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Nonaxisymmetric Instability of a Streaming Annular Fluid Jet Surrounding a Tar Cylinder under General Tenuous Varying Magnetic Fields

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The magnetodynamic stability of streaming annular jet surrounding a tar mantle under inertia and electromagnetic forces with varying magnetic fields, is developed. A general eigenvalue relation is derived and discussed. The axial interior and exterior fields have strong stabilizing influences for symmetric $m = 0$ and asymmetric $m \neq 0$ modes. The azimuthal varying field is purely destabilizing for $m = 0$ but in the $m \neq 0$ it is stabilizing or destabilizing according to restrictions. The streaming has a strong destabilizing influence in all modes for all wavelengths. Its influence increases the MFD unstable domains and decreases those of stability. As the tenuous azimuthal magnetic field influence is superior to those of axial fields, the MFD unstable domains are increasing with increasing q (the tar cylinder radius normalized with respect to that of the fluid) values and vice versa. If the unperturbed fluid velocity is smaller than the Alfven wave velocity, the model destabilizing character is suppressed and stability arises.

Keywords: electromagnetic forces, axial fields, hydrodynamic, hydromagnetic

1. Introduction

The instability of a full fluid jet in the hydrodynamic and hydromagnetic versions has been documented and mastered through numerous investigations [1] to [3]. That is due to its practical applications in several domains such as fuel atomization, spinning of synthetic fibers, the spray drying, inkjet printer . . . etc.

The stability of an annular fluid jet is also interesting to be discussed. For its crucial applications in the astrophysics, Kendall [4] has recently explained and made very neat experiments with modern equipments to deduce and study the capillary stability of that model which is a gas jet coaxial with a liquid jet. Radwan has discussed the stability of a viscous [5] and a streaming [7] hollow jet. The capillary instability of an annular fluid jet around a solid axis has been elaborated by Radwan [8] on utilizing the lagrangian principle of energy. He [8] found that the model is only unstable for small asymmetric disturbances and that the area under the instability curves are decreasing with increasing q values, but the capillary instability will never be suppressed. Recently Radwan [10] examined the influence of a constant magnetic field on the forgoing study [8]. It is found that the magnetic field is stabilizing and could suppress the capillary instability.

The eneavours of the present work is to investigate the magneto-dynamic instability and oscillation of a streaming annular fluid jet surrounding a tar mantle under general varying tenuous magnetic fields for all possible (symmetric and asymmetric) modes of perturbations.

For the importance of the annular fluid jet stability discussions, in general, in several domains of applications we may refer to Kendall [4].

2. Formulation of the problem

Consider a circular fluid jet of uniform density ρ radius a and streams with uiform velocity $(0, 0, U)$; coaxial and concentric with a tar cylinder of radius q a where $0 \leq q < 1$. This model is ambient with a tenuous medium of negligible inertia. The fluid is assumed to be non-viscous, incompressible and perfectly conducting. The model is penetrated by the magnetic field $(0, 0, H_o)$ and embedded in the general tenuous varying magnetic field $\underline{H}_o^s = (0, \beta H_o a r^{-1}, \alpha H_o)$ where H_o is the intensity of the magnetic field interior the annular jet and β, α are parameters satisfy certain restrictions (see equation 9). The components of the streaming velocity and the magnetic fields are taken along utilized cylindrical polar coordinates r, φ, z-system with the z-axis coinciding with the axis of the annular jet. The model is acting upon the inertia and electromagnetic forces.

The basic equations to be used here are the combination of the ordinary fluid dynamic equations with those of Maxwell concern the electromagnetic theory. For the problem at hand these equations are given by in the fluid

$$\rho\left(\frac{\partial \underline{u}}{\partial t} + (\underline{u}\cdot\underline{\nabla})\,\underline{u}\right) = -\nabla p + \mu\,(\underline{\nabla}\times\underline{H})\times\underline{H} \tag{1}$$

$$\underline{\nabla}\cdot\underline{u} = 0 \tag{2}$$

$$\underline{\nabla}\cdot\underline{H} = 0 \tag{3}$$

$$\frac{\partial \underline{H}}{\partial t} = \underline{\nabla}\times(\underline{u}\times\underline{H}) \tag{4}$$

in the tenuous

$$\underline{\nabla}\cdot\underline{H}^s = 0 \tag{5}$$

$$\underline{\nabla}\times\underline{H}^s = 0 \tag{6}$$

Here $\underline{u}$ and p are the fluid velocity vector and kinetic pressure, μ is the coefficient of the magnetic permeability and $\underline{H}$ is the magnetic field intensity. Equations (3) and (5) are the conservation of the magnetic fluxies interior and exterior the annular jet. Equation (4) is that of evolution of the magnetic field and equation (2) is the equation of continuity for incompressible fluid.

The unperturbed state is studied and as a result

$$\underline{\nabla}\left(p_o + \left(\frac{\mu}{2}\right)(\underline{H}_o\cdot\underline{H}_o)\right) = 0 \tag{7}$$

Since the acting forces in the initial state would be balanced at $r = a$, the unperturbed fluid pressure is being

$$p_o = \left(\frac{\mu H_o^2}{2}\right)\left(\alpha^2 + \beta^2 - 1\right) \tag{8}$$

with the restrictions

$$\alpha^2 + \beta^2 \geq 1 \tag{9}$$

where the equality occurs as a limiting case of zero fluid pressure.

3. Perturbation analysis

The relevant perturbation equations derived from equations (1) to (6) due to small departures from the initial state are accomplished by substituting the expansions $Q(r, \varphi, z; t) = Q_o(r) + \varepsilon_o Q_1(r, \varphi, z; t)$ and retaining first order terms only n small fluctuating variable $Q_1(r, \varphi, z; t)$. Here ε_o is the amplitude of the perturbation at time $t = 0$ while Q stands for $\underline{u}= \left(u_r, u_\varphi, u_z\right)$, p, $\underline{H}, \underline{H}^s$ and the radial distance of the fluid jet. For a single Fourier term, the latter is described by

$$r = a + \varepsilon_o \eta \tag{10}$$

with

$$\eta = exp\left[i(kz + m\varphi - kct)\right] \tag{11}$$

Here η is the elevation of the surface wave measured from the unperturbed position, k (any real number) is the longitudinal wave-number, m (an integer) is the azimuthal wavenumber and kc is the oscillation frequency at time t.

In magnetodynamic approximation, the relevant perturbation equations are given as :

in the fluid

$$\rho\left(\frac{\partial \underline{u}_1}{\partial t} + U\frac{\partial \underline{u}_1}{\partial z}\right) = -\underline{\nabla} p_1 + \mu\left[(\underline{\nabla} \times \underline{H}_1) \times \underline{H}_o + (\underline{\nabla} \times \underline{H}_o) \times \underline{H}_1\right] \tag{12}$$

$$\underline{\nabla} \cdot \underline{u}_1 = 0 \tag{13}$$

$$\underline{\nabla} \cdot \underline{H}_1 = 0 \tag{14}$$

$$\frac{\partial \underline{H}_1}{\partial t} = \underline{\nabla} \times (\underline{u}_1 \times \underline{H}_1) + \underline{\nabla} \times (\underline{u}_o \times \underline{H}_1) \tag{15}$$

in the tenuous

$$\underline{\nabla H}_1^s = 0 \tag{16}$$

$$\underline{\nabla} \times \underline{H}_1^s = 0 \tag{17}$$

Consequent to the deformation (10) and based the $(\varphi, z; t)$ dependence (cf. equation (11)) and according to the linear perturbation technique, as usual for the stability problems of cylindrical models, every perturbed quantity can be expressed as $exp\left[i(kz + m\varphi - kct\right]$ times an amplitude function of r. Thence the linearized system (12) to (17) is simplified and solved. Apart from the singular solutions we have obtained

$$\begin{aligned}
\underline{H}_1 &= \left(\frac{H_o}{(U-c)}\right)\left(u_{1r}\underline{e}_r + u_{1\varphi}\underline{e}_\varphi + u_{1z}\underline{e}_z\right) && (18)\\
p_1 &= ik\rho(c-U)\left[AI_m(kr) + BK_m(kr)\right] exp\left[i(kz + m\varphi - kct)\right] && (19)\\
u_{1r} &= k\left(AI'_m(kr) + BK'_m(kr)\right) exp\left[i(kz + m\varphi - kct)\right] &&\\
u_{1\varphi} &= \frac{im}{r}k\left(AI_m(kr) + BK_m(kr)\right) exp\left[i(kz + m\varphi - kct)\right] && (20)\\
u_{1z} &= ik\left(AI_m(kr) + BK_m(kr)\right) exp\left[i(kz + m\varphi - kct)\right] &&\\
\underline{H}_1^s &= \underline{\nabla}\Phi_1^s &&\\
\Phi_1^s &= CK_m(kr) exp\left[i(kz + m\varphi - kct)\right] && (21)
\end{aligned}$$

where A, B and C are arbitrary constants of integrations, $I_m(kr)$ and $K_m(kr)$ are modified Bessel functions of the first and second kind of order m; and $\underline{e}_r, \underline{e}_\varphi, \underline{e}_z$ are three unit vectors in the cylindrical (r, φ, z).

4. Eigenvalue relation

The solution of the perturbation equation (12) to (17) represented by (18) to (21) must satisfy appropriate conditions. Under the present circumstances these boundary conditions are the following :

1. The kinematic boundary conditoin states that the normal component of the velocity vector$\underline{u}$($i.e.\ \underline{N}\cdot\underline{u}$) must be compatible with the velocity of the (fluid-tenuous) surface across the interface (10) at $r = a$. This reads

$$\underline{N}_o.\underline{u}_1 + \underline{N}_1\cdot\underline{u}_o = \frac{\partial r}{\partial t} \quad \text{at} \quad r = a \qquad (22)$$

   with

$$\underline{N} = \underline{N}_o + \varepsilon_o\underline{N}_1 = (1,0,0) + \varepsilon_o\left(0, -\frac{im}{a}, -ik\right)\eta \qquad (23)$$

   is the outward unit vector normal to the perturbed interface (10). Therefore

$$AI'_m(ka) + BK'_m(ka) = i(U-c) \qquad (24)$$

2. The normal component of the velocity $\underline{N}\cdot\underline{u}$ must vanish at $r = qa$. This condition yields

$$AI'_m(qka) + BK'_m(qka) = 0 \qquad (25)$$

   Equations (24) and (25) give

$$\begin{aligned}
A &= \frac{i(U-c)K'_m(y)}{L^m_{xy}} && (26)\\
B &= \frac{i(U-c)I'_m(y)}{L^m_{xy}} && (27)
\end{aligned}$$

with

$$L_{xy}^m = I'm(x)K'_m(y) - I'_m(y)K'_m(x) \tag{28}$$

where $x = (ka)$ and $y = (qx)$ are the dimensionless longitudinal wavenumbers.

3. The normal component of the magnetic field must be continuous across the fluid-tenuous interface (10) at $r = a$,

$$\underline{N}_o.\underline{H}_1 + \underline{N}_1 \cdot \underline{H}_o = (\underline{N}_o \cdot \underline{H}_1 + \underline{N}_1.\underline{H}_o)^s \quad \text{at} \quad r = a \tag{29}$$

from which we get

$$C = iH_o\frac{(m\beta + \alpha x)}{(xK'_m(x))} \tag{30}$$

4. The normal component of the total stress tensor must also be continuous across the disturbed interface (11) at $r = a$

$$p_1 + \eta\frac{\partial p_o}{\partial r} + \left(\frac{\mu}{2}\right)\left(2\underline{H}_o \cdot \underline{H}_1 + \eta\frac{\partial\left(\underline{H}_o \cdot \underline{H}_o\right)}{\partial r}\right) \tag{31}$$
$$= \left(\frac{\mu}{2}\right)\left(2\underline{H}_0^s \cdot \underline{H}_1^s + \eta\frac{\partial\left(\underline{H}_o^s \cdot \underline{H}_o^s\right)}{\partial r}\right) \text{ at } r = a$$

This condition, taking into account the solutions in the present section and in the preceding sections, degenerates to following eigenvalue relation

$$k^2(U-c)^2 = \frac{\mu H_o^2}{\rho a^2}\left\{x^2 - \left[\beta^2 + (m\beta + \alpha x)^2\,\frac{K_m(x)}{xK'_m(x)}\right]F_m(x,y)\right\} \tag{32}$$

with

$$F_m(x,y) = x\frac{L_{xy}^m}{L_y^m},\ x = ka,\ y = qx \tag{33}$$

$$L_{xy}^m = I'_m(x)K'_m(y) - I'_m(y)K'_m(x) \tag{34}$$

$$L_y^m = I_m(x)K'_m(y) - I'_m(y)K_m(x) \tag{35}$$

5. Discussions and conclusion

The desired magnetodynamic dispersion relation (32) relates the oscillation frequency $\omega = kc$ or rather the temporal amplification $\sigma = -ikc$ with the longitudinal wavenumber x and y normalized with respect to the scale length a of the problem, the azimuthal wavenumber m, the parameters β, α of the tenuous varying magnetic field, the geometric factor q which is the radius of the tar cylinder normalized with respect to a, the compound functions $F_m(x,\,y)$, L_{xy}^m and L_y^m and with the parameters μ, U, ρ, a and H_o of the problem. One has to refer to the fact that the fundamental natural quantity $\left(\frac{\mu H_o^2}{\rho a^2}\right)^{\frac{1}{2}}$ has a unit of time, so the dimensionless growth rate and oscillation frequency are given by $\frac{\sigma}{\left(\mu H_o^2/\rho a^2\right)^{\frac{1}{2}}}$ and $\frac{\omega}{\left(\mu H_o^2/\rho a^2\right)^{\frac{1}{2}}}$ respectively.

Before we are going to discuss the eigenvalue relation (32) in its general form, we intend to involve ourselves in some limiting cases. If we let q tends to zero i.e. $y \to 0$, in such a case the model of a streaming full fluid jet ambient with the tenuous medium pervaded by a general varying magnetic field and acting upon it the intertia and electromagnetic forces. The corresponding eigenvalues relation can be obtained from (32) by utilizing the asymptotic behaviour of the cylindrical functions appearing in it. As q tends to zero, using the asymptotic behaviour of modified Bessel functions

$$\lim_{q\to 0} I'_m(y) \to 0 \tag{36}$$

$$\lim_{q\to 0} K'_m(y) \to \infty \tag{37}$$

we obtain

$$\frac{k^2(U-c)^2}{\frac{\mu H_o^2}{(\rho a^2)}} = x^2 - \frac{\beta^2 x I'_m(x)}{I_m(x)} - (m\beta + \alpha x)^2 \frac{I'_m(x) K_m(x)}{I_m(x) K'_m(x)} \tag{38}$$

In absence of the streaming character, the relation (38) recovers our recent result [8] if we neglect the contribution of the surface tension there. For detail discussions we may refer to the analytical and numerical analysis of ref. [8].

If we consider a very thin fluid layer surrounding a very thick tar mantle i.e. we let $q \to 1$ from below, the associated eigenvalue relation can be identified from (32) by using the asymptotic behaviour of modified Bessel functions and their derivatives as $q \to 1$. Using a series development for I_m, I'_m, K_m, K'_m around the value qx and neglecting terms of second order of $(1-q)$, the eigenvalue relation (32) in the rotationally axisymmetric mode degenerates to

$$\frac{k^2(U-c)^2}{\frac{\mu H_o^2}{(\rho a^2)}} = x^2 + (1-q)\left(\beta^2 + \alpha^2 \frac{x k_0}{K'_0}\right) \frac{x^2 \left(I_1 K'_1 - I'_1 K_1\right)}{\left(I_0 K_1 - K_0 I_1\right)} \tag{39}$$

where the arguments of all modified Bessel functions are x. In view of the well-known Wronskian relation

$$W\left(I_m(x), K_m(x)\right) = I_m(x) K'_m(x) - I'_m(x) K_m(x) = -x^{-1} \tag{40}$$

and the recurrence relations

$$I'_0(x) = I_1(x) \tag{41}$$

$$K'_0(x) = -K_1(x) \tag{42}$$

equation (4) reduces to

$$\frac{k^2(U-c)^2}{\frac{\mu H_o^2}{(\rho a^2)}} = x^2 + \left\{1 - (1-q)\left[\beta^2 - \alpha^2 \frac{x K_0(x)}{K_1(x)}\right]\right\} \tag{43}$$

we have here the following three different subclasses.

1. If the model is very thin cylindrical fluid shells and surrounding a very thick mantle and pervaded by and embedded with axial magnetic fields then $\beta = 0$. The discussion of (43) in this case shows, as $U = 0$, that the model is stable for all α and $q \to 1$ values for all (short and long) wavelength. This is physically plausible because the longitudinal uniform fields give a measure of rigidity to the conducting fluid and that of course the main basic reason for the stabilizing character.

2. If the model is very thin cylindrical fluid shells having very thick tar mantle and penetrated by the uniform axial field $H_o \underline{e}_z$ and ambient with the azimuthal tenuous varying magnetic field $\underline{H}_o^s = \left(\dfrac{\beta a H_o}{r}\right) \underline{e}_\varphi$ then $\alpha = 0$. In such a case the temporal amplification $(-ikc)$ is depleted as q tends to unity from below depending on the value of β. For $q = 0.90$ we get high stability for small values of β but high instability for large values of β. It is found also that for $q = 0.99$, the model is stable for all values of β and the stability becomes slower and slower as β is increased. This is verified since in the last case $(q = 0.99)$ we find $\beta^2(1-q) \leq 1$ and if $\beta^2(1-q)$ equals unity we must take terms in the relation (43) of order $(1-q)^2$ or even more. It is worthwhile to mention here that the increasing the thickness of the solid mantle leads to the increasing of the stability with respect to the full fluid jet for the corresponding values of $\beta = 1.0, 20.0$.

3. The third subclass is the general case in which $\alpha \neq 0, \beta \neq 0$ and the model is a very this fluid shells surrounding a very thick tar axis. This model is stable for all α, β and $x \neq 0$ values if the restriction $\left(\dfrac{\beta}{a}\right) < \left(\dfrac{x K_0(x)}{K_1(x)}\right)^{\frac{1}{2}}$ is satisfied as $U = 0$.

Form the discussions of the above mentioned foregoing three cases and their results, we may conclude that the instability states and those of stability are depending on α, β and q values whether the deformation is axisymmetric or / and asymmetric. In contrast to the same problem acting upon it through capillary force only as we have obtained [6] where it is found that the considered system is unstable only for small wavenumbers in the axisymmetric mode $m = 0$. Moreover the area under the instability curves are found to be decreasing with increasing q values (in the range $0 \leq q < 1$) but the instability is never be suppressed whatever is the largest value of q.

Now, let us return to the general eigenvalue relation (32) which is valid to all possible ($m = 0$ and $m \neq 0$) modes of perturbation.

The influence of the longitudinal magnetic field pervaded interior the annular jet is represented by the term x^2 following $\left(\dfrac{\mu H_o^2}{\rho a^2}\right)$. It has a strong stabilizing effect for all (short and long) wavelengths whether the perturbation is non-axisymmetric $m \neq 0$ or / and axisymmetric one $m = 0$ and that is independent of q values.

In order to identify the tenuous varying magnetic field influence we have to determine the behaviour of the different cylindrical functions occurring in the relation (32). As is well known, for every non-zero real values of x that $I_m(x)$ is positive and monotonic increasing while $K_m(x)$ is monotonic decreasing but never negative. Using that together with the recurrence relations

$$2I'_m(x) = I_{m-1}(x) + I_{m+1}(x) \tag{44}$$

$$2K'_m(x) = -K_{m-1}(x) - K_{m+1}(x) \tag{45}$$

one can show that $I'_m(x)$ is always positive while $K'_m(x)$ is never positive. In view of these results, following inequalities (note that y is less than x):

$$I'_m(x) > I'_m(y) \tag{46}$$

$$|K'_m(x)| < |K'_m(y)| \tag{47}$$

$$I'_m(x)\,|K'_m(y)| > I'_m(y)\,|K'_m(x)| \tag{48}$$

are satisfied for every non-zero real values of x and y. therefore the compound I^m_{xy} and L^m_y are being

$$L^m_{xy} < 0 \tag{49}$$

$$L^m_y < 0 \tag{50}$$

for every non-zero real values of x and y for all possible ($m = 0$ and $m \neq 0$) modes of perturbation. From the foregoing discussions we may deduce that

$$F_m(x, y) > 0 \tag{51}$$

and never changes sign for every non-zero real values of x and y and for every integer value of m.

By the use of inequality (43) and the fact that $\left(\dfrac{K_m(x)}{K'_m(x)}\right)$ is always negative (see (45)) : the analytical discussions of the relation (32) show that the longitudinal tenuous magnetic field is strongly stabilizing for all q, x and m values. The azimuthal tenuous varying magnetic field is strongly destabilizing in the symmetric mode $m = 0$ which in the asymmetric $m \neq 0$, it is stabilizing if the restrictions

$$mK_m(x) \geq x\,|K'_m(x)| \tag{52}$$

are satisfied and vice versa. Therefore the azimuthal tenuous magnetic field is destabilizing or stabilizing according to some restrictions.

The general tenuous varying magnetic field is absolutely destabilizing if $m\beta + \alpha x = 0$. The latter restriction is satisfied if the coefficient of the perturbed tenuous magnetic field vanishes, see equation (30). Also the destabilizing influence of the azimuthal field is largest, when the perturbed tenuous field is orthogonal to the unperturbed basic tenuous field $\underline{H}^s_o$ at the boundary surface $r = a$. This can be shown as follows:

From the obtained solution we have established, apart from the singular solutions and the value of C, that

$$\underline{H}^s_o = \left(0, \beta a H_o r^{-1}, \alpha H_o\right)$$

$\underline{H}^s_1 = C\underline{\nabla}\left\{K_m(kr)exp\left[i(kz + m\varphi - ikct)\right]\right\}$ from which the direction numbers at $r = a$ of $\underline{H}^s_o$ are $(0, \beta, \alpha)$ and those of $\underline{H}^s_1$ are (H^s_{1r}, m, x). Therefore, we have $\left(\underline{H}_o \cdot \underline{H}_1\right)^s$ is proportional to $m\beta + \alpha x$, which equals zero if $\underline{H}^s_o$ is orthogonal to $\underline{H}^s_1$.

To sum up the non-streaming jet having a tar mantle pervaded by uniform field and ambient which general varying tenuous magnetic field is not absolutely stable not only in the symmetric mode $m = 0$ but also in the asymmetric modes $m \neq 0$ of perturbations. In contrast to the same problem subject to surface tension only [6], it is found that the model is absolutely stable in all asymmetric modes $m \neq 0$ while it is unstable to small wavenumbers in the axisymmetric disturbance $m = 0$.

From the analytical discussions of the eigenvalue relation (32), it is found that the streaming has a strong destabilizing influence for all q values for all (short and long wavelengths whether the perturbation is symmetric or / and asymmetric modes. This is also true even for other cylindrical configurations and other acting forces, see refs. [7] and [9]. Therefore, the streaming has the influence of enlarging the instability regions but decreasing the stable domains. This means that the magneto-dynamic instability of an annular jet having a tar mantle becomes more worse as fluid is uniformly streaming.

1. order to verify and confirm the foregoing analytical results: to clarify the influence of q and $U^* = \dfrac{-ik}{\left(\dfrac{\mu H_o^2}{\rho a^2}\right)^{\frac{1}{2}}}$ on the magnetodynamic instability of the annular jet, the eigenvalue relation (32) has been analysed numerically.

The criterion (32) has been calculated in computer in order to obtain the values of $\sigma = -ikc$ or $\omega = kc$ in the natural unit $\left(\dfrac{\mu H_o^2}{\rho a^2}\right)^{\dfrac{1}{2}}$, corresponding to the instability states or those of stability respectively, for various values of x. This is carried out for different values of (α,β) and q for numerous values of U^*. The numerical data are collected and tabulated and presented graphically, there are many features in these numerical presentations.

1. **In the rotationally axisymmetric mode** $m = 0$

   The numerical data are presented graphically for different pairs of values of (α, β) = (1, 1), (2, 1), (2, 2) and (10, 1). For every pair (α, β) values, several values of U^* are considered and at the same time numerous fluid cylindrical shells (of different thickness) surrounding the tar mantle are assumed. The values of U^* are taken to be 0.2, 0.4 and 0.7 while the geometric factor q (the ratio between the radii of the solid cylinder and the fluid jet) values are 0.1, 0.5, 0.9 and 0.99. from these numerical data we deduce the following :

   As $\alpha = \beta = 1$ and 2 for a given set values of U^*, the unstable domain are found to be decreasing with increasing q values while the stable domains are extensively increasing. This means that as the fluid cylindrical shell is thin and surrounding a very thick tar mantle, it is more mangetodynamic stable than the other cases as long as $\alpha = \beta$ whatever is the value of U^*. As $\alpha > \beta$ for a given values of U^*, it is found that the domains of instability are increasing with increasing q values. Therefore, a very thin shell surrounding a very thick tar mantle is magnetodynamic less stable than the other cases as long as $\alpha > \beta$ whatever is the value of U^* . We predict that the motion is not slowly and experimentally the observations are not easy in comparison with the case of MHD full fluid jet (as $q = 0$).

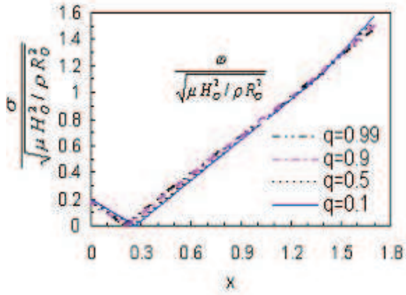

Figure 1 The numerical results for $m = 0.0$ and $U* = 0.2\ \alpha,\ \beta = (1, 1)$

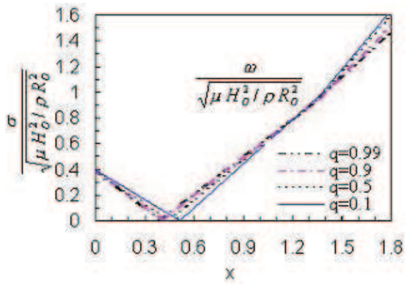

Figure 2 The numerical results for $m = 0.0$ and $U* = 0.4\ \alpha$, $\beta = (1, 1)$

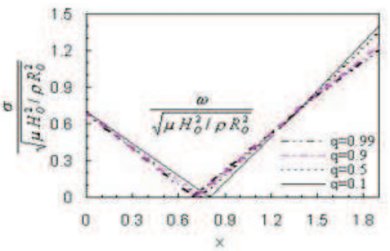

Figure 3 The numerical results for m = 0.0, ,and U*= 0.7α, $\beta = (1, 1)$)

Moreover, from the above discussions we may conclude also that the Lorentz force is not purely stabilizing, in particular as $\beta \neq 0$, but it is stabilizing or destabilizing according to some restrictions. This is due to the fact that the azimuthal tenuous magnetic field is not uniform and consequently due to the influence of that field the magnetodynamic unstable domains are never be suppressed.

For given value of q and (α, β), it is found that the streaming has a strong destabilizing influence. This is very clear from the numerical representation where it is found that the unstable domains are increasing while those of stable are decreasing with increasing U^* values. This result is true whether the components of the tenuous magnetic field $\underline{H}_o^s$ are equal or not and also whether the cylindrical shells are thick or thin.

2. **In the non-axisymmetric disturbances** $m = \mp 1$

The numerical data are presented numerically for different values of (α, β) as (α, β)=(1, 1), (1, 2) and (5, 5). Then for every pair of these values numerous values of U^* and also of q are considered. Similar discussions can be carried out here for the present case $m = \mp 1$ as in the case of $m = 0$. However it is remarkable that there are very little domains of instabilities while those of stability are very wide and very high, in particular for very thin cylindrical fluid shells surrounding very thick tar mantles.

The domains of instability, for the same values of α and β, are quickly decreased with increasing q values. While they are increasing with increasing U^* values. This means that q has a stabilizing influence while U^* is strongly destabilizing. This confirms the analytical discussions.

As $\beta > \alpha$, the domains of instabilities are much larger than those of the cases in which $\alpha = \beta$. This is physically plausible because the tenuous azimuthal varying magnetic field is destabilizing whether the disturbance modes are axisymmetric (sausage) $m = 0$ or / and non-axisymmetric modes $m \neq 0$.

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