{i=1}^{3} l_i \beta_i \, m_p + \sum_{j=1}^{4} l_j \beta_j \, m_p \tag{4}$$

where:

$$\begin{aligned}
l_2 &= l_3 = b \\
l_1 &= b - 2s \\
\beta_2 &= \beta_3 = \theta \\
\beta_1 &= 2\theta \\
l_j &= a/sin\gamma \\
\beta_j &= \arctan\left[\tan\gamma \cos\gamma\right]
\end{aligned}$$

Variation of plastic strain energy dissipated at plastic zones of membrane stresses amounts:

$$\delta W_m \; = \; (N_x \delta \varepsilon_x^p + N_y \delta \varepsilon_y^p) A_p \tag{5}$$

where: N_x, N_y are membrane forces per unit length, A_p is an area of membrane stresses plastic zones. Membrane forces N_x, N_y can be determined using the associated flow rule for Huber–Mises yield criterion.

Variation of virtual work of external forces amounts:

$$\delta W_{ext} = 2Nab \sin\theta \delta\theta = 2Nb\Delta\delta\theta \tag{6}$$

where N is a force per unit length ($N = P/t$).

Taking into account (3) to (6) in (2), a relation of compressive external force N or P in terms of deflection Δ or rotation angle θ (represented graphically as a *failure curve*) is evaluated.

$$\begin{aligned}
N \;\; &= \;\; 2m_p \frac{b-s}{b\Delta} + \frac{2a}{b \cdot \Delta \sin\gamma} m_p \frac{\cos\gamma}{1 - \sin^2\theta \sin^2\gamma} \\
&\quad + \frac{1}{2}(a/b)\frac{s}{\Delta}\left(N_x \alpha_x + N_y \alpha_y\right)
\end{aligned} \tag{7}$$

where:

$$
\begin{aligned}
N_x &= \frac{N_0}{\sqrt{1+\left(\frac{2\alpha_y+\alpha_x}{2\alpha_x+\alpha_y}\right)^2 - \frac{2\alpha_y+\alpha_x}{2\alpha_x+\alpha_y}}} \\
N_y &= \frac{N_0\left(2\alpha_y+\alpha_x\right)}{\sqrt{1+\left(\frac{2\alpha_y+\alpha_x}{2\alpha_x+\alpha_y}\right)^2 - \frac{2\alpha_y+\alpha_x}{2\alpha_x+\alpha_y}}\left(2\alpha_x+\alpha_y\right)} \\
\alpha_x = \alpha_x(\theta) &= \frac{a\sin\theta\cos\theta}{s\sqrt{\left(\frac{a\sin\theta}{s}\right)^2+1}} \\
\alpha_y = \alpha_y(\theta) &= \sin\theta \\
N_0 &= \sigma_{pl}\cdot t \\
m_p &= \frac{\sigma_{pl}t^2}{4} \\
t &- \text{plate thickness} \\
\sigma_{pl} &- \text{material yield stress}
\end{aligned}
$$

3.2. *Channel–section column subject to uniform compression – CF1 mechanism*

A detailed geometry of the CF1 mechanism is shown in Fig. 6. It is termed as three–hinge flange mechanism, since one can distinguish three different yield lines, namely [10,17]:

- line 1 of lengh $l_1 = b$,
- line 2 of lengh $l_2 = b_1$,
- line 3 of length $l_3 = b_1/\cos\beta$.

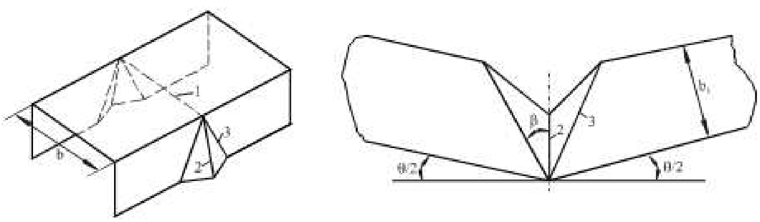

Figure 6 Mechanism CF1

In this case the total energy of plastic deformation takes form:

$$W = m_p \sum_{i=1}^{3} l_i'\beta_i \tag{8}$$

where:

$$\begin{aligned} l'_2 &= 2l_2 \\ l'_3 &= 4l_3 \\ \beta_1 &= \theta \\ \beta_2 &= 2\arccos\left[\cos\frac{\theta}{2} - \frac{\sin\theta/2}{\tan\beta}\right] \\ \sin\beta_3 &= \frac{\sin[\arccos\frac{1}{\tan\beta\sin(\theta/2)+\cos(\theta/2)}]}{\sin\beta} \end{aligned}$$

After numerical derivation of the energy (8) with respect to the parameter θ one obtains the current bending moment at the global plastic hinge in terms of θ and – after simple recalculation – the compressive load versus the shortening of the column. This method has been successfully used for the upper bound estimation of column load capacity by Kołakowski and Kotełko [17].

4. Equilibrium strip method

The equilibrium strip method treats the plastic mechanism as a compatible collection of strips of infinitesimal or unit width parallel to the direction of applied force. On the basis of free–body–diagram of a separated strip an equilibrium equation is formulated and then, those equations are integrated across walls of the plastic mechanism, in order to obtain simultaneous equilibrium equation for the mechanism as a whole [2]. An application of this method is restricted to the analysis of local plastic mechanisms build of stationary yield lines only. It is widely used in investigations of plated columns under compression and delivers a direct relation between an applied compressive force and the deflection of the column.

Tab. 1 shows the geometry of plastic mechanisms in thin plates (mentioned above) together with the relations for current load in terms of deflection at the stage of failure. The relations, elaborated (among others) by Kato [4] and Mahendran [6] and quoted by Ungureanu et al. [3, 18] are based on the equilibrium strip method and take into account initial imperfections.

Table 1 Load–deflection relations for thin plates – equilibrium strip method [11,18]

No.	Mechanism	Load – deflection relation
1	Mechanism "pyramid" type Fig. 4	$N/N_y = \sigma/\sigma_{pl} = \frac{1}{2}\left\{1 - 2\frac{\Delta+\Delta_i}{t} + \sqrt{1+4\left(\frac{\Delta+\Delta_i}{t}\right)^2}\right\}$
2	Mechanism "pitched roof" type Fig. 3	$N/N_y = \sigma/\sigma_{pl} = \left\{\begin{array}{l} 1 - 0.355\frac{\Delta+\Delta_i}{t} + 0.056\left(\frac{\Delta+\Delta_i}{t}\right)^2 \\ -0.003\left(\frac{\Delta+\Delta_i}{t}\right)^3 \end{array}\right\}$ $\mathrm{N}_y = \sigma_{pl}\cdot t;\quad N = \sigma\cdot t$

For the channel–section column, in which CF1 plastic mechanism is formed, an algorithm determining a current compressive force P in terms of the deflection δ

was elaborated Murray and Khoo [9], applying the equilibrium strip method. Their solution is also quoted by Ungureanu and Dubina [12]. The total compressive force consists of :

$$P = P_w + 2P_f$$

where P_w and P_f are forces acting in the web and flange, respectively. Force P_f takes form:

$$P_f = \frac{\sigma_{pl} \cdot tb}{2}\left[\sqrt{\frac{2\Delta}{k_1 t}+1} - \frac{2\Delta}{k_1 t} + \frac{k_1 t}{2\Delta}\ln\left(\sqrt{\frac{2\Delta}{k_1 t}+1} - \frac{2\Delta}{k_1 t}\right)\right] \tag{9}$$

where Δ is the local deflection of the flange [18] and $k_1 = 1 + \sec^2\beta$. Product of P_f and its distance from the neutral axis of the whole section is as follows:

$$P_f e_f = \frac{\sigma_{pl} \cdot t^3 b^2 k_1}{12\Delta^2}\left[\left(\frac{2\Delta}{k_1 t}+1\right)^{\frac{3}{2}} - 1 - \left(\frac{2\Delta}{k_1 t}\right)^3\right] \tag{10}$$

The total compressive force P in terms of deflection δ is expressed by the following formula:

$$P = \frac{1}{2\bar{A}^2\bar{C}}\left[\delta + e - \bar{A}\bar{D} - \bar{B} - G\right] \tag{11}$$

where:

$$\begin{aligned}
G &= \sqrt{\left(\delta + e - \bar{A}\bar{D} - \bar{B}\right)^2 - 4\bar{A}^2\bar{C}\cdot\left(\bar{C}\bar{B}^2 + \bar{B}\bar{D} + \bar{F}\right)} \\
\bar{A} &= -\frac{1}{\sigma_{pl} t} \\
\bar{B} &= \frac{t}{2} + \frac{P_f}{\sigma_{pl} b_1} \\
\bar{C} &= -\sigma_{pl} b_1 \\
\bar{D} &= 2\sigma_{pl} b_1 \cdot t \\
\bar{F} &= 2P_f e_f - \frac{\sigma_{pl} b_1 t^2}{2} \\
\delta &= \frac{\Delta^2 \cdot L}{2b^2 \cdot \tan\beta}
\end{aligned}$$

5. Exemplary numerical results

The case study was performed for two subjects: steel thin plate under uniform compression and steel channel section column under axial compression.

The analytical results were verified in numerical analysis performed with FEM application and ANSYS version 10 package. In FEM model shell element (SHELL181) was applied. It was four nodes element with six degrees of freedom in each nodes (translations in x, y and z directions of local coordinate system and rotations about these axis respectively). The element formulation is based on logarithmic strain and true stress measures and it is well suited for nonlinear large strain applications. The element allows to apply different material descriptions. In the analysis the bilinear characteristic was applied but the strain– hardening was neglected, so that elastic – perfectly plastic material model was used. The calculations were performed in two steps. First it was eigen–buckling analysis to obtain buckling mode of the member. The first buckling mode was updated on to model geometry as the initial imperfection of its walls. In the second step the principal nonlinear analysis was performed. The applied load was increased in constant sub–steps and the deflections were determined for each step.

For the plate under uniform compression the analytical solutions based on formulae given in Tab. 1 for mechanisms type "pyramid" and "pitched roof" have been compared with the solution based on the energy method for the "pitched roof" mechanism – formula (7). Exemplary results together with FE loading path are shown in Fig. 7. The FE results concern the plate simply supported at all edges. Failure curves obtained by the energy method correspond to three different values of the mechanism parameter s (Fig. 3). The failure curve for the pitched–roof mechanism obtained by the equilibrium strip method was evaluated for the mechanism parameter $\alpha = 30^o$ (pos. 2 Tab. 1). The failure curve for the "pyramid" type mechanism was also obtained using the equilibrium strip method. (Tab. 1).

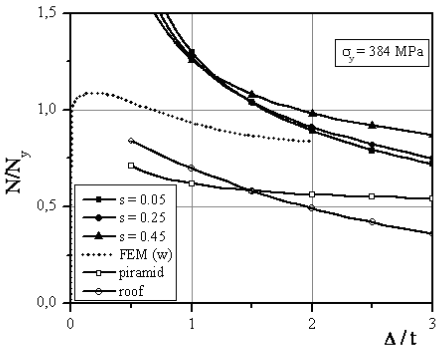

Figure 7 Loading path and failure curves of rectangular steel plate of dimension 1600 × 800 × 50 [mm]; $\Delta_i = 0$

Fig. 7. shows the loading path up to the relative deflection of magnitude 3. Fig. 8. represents the comparison of analytical solutions only, for the same plate, but for large relative deflections, beyond 3. A continuous line represents the energy method solution for the pitched roof mechanism ($s = 0.25$), two dotted lines – the equilibrium strip method solution for "pyramid" type and pitched roof mechanism, respectively. Fig. 9. represents the exemplification of the upper bund approach consisting in the compilation of the FE extrapolated post–buckling path and the failure curve obtained for the pitched roof mechanism using the energy method (re. 7). It should be underlined here that the post–buckling path can be obtained also by an anlitycal solution [17].But even if the FE simulation is applied, it may be achieved in relatively short time, up to the first yield, without very time–consuming calculations in the non–linear stage.

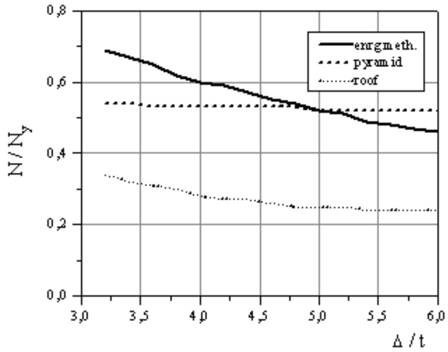

Figure 8 Failure curves of rectangular steel plate of dimension 1600 × 800 × 50 [mm]

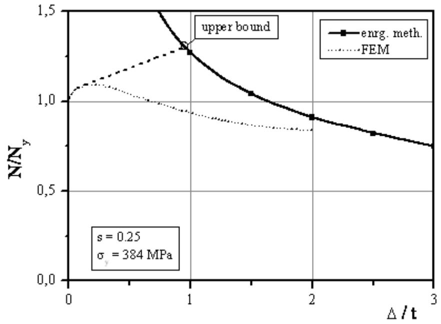

Figure 9 Loading path of rectangular steel plate of dimension 1600 × 800 × 50 [mm], (pitched roof mechanism for energy method – $s = 0.25$)

The qualitative agreement of the FE unloading path with the analytical failure curve is relatively good. The discrepancy between the upper bound ultimate load and FE maximum load amounts about 18%. Fig. 10 shows comparative diagrams of the analysis performed for steel thin-walled column, in which the plastic mechanism CF1 is formed. Analytical and FE results are compared with results of experiment carried out by Murray and Khoo [9]. The analytical results are obtained using the energy method described in paragraph 3.2 (curve 1) and using relation (11) based on the equilibrium strip method (curve 2).

6. Concluding remarks

In the case of a thin plate under uniform compression the analytical equilibrium strip underestimates strongly the load–capacity in comparison with the energy analytical

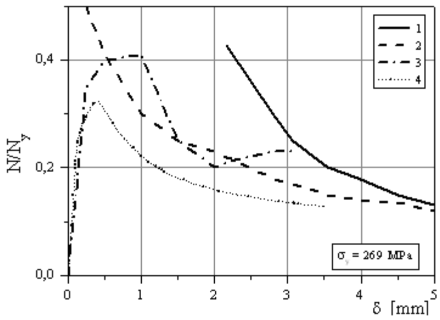

Figure 10 Load-deflection diagrams for steel channel section column: 1 – energy method, 2 – equilibrium strip method, 3 – experimental results [9], 4 – FE results, height of the web = 102 mm, width of the flange = 38.33 mm, wall thickness = 1 mm

solution and also with FE results. A relatively good agreement of two analytical methods applied is obtained for large deflection only.

In the case of channel–section column under compression, comparison of diagrams shown in Fig. 10 indicates that both analytical methods give convergent results also for large deflections only, when the plastic mechanism of failure is entirely developed. Unfortunately, experimental results obtained by Murray and Khoo [9] are limited to relatively small deflections. FE results underestimate the ultimate load, although the unloading FE path is of the same character as analytical failure curves and experimental curve. It should be admitted that ideal pin joint ends were probably not realized in experiment, while they were assumed in FE calculations.

Generally, the energy method delivers an estimation of the load–capacity higher than the equilibrium strip method, except a load–capacity in the final stage of failure (for large deflection). It coincides with Flockhart et al. research results [8] but indicates much more substantial discrepancies between two methods under investigation. Thus, further research into a realistic application of both analytical methods (particularly equilibrium strip method) for different plated structures should be continued.

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