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New Methods of the Assur Groups Structural Synthesis

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The paper presents the synthesis of the Assur groups different classes, single– or multi–contours conducted with three new methods (M1, M2, M3). Their usage every time leads to the Assur group with zero degree of freedom. Method M1 was used to synthesis of Assur group class III and higher; M2 to creation of multi contour Assur groups; M3 to creation of kinematic chains composed of Assur groups.

Keywords: Structural synthesis, Assur groups

1. Introduction

Assur group it's kinematic chain with zero degree of freedom. After connection with the external kinematic pairs to the base and drive link (links) create the mechanism. Number of links and number of kinematic pairs in the received Assur groups is compliant with the structural pattern:

$$3n_r - 2p_5 = 0 \tag{1}$$

where n_r – number of moving links, p_5 – number of the kinematic pairs 5 class. Relationship between n_r and p_5 is presented in the Tab. 1.

Table 1 Relationship between n_r and p_5 in the flat Assur groups

n_r	4	6	8	10	12	14	16	18	...
p_5	6	9	12	15	18	21	24	27	...

2. Assur groups

The Assur group given classes which consist of the lowest number of links and kinematic pairs is called simple Assur group [3]. In the each class of the Assur groups it's possible to construct such group [4], using base contour presented by I.I. Artobolewskij. Working on the basis of the simple Assur groups it's possible to construct more complex groups which construction process can be described by the following rule [6]:

M1. Change k–conjunctive link to $(k+1)$–conjunctive link and add two double–conjunctive links.

Application of this method, increase the Assur group series by one [5]. The presented M1 method is analogical to proposed by the L. W. Assura [2].

On the Fig. 1b, c presented are the simple Assur groups class III series 4, 5 constructed on the basis of the most simple Assur group class III series 3 /series of the Assur group equals number of the external kinematic pairs/ (Fig. 1a) by replacement of the following double–conjunctive links with the triple–conjunctive links and every time addition of the two double–conjunctive links. For instance the Assur group presented on the Fig. 1b was constructed by replacement of the double–conjunctive links 4 with the triple–conjunctive link and addition of the links 5 and 6.

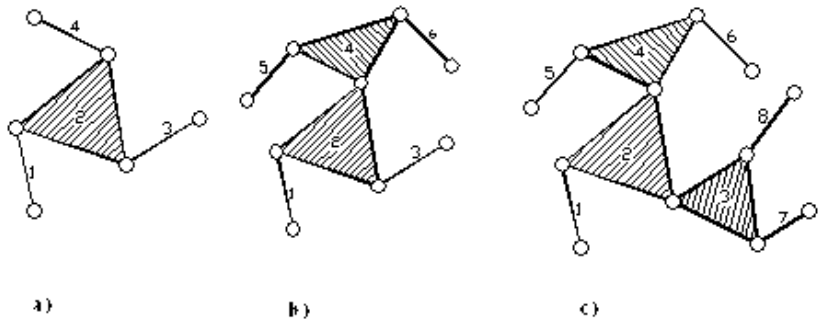

Figure 1 Assur groups class III: a) series 3, b) series 4, c) series

By application of the M1 method created were the Assur groups class III series $4 \div 10$. Following number of the Assur groups were received:

- one series 4 (Fig. 1b),
- one series 5 (Fig. 1c),
- two series 6,
- two series 7,
- four series 8,

- six series 9,
- eleven series 10.

The Assur groups class III includes only double–conjunctive and triple–conjunctive links.

In the construction of the Assur group series $j(j \geq 3)$ occurs:

- $n_r = 2(j-1)$ of moving links,
- $p_5 = 3(j-1)$ of kinematic pairs,
- $n_3 = j-2$ triple–conjunctive links,
- $n_2 = j$ double–conjunctive links.

Symbols assumed:

n_2 – number of double–conjunctive links,
n_3 – number of triple–conjunctive links,
n_4 – number of quadruple–conjunctive links.

The Assur groups class IV constructed on the basis of the Assur group class IV series 2 are presented on the Fig. 2. The Assur group class IV series 3 (Fig. 2b) was created by replacement of the triple–conjunctive link 1 with quadruple–conjunctive link and addition of the links 5 and 6. By replacement of the double–conjunctive link 2 with the triple–conjunctive link and addition of the links 7 and 8 the Assur group class 4 series 4 has been constructed (Fig. 2c).

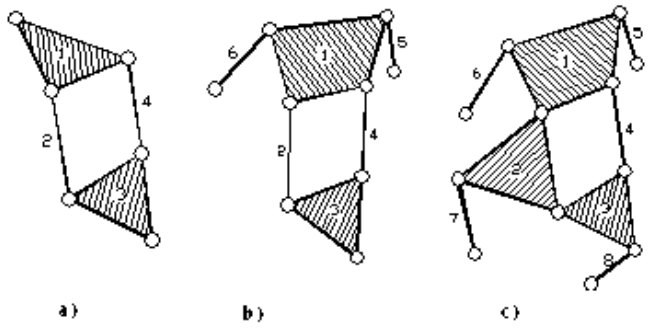

Figure 2 The Assur groups class IV: a) series 2, b) series 3, c) series 4

Fig. 3 presents three simple kinematic chains class IV series 3 with the same number of links $n_r = 6$ and kinematic pairs $p_5 = 9$ however with different structure. Two Assur groups consist of three double–conjunctive links and three triple–conjunctive links (Fig. 3a). Therefore its possible to create Assur groups with the same number of kinematic pairs and links and different structure. The third Assur group contains four double–conjunctive links, single triple–conjunctive link and single quadruple–conjunctive link (Fig. 3b).

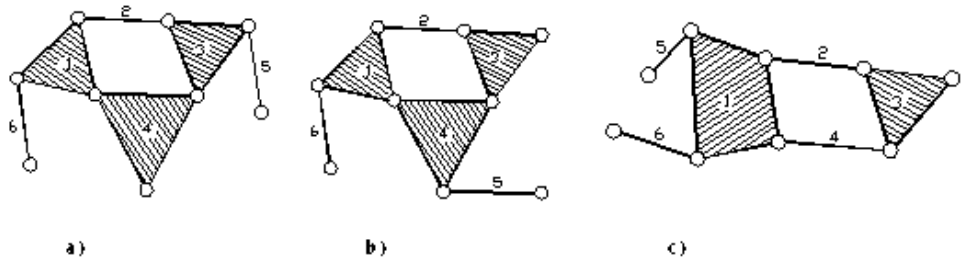

Figure 3 The Assur groups class IV series 3 $n_r = 6$, $p_5 = 9$, a) $n_2 = 3$, $n_3 = 3$, b) $n_2 = 4$, $n_3 = 1$, $n_4 = 1$

In the tAB. 2 presented are data concerning the received Assur groups class IV due to the application of the M1 method.

Table 2 Collation of data concerning the simple Assur groups class IV

Series	n_r	p_5	n_2	n_3	n_4	*Number of Assur groups*
2	4	6	2	2	0	1
3	6	9	3	3	0	2
			4	1	1	1
4	8	12	4	4	0	4
			5	2	1	3
			6	0	2	1
5	10	15	5	5	0	6
			6	3	1	8
			7	1	2	1
6	12	18	6	6	0	12
			7	4	1	42
			8	2	2	15

In the simple Assur groups class IV series $j (j \geq 2)$ occurs:

- moving links $n_r = 2\,j$,
- kinematic pairs $p_5 = 3\,j$.

If the Assur group class IV series j includes:

- j number of the double–conjunctive links then other links are the triple–conjunctive links in a number of $n_3 = j$,
- $j + 1$ number of the double–conjunctive links then other links are: the triple–conjunctive links in a number of $n_3 = j - 2$ and the single quadruple–conjunctive link n_4,

- $j + 2$ number of double–conjunctive links the other links are: – $j - 4$ number of triple–conjunctive links and two quadruple–conjunctive links.

By using the M1 method, analogically we can create Assur groups class V, VI and higher and formulate conclusions concerning relations between series and number of double–, triple– and quadruple–conjunctive links which comprise given Assur group.

3. Multi–contour kinematic chains

By using Assur groups class III, IV and higher we can conduct synthesis of multi–contour Assur groups that is these which in their structure consist of at least two contours. In multi–contour Assur groups k–conjunctive links ($k \geq 3$) are always contour links that is these links are part of at least single contour.

The author's method M2 has been created which application leads to creation of multi–contour Assur groups. The method use instruction:

M2 – Replace the k–conjunctive link in the Assur group with the (k+1)–conjunctive link and connect it with the external kinematic pair.

In the Assur group class IV series 3 (Fig. 4a) in accordance with the M2 method, the double–conjunctive link 5 was replaced with triple–conjunctive link and connected by the external kinematic pair with the link 6. That's how the double–contour Assur group class IV series 2 was created (Fig. 4b).

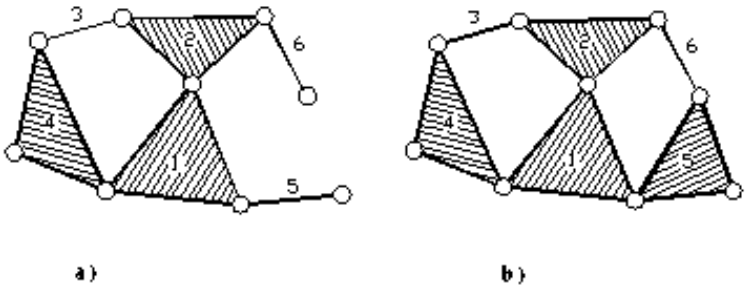

Figure 4 The double contour Assur group class IV series 2 (b) created from Assur group class IV series 3 (a)

Typical feature of using the M2 method for contours creation in Assur groups is reduction its series by one with each usage. Through analysis of multi–contour Assur groups, formulated were conclusions concerning relations between number of contours, series and number of links and kinematic pairs. In the case of the Assur groups series j :

- single–contour – $n_r = 2j$, $p_5 = 3j$,

- double–contour – $n_r = 2(j+1)$, $p_5 = 3(j+1)$,
- triple–contour – $n_r = 2(j+2)$, $p_5 = 3(j+2)$,
- quadruple–contour – $n_r = 2(j+3)$, $p_5 = 3(j+3)$.

In generalized form the following conclusions can be presented as following:

In l–contour Assur groups series j, number of links $n_r = 2(j+l-1)$, and number of kinematic pairs $p_5 = 3(j+l-1)$.

Multi-contour Assur groups can be created on the basis of different Assur Groups. For instance Assur group class V series 4 (Fig. 5a), may by created from both Assur group class IV series 5 (Fig. 5b) and Assur group class V series 5 (Fig. 5a).

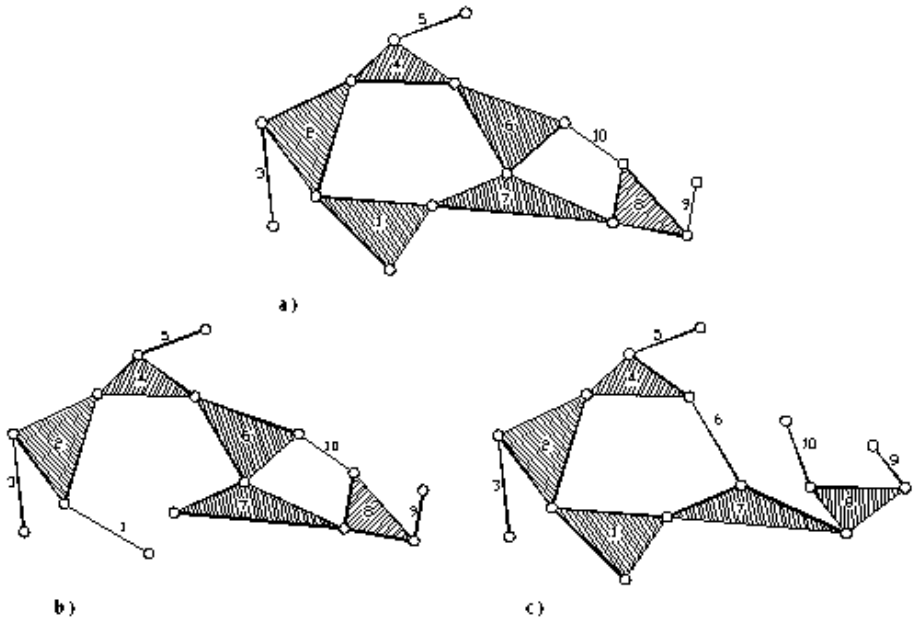

Figure 5 The double contour Assur group class V series 4 (a), created from Assur group class IV series 5 (b), or Assur group class V series 5 (c)

4. Kinematic chains comprised of Assur groups

By using Assur groups we can create kinematic chains with zero degree of freedom. Method of creation of this type of kinematic chains was formulated analogically to the M2 method:

M3 – In order to create kinematic chain comprised of Assur groups its required to replace the k–conjunctive link in one of Assur groups with

the (k+1)–conjunctive link and connect it with other Assur group by external kinematic pair.

The Assur group class III series 3 (Fig. 6a) and the Assur group class IV series 2 (Fig. 6b) will results in construction of the kinematic chain (Fig. 6c). To conduct creation of such construction the double–conjunctive link 3 in the Assur group class III was replaced with the external kinematic pair.

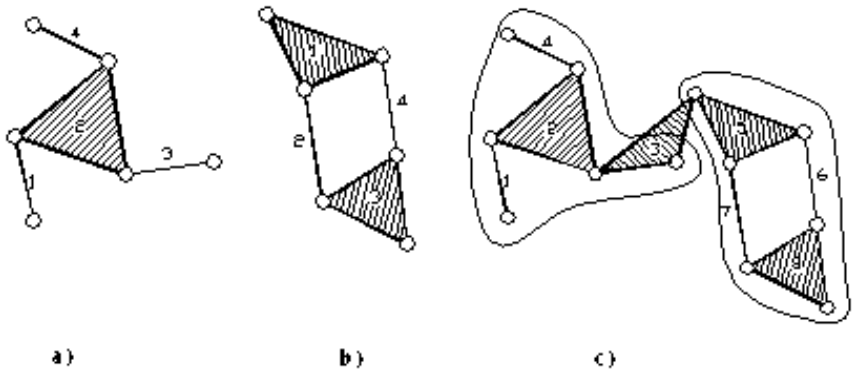

Figure 6 Kinematic chain (c) comprised of the Assur group class III series 3 (a) and Assur group class IV series 2 (b)

Combination of Assur groups can be conducted through replacement of internal or external link. In the case presented on Fig. 7 replaced was external double–conjunctive link. The example of combination through internal link is presented on Fig. 7.

Number of links and kinematic pairs in such created kinematic chain is sum of number of links and kinematic pairs of component Assur groups:

$n_{r(l)} = n_{r(1)} + n_{r(2)}$

$p_{5(l)} = p_{5(1)} + p_{5(2)}$

Number of double–, triple– and quadruple–conjunctive links which are part of received kinematic chain depends on conducted replacement of link during combination of Assur groups.

If during combination of two Assur groups replaced is link:

- 1) double–conjunctive with triple–conjunctive than:

  $n_{2(l)} = n_{2(1)} + n_{2(2)} - 1$

  $n_{3(l)} = n_{3(1)} + n_{3(2)} + 1$

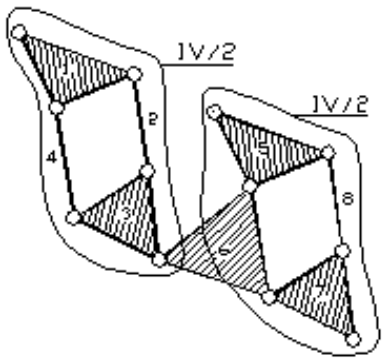

Figure 7 Kinematic chain comprised of two Assur groups class IV series 2

- 2) triple–conjunctive with quadruple -conjunctive than:

$$n_{2(l)} = n_{2(1)} + n_{2(2)}$$
$$n_{3(l)} = n_{3(1)} + n_{3(2)} - 1$$
$$n_{4(l)} = 1$$

Analogical relations can be formulated for kinematic chains created from combination of three, four or higher number of Assur groups class III. It is necessary to emphasize that during combination of i Assur groups number of quadruple–conjunctive links can change from 0 to $i - 1$.

5. Conclusions

Presented work is dedicated to methodology of synthesis of Assur groups class III to VI, multi–contour Assur groups and kinematic chains comprised of Assur groups of different classes and series.

For Assur groups of particular classes, received by application of the M1 method formulated were conclusions concerning relations between series and number of: links, kinematic pairs.

The M2 method was utilized to multi–contour Assur groups construction. For this group of Assur groups defined were relations between number of links and kinematic pairs and series of particular Assur groups double–contour, triple–contour, etc.

It is necessary to mention that created multi–contour Assur groups should be examined for structural rationality that is its required to analyze, if received Assur groups don't comprise rigid and overrigid kinematic chains

The last formulated method (M3) were used to combine Assur groups in kinematic chains, which after connecting to the base and drive links create the mechanism. Presented is relation between number of links and kinematic pairs in component Assur groups and created with them chains

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