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Deformation in a Generalized Thermoelastic Medium with Hydrostatic Initial Stress Subjected to Different Sources

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In the present problem we study the deformation of a rotating generalized thermoelastic medium with hydrostatic initial stress subjected to three different type of sources. The components of displacement, force stress and temperature distribution are obtained in Laplace and Fourier domain by applying integral transforms. The general solution obtained is applied to a specific problem of a half–space subjected to concentrated force, distributed force and a moving source. These components are then obtained in the physical domain by applying a numerical inversion method. Some particular cases are also discussed in context of the problem. The results are also presented graphically to show the effect of rotation and hydrostatic initial stress.

Keywords: Rotation, Hydrostatic, initial stress, Generalized thermoelasticity, Laplace and Fourier transforms, temperature distribution.

1. Introduction

Generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory of thermoelasticity in which the parabolic type heat conduction equation is based on Fourier's law of heat conduction. This newly emerged theory which admits finite speed of heat propagation is now referred to as the hyperbolic thermoelasticity theory, Chandrasekharaiah [6], since the heat equation for rigid conductor is hyperbolic–type differential equation.

There are two important generalized theories of thermoelasticity. The first is due to Lord, Shulman [16]. The second generalization to the coupled theory of thermoelasticity what is known as the theory of thermoelasticity with two relaxation times or the theory of temperature–rate–dependent thermoelasticity. Muller [18], in a review of the thermodynamics of thermoelastic solid, proposed an entropy production inequality, with the help of which he consider restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws [14]. Green and Lindsay(G–L) obtained another version of the constitutive equations [15]. These equations were also obtained independently and more explicitly by Suhubi [35]. This theory contains two constants that act as relaxation times and modify all the equations of the coupled theory, not only the heat equations. The classical Fourier law violated if the medium under consideration has a centre of symmetry.

Barber and Martin–Moran [2] discussed Green's functions for transient thermoelastic contact problems for the half–plane. Barber [3] studied thermoelastic displacements and stresses due to a heat source moving over the surface of a half plane. Sherief [34] obtained components of stress and temperature distributions in a thermoelastic medium due to a continuous source. Dhaliwal *et al.* [13] investigated thermoelastic interactions caused by a continuous line heat source in a homogeneous isotropic unbounded solid. Chandrasekharaiah and Srinath [5] studied thermoelastic interactions due to a continuous point heat source in a homogeneous and isotropic unbounded body. Sharma *et al.* [26] investigated the disturbance due to a time–harmonic normal point load in a homogeneous isotropic thermoelastic half–space. Sharma and Chauhan [28] discussed mechanical and thermal sources in a generalised thermoelastic half–space. Sharma, Sharma and Gupta [27], investigated the steady–state response of an applied load moving with constant speed for infinite long time over the top surface of a homogeneous thermoelastic layer lying over an infinite half–space. Recently Deswal and Choudhary [11] studied a two–dimensional problem due to moving load in generalized thermoelastic solid with diffusion.

The development of initial stresses in the medium is due to many reasons, for example resulting from difference of temperature, process of quenching, shot pinning and cold working, slow process of creep, differential external forces, gravity variations etc. The Earth is assumed to be under high initial stresses. It is therefore of much interest to study the influence of these stresses on the propagation of stress waves. Biot [4] showed the acoustic propagation under initial stresses which is fundamentally different from that under stress free state. He has obtained the velocities of longitudinal and transverse waves along the co–ordinate axis only.

The wave propagation in solids under initial stresses has been studied by many authors for various models. The study of reflection and refraction phenomena of plane waves in unbounded medium under initial stresses is due to Chattopadhyay *et. al.* [8], Sidhu and Singh [23] and Dey *et. al.* [12]. Montanaro [17] investigated the isotropic linear thermoelasticity with hydrostatic initial stress. Singh *et. al.* [24], Singh [25] and Othman and Song [19] studied the reflection of thermoelastic waves from free surface under hydrostatic initial stress in context of different theories of generalized thermoelasticity.

Some researchers in past have investigated different problem of rotating media. Chand *et. al.* [7] presented an investigation on the distribution of deformation, stresses and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting elastic half space. Many authors (Schoenberg and Censor [22], Clarke and Burdness [9], Destrade [10] studied the effect of rotation on elastic waves. Roychoudhuri and Mukhopadhyay [21] studied the effect of rotation and relaxation times on plane waves in generalized thermo–visco–elasticity. Ting [36] investigated the interfacial waves in a rotating anisotropic elastic half space. Sharma and his co–workers [29–32] discussed effect of rotation on different type of waves propagating in a thermoelastic medium. Othman and Song [20] presented the effect of rotation in magneto thermoelastic medium. Recently Ailawalia and Narah [1] discussed the effect of rotation due to moving load at the interface of elastic half space and generalized thermoelastic half space.

In the present investigation we have obtained the expressions for displacement, force stress and temperature distribution in a rotating generalized thermoelastic medium with hydrostatic initial stress by applying Laplace and Fourier transforms subjected to different sources. Such types of problems in the rotating medium are very important in many dynamical systems. Some particular cases are also derived from the present investigation.

2. Formulation of the problem

We consider a homogeneous generalized thermoelastic half–space with hydrostatic initial stress rotating uniformly with angular velocity $\vec{\Omega} = \Omega\hat{n}$, where $\hat{n}$ is a unit vector representing the direction of the axis of rotation. All quantities considered are functions of the time variable t and of the coordinates x and y. The displacement equation of motion in the rotating frame has two additional terms (Schoenberg and Censor, [22]): centripetal acceleration, $\vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right)$ due to time varying motion only and $2\vec{\Omega}\times\vec{u}$ where $\vec{u} = (u_1\, , u_2,\ 0)$ the dynamic displacement vector and angular velocity is $\vec{\Omega} = (0\ , \Omega,\ 0)$. These terms do not appear in non–rotating media.

We consider a mechanical normal source acting at the plane surface of generalized thermoelastic half space with hydrostatic initial stress. A rectangular coordinate system $(x\ , y,\ z)$ having origin on the surface $y = 0$ and axis pointing vertically into the medium is considered.

3. Basic equations and their solutions

For a two dimensional problem ($xy-$plane) all quantities depends only on space coordinates x, y and time t. The field equations and constitutive relations in generalized linear thermoelasticity with rotation and without body forces and heat sources are given by Lord and Shulman [16], Green and Lindsay [15] and Montanaro [17] as,

$$\begin{aligned}(\lambda+2\mu)\frac{\partial^2 u_1}{\partial x^2}+\left(\lambda+\mu+\frac{p}{2}\right)\frac{\partial^2 u_2}{\partial x\partial y}+\left(\mu-\frac{p}{2}\right)\frac{\partial^2 u_1}{\partial y^2}-\upsilon\left(1+\vartheta_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}\\ =\rho\left[\frac{\partial^2 u_1}{\partial t^2}-\Omega^2 u_1-2\Omega\,\frac{\partial u_2}{\partial t}\right]\end{aligned} \tag{1}$$

$$\begin{aligned}(\lambda+2\mu)\frac{\partial^2 u_2}{\partial y^2}+\left(\lambda+\mu+\frac{p}{2}\right)\frac{\partial^2 u_1}{\partial x\partial y}+\left(\mu-\frac{p}{2}\right)\frac{\partial^2 u_2}{\partial x^2}-\upsilon\left(1+\vartheta_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}\\ =\rho\left[\frac{\partial^2 u_2}{\partial t^2}-\Omega^2 u_2-2\Omega\frac{\partial u_1}{\partial t}\right]\end{aligned} \tag{2}$$

$$\begin{aligned}K^*\left(n^*+t_1\frac{\partial}{\partial t}\right)\nabla^2 T &= \rho C_E\left(n_1\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)T\\ &\quad+\upsilon T_0\left(n_1\frac{\partial}{\partial t}+n_0\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla\cdot\vec{u})\end{aligned} \tag{3}$$

$$t_{21} = \left(\mu+\frac{p}{2}\right)\frac{\partial u_2}{\partial x}+\left(\mu-\frac{p}{2}\right)\frac{\partial u_1}{\partial y} \tag{4}$$

$$t_{22} = -p+\lambda\frac{\partial u_1}{\partial x}+(\lambda+2\mu)\frac{\partial u_2}{\partial y}-\upsilon\left(1+\vartheta_0\frac{\partial}{\partial t}\right)T \tag{5}$$

Introducing dimensionless variables defined by,

$$\begin{aligned}&x_i'=\frac{\omega^*}{c_0}x_i \quad u_i'=\tfrac{\rho c_0\omega^*}{\upsilon T_0}u_i \quad t'=\omega^* t\\ &\tau_0'=\omega^*\tau_0 \quad \vartheta_0'=\omega^*\vartheta_0 \quad T'=\frac{T}{T_0}\\ &t_{ij}'=\frac{t_{ij}}{\upsilon T_0} \quad \Omega'=\tfrac{\Omega}{\omega^*} \quad p'=\frac{p}{\upsilon T_0}\end{aligned} \tag{6}$$

where

$$\omega^*=\rho C_E c_0^2/K^*,\ \rho c_0^2=\lambda+2\mu$$

in equations (1)–(3), we obtain the equations of motion in dimensionless form.

We define displacement potentials ϕ and ψ which are related to displacement components u_1 and u_2 as,

$$u_1=\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial y},\quad u_2=\frac{\partial\phi}{\partial y}+\frac{\partial\psi}{\partial x}, \tag{7}$$

in the resulting dimensionless equations, and then applying the Laplace and Fourier transform defined by,

$$\bar{f}(x,y,p) = \int_0^\infty f(x,y,t)e^{-st}dt \tag{8}$$

$$\tilde{f}(\xi\ ,y,p) = \int_{-\infty}^{\infty}\bar{f}(x,y,p)\,e^{i\xi x}dx, \tag{9}$$

we get, (after neglecting the primes),

$$\left[\frac{d^2}{dy^2} - \xi^2 + \Omega^2 - s^2\right]\tilde{\phi} + 2\Omega s\tilde{\psi} - (1 + \vartheta_0 s)\,\tilde{T} = 0 \tag{10}$$

$$\left[\frac{d^2}{dy^2} - \xi^2 + a_1\Omega^2 - a_1 s^2\right]\tilde{\psi} - 2\Omega a_1 s\tilde{\phi} = 0 \tag{11}$$

$$\left[\frac{d}{dy^2} - \xi^2 - s\left(\frac{n_1 + \tau_0 s}{n^* + t_1 s}\right)\right]\tilde{T} - \frac{n_1 + \tau_0 n_0 s}{n^* + t_1 s}\,(\varepsilon s)\left[\frac{d}{dy^2} - \xi^2\right]\tilde{\phi} = 0 \tag{12}$$

Eliminating $\tilde{\phi}$ and $\tilde{\psi}$, from equations (10)–(12) we obtain,

$$\left[\nabla^6 - A\nabla^4 + B\nabla^2 - C\right]\tilde{\phi} = 0 \tag{13}$$

where

$$\begin{aligned}
\nabla &= \frac{d}{dy}\\
a_1 &= \frac{\rho c_0^2}{\mu - \frac{p}{2}}\\
\in &= \frac{\upsilon^2 T_0}{\rho K^* \omega^*}\\
e_1 &= \xi^2 + s\left(\frac{n_1 + \tau_0 s}{n^* + t_1 s}\right)\\
e_2 &= \xi^2 - \Omega^2 + s^2\\
e_3 &= \varepsilon s\,(1 + \vartheta_0 s)\left(\frac{n_1 + n_0\tau_0 s}{n^* + t_1 s}\right)\\
e_4 &= \xi^2 - a_1\Omega^2 + a_1 s^2\\
A &= e_1 + e_2 + e_3 + e_4\\
B &= e_4\,(e_1 + e_2 + e_3) + e_1 e_2 + e_3\xi^2 + 4a_1\Omega^2 s^2\\
C &= e_4\left(e_1 e_2 + e_3\xi^2\right) + 4e_1 a_1 s^2\Omega^2
\end{aligned} \tag{14}$$

The solutions of equation (13) satisfying the radiation conditions that $\tilde{\phi}$, $\tilde{\psi}$, $\tilde{T} \to 0$ as $y \to \infty$ are,

$$\tilde{\phi} = D_1 e^{-\phi_1 y} + D_2 e^{-\phi_2 y} + D_3 e^{-\phi_3 y} \tag{15}$$

$$\tilde{\psi} = a_1^* D_1 e^{-\phi_1 y} + a_2^* D_2 e^{-\phi_2 y} + a_3^* D_3 e^{-\phi_3 y} \tag{16}$$

$$\tilde{T} = b_1^* D_1 e^{-\phi_1 y} + b_2^* D_2 e^{-\phi_2 y} + b_3^* D_3 e^{-\phi_3 y} \tag{17}$$

where ϕ_i^2 are the roots of equation (13) and a_i^*, b_i^* are coupling constants defined by ,

$$a_i^* = \frac{\phi_i^4 - (e_1 + e_2 + e_3)\,\phi_i^2 + \left(e_1 e_2 + e_3\xi^2\right)}{2\Omega s\,(\phi_i^2 - e_1)} \tag{18}$$

$$b_i^* = \varepsilon s\left(\frac{n_1 + n_0\tau_0 s}{n^* + t_1 s}\right)\left(\frac{\phi_i^2 - \xi^2}{\phi_i^2 - e_1}\right) \tag{19}$$

$$i = 1, 2, 3$$

4. Boundary conditions

The boundary conditions at the plane surface $y = 0$ subjected to an arbitrary normal source $F(x,t)$ are,

$$t_{22} = F(x,t) \qquad t_{21} = 0 \qquad T = 0 \tag{20}$$

Using equations (4)–(7) in the boundary conditions (20), we obtain the boundary conditions in the dimensionless form. On suppressing the primes and applying the Laplace and Fourier transform defined by (8) and (9) on the dimensionless boundary conditions and using (15)–(17), in the resulting transformed boundary conditions, we get the transformed expressions for displacement, force stress , and temperature distribution in a rotating generalized thermoelastic medium with hydrostatic initial stress as,

$$\tilde{u}_1 = \frac{\left(\sum\limits_{m=1}^{3} b_m \Delta_m e^{-\phi_m y}\right)}{\Delta} \tag{21}$$

$$\tilde{u}_2 = \frac{\left(\sum\limits_{m=1}^{3} l_m \Delta_m e^{-\phi_m y}\right)}{\Delta} \tag{22}$$

$$\tilde{t}_{21} = \frac{\left(\sum\limits_{m=1}^{3} q_m \Delta_m e^{-\phi_m y}\right)}{\Delta} \tag{23}$$

$$\tilde{t}_{22} = \frac{\left(\sum\limits_{m=1}^{3} r_m \Delta_m e^{-\phi_m y}\right)}{\Delta} \tag{24}$$

$$\tilde{T} = \frac{\left(\sum\limits_{m=1}^{3} b_m^* \Delta_m e^{-\phi_m y}\right)}{\Delta} \tag{25}$$

where

$$\Delta = \left[\frac{r_1\Delta_1 + r_2\Delta_2 + r_3\Delta_3}{p - \tilde{F}(\xi,s)}\right] \qquad \Delta_1 = \left(p - \tilde{F}(\xi,s)\right)[q_2 b_3^* - b_2^* q_3]$$

$$\Delta_2 = -\left(p - \tilde{F}(\xi,s)\right)[q_1 b_3^* - b_1^* q_3] \qquad \Delta_3 = \left(p - \tilde{F}(\xi,s)\right)[q_1 b_2^* - b_1^* q_2]$$

$$q_i = -\left[\frac{\left(\mu - \frac{\upsilon T_0 p}{2}\right) a_i^* \phi_i^2}{\rho c_0^2} + \frac{\left(\mu + \frac{\upsilon T_0 p}{2}\right) a_i^* \xi^2}{\rho c_0^2} - \frac{2i\xi\mu\phi_i}{\rho c_0^2}\right] \tag{26}$$

$$r_i = \phi_i^2 - \frac{\lambda\xi^2}{\rho c_0^2} - \frac{2i\xi(\lambda+\mu) a_i^* \phi_i}{\rho c_0^2} - (1 + \vartheta_0 s) b_i^*, \qquad i = m = 1,2,3$$

$$b_i = a_i^* \phi_i - i\xi \qquad l_i = -\left(a_i^* i\xi + \phi_i\right)$$

$$\tilde{F}(\xi,s) = \frac{\sigma_1}{p^2 t_0}\left(1 - e^{-st_0}\right)$$

5. Particular cases

1. Neglecting angular velocity (i.e. $\vec{\Omega} = 0$), we obtain transformed components of displacement, stress forces and temperature distribution in a non–rotating generalized thermoelastic medium with hydrostatic initial stress as,

$$\tilde{u}_1 = \frac{\left(\sum_{m=1}^{3} b'_m \Delta_m^{(1)} e^{-\phi'_m y}\right)}{\Delta^{(1)}} \tag{27}$$

$$\tilde{u}_2 = \frac{\left(\sum_{m=1}^{3} l'_m \Delta_m^{(1)} e^{-\phi'_m y}\right)}{\Delta^{(1)}} \tag{28}$$

$$\tilde{t}_{21} = \frac{\left(\sum_{m=1}^{3} q'_m \Delta_m^{(1)} e^{-\phi'_m y}\right)}{\Delta^{(1)}} \tag{29}$$

$$\tilde{t}_{22} = \frac{\left(\sum_{m=1}^{3} r'_m \Delta_m^{(1)} e^{-\phi'_m y}\right)}{\Delta^{(1)}} \tag{30}$$

$$\tilde{T} = \frac{\left(\sum_{m=1}^{3} b\prime^*_m \Delta_m^{(1)} e^{-\phi'_m y}\right)}{\Delta^{(1)}} \tag{31}$$

where

$$\begin{aligned}
\Delta^{(1)} &= r'_3 \frac{\Delta_3^{(1)}}{p - \tilde{F}(\xi, s)} - q'_3 \left(r'_1 b'^*_2 - b'^*_1 r'_2\right) \\
\Delta_1^{(1)} &= -\left[p - \tilde{F}(\xi, s)\right] b'^*_2 q'_3 \\
\Delta_2^{(1)} &= \left[p - \tilde{F}(\xi, s)\right] b'^*_1 q'_3 \\
\Delta_3^{(1)} &= \left[p - \tilde{F}(\xi, s)\right] \left[q'_1 b'^*_2 - b'^*_1 q'_2\right] \\
r'_{1,2} &= \phi'^2_{1,2} - \frac{\lambda \xi^2}{\rho c_0^2} - (1 + \vartheta_0 p)\, b'^*_{1,2} \\
r'_3 &= -\frac{2i\xi (\lambda + \mu)\, \phi'_3}{\rho c_0^2} \\
b'_{1,2} &= -i\xi \qquad b'_3 = \phi'_3 \\
q'_{1,2} &= \frac{2i\xi\mu\phi'_{1,2}}{\rho c_0^2} b'^*_i = \frac{\phi'^2 - e'_2}{1 + \vartheta_0 s} \\
q''_3 &= -\left[\frac{\left(\mu - \frac{\upsilon T_0 p}{2}\right)\phi'^2_3}{\rho c_0^2} + \frac{\left(\mu + \frac{\upsilon T_0 p}{2}\right)\xi^2}{\rho c_0^2}\right] \\
l'_{1,2} &= -\phi'_{1,2} \qquad l'_3 = -i\xi
\end{aligned} \tag{32}$$

$$\begin{aligned} A_1 &= e_1 + e_2' + e_3 \qquad & B_1 &= \left(e_1 e_2' + e_3 \xi^2\right) \\ \phi_{1,2}' &= \frac{A_1 \pm \sqrt{A_1 - 4B_1}}{2} \\ \phi_3'^2 &= \xi^2 + \frac{\rho c_0^2 s^2}{\left(\mu - \frac{vT_0 p}{2}\right)} & e_2' &= \xi^2 + s^2 \end{aligned}$$

2. Neglecting both angular velocity and hydrostatic initial stress (i.e. $\vec{\Omega} = p = 0$), we get the expressions for displacement, force stresses and temperature distribution in non–rotating thermoelastic medium as,

$$\tilde{u}_1 = \frac{\left(\sum_{m=1}^{3} b_m'' \Delta_m^{(2)} e^{-\phi_m'' y}\right)}{\Delta^{(2)}} \tag{33}$$

$$\tilde{u}_2 = \frac{\left(\sum_{m=1}^{3} l_m'' \Delta_m^{(2)} e^{-\phi_m'' y}\right)}{\Delta^{(2)}} \tag{34}$$

$$\tilde{t}_{21} = \frac{\left(\sum_{m=1}^{3} q_m'' \Delta_m^{(2)} e^{-\phi_m'' y}\right)}{\Delta^{(2)}} \tag{35}$$

$$\tilde{t}_{22} = \frac{\left(\sum_{m=1}^{3} r_m'' \Delta_m^{(2)} e^{-\phi_m'' y}\right)}{\Delta^{(2)}} \tag{36}$$

$$\tilde{T} = \frac{\left(\sum_{m=1}^{2} b_m''^{*} \Delta_m^{(2)} e^{-\phi_m'' y}\right)}{\Delta^{(2)}} \tag{37}$$

where

$$\begin{aligned} \Delta^{(2)} &= -\left[\frac{r_3'' \Delta_3^{(2)}}{\tilde{F}(\xi, s)} + q_3'' \left(r_1'' b_2''^{*} - b_1''^{*} r_2''\right)\right] \\ \Delta_1^{(2)} &= \tilde{F}(\xi, s)\, b_2''^{*} q_3'' \\ \Delta_2^{(2)} &= -\frac{\Delta_1^{(2)} b_1''^{*}}{b_2''^{*}} \\ \Delta_3^{(2)} &= -\tilde{F}(\xi, s)\left[q_1'' b_2''^{*} - b_1''^{*} q_2''\right] \\ A_2 &= e_1 + e_2' + e_3 \\ B_2 &= e_1 e_2' + e_3 \xi^2 \\ \phi_{1,2}''^2 &= \frac{A_2 \pm \sqrt{A_2^2 - 4B_2}}{2} \\ \phi_3''^2 &= \xi^2 + \frac{s^2 \rho c_0^2}{\mu} \end{aligned}$$

$$
\begin{aligned}
b_{1,2}^{\prime\prime *} &= \frac{\phi_{1,2}^{\prime\prime 2} - e_2^{\prime}}{1+\vartheta_0 s} \\
r_{1,2}^{\prime\prime} &= \phi_{1,2}^{\prime\prime 2} - \frac{\lambda \xi^2}{\rho c_0^2} - (1+\vartheta_0 p)\, b_{1,2}^{\prime\prime *} \\
r_3^{\prime\prime} &= -2i\xi\phi_3^{\prime\prime}\left(\frac{\lambda+\mu}{\rho c_0^2}\right) \\
q_{1,2}^{\prime\prime} &= \frac{2i\xi\mu\phi_{1,2}^{\prime\prime}}{\rho c_0^2} \\
s_3^{\prime\prime} &= -\frac{\left(\phi_3^{\prime\prime 2}+\xi^2\right)\mu}{\rho c_0^2} \\
b_{1,2}^{\prime\prime} &= -i\xi \\
b_3^{\prime\prime} &= \phi_3^{\prime\prime} \\
l_{1,2}^{\prime\prime} &= -\phi_{1,2}^{\prime\prime} \\
l_3^{\prime\prime} &= -i\xi
\end{aligned}
$$

3. Neglecting hydrostatic initial stress (i.e. $p = 0$), we obtain transformed components of displacement, stress forces and temperature distribution in a rotating generalized thermoelastic medium.

6. Types of sources

6.1. *Concentrated source*

For a concentrated source we take

$$F(x,t) = \delta(x)\,\delta(t)$$

such that

$$\tilde{F}(\xi,s) = 1 \tag{38}$$

6.2. *Distributed sources*

6.2.1. *Uniformly distributed source*

The solution due to a uniformly distributed source in normal direction is obtained by setting

$$F(x,t) = \phi(x)\,\delta(t)$$

and

$$\tilde{F}(\xi,s) = \tilde{\phi}(\xi)$$

where

$$\phi(x) = \begin{cases} 1 & \text{if} \quad |x| \le a \\ 0 & \text{if} \quad |x| > a \end{cases} \tag{39}$$

in equation (20) The Fourier transform with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width 2a applied at the origin of the coordinate system $(x = y = 0)$ becomes

$$\tilde{\phi}(\xi) = \left[\frac{2\sin(\xi)}{\xi}\right], \quad \xi \neq 0 \tag{40}$$

6.2.2. *Linearly distributed source*

The solution due to a linearly distributed source in normal direction is obtained by substituting

$$\phi\left(x\right)=\begin{cases}1-\frac{|x|}{a} & \text{if} \quad |x|\leq a\\ 0 & \text{if} \quad |x|>a\end{cases} \tag{41}$$

in equation (20). The Fourier transform of $\phi\left(x\right)$ is

$$\tilde{\phi}\left(\xi\right)=\frac{2\left[1-\cos\left(\xi\right)\right]}{\xi^{2}} \tag{42}$$

The expressions for the components of displacement, force stress and temperature distribution are obtained as in equations (21)-(25), (27)–(31) and (33)–(37), by replacing $\tilde{\phi}\left(\xi\right)$ as $[2\sin\left(\xi\right)/\xi]$ and $2[1-\cos\left(\xi\right)]/\xi^{2}$,in the case of a uniformly distributed force and linearly distributed force in equations (26), (32) and (38) for load in normal direction.

6.3. *Moving source*

In case of a source moving along the $x-$axis with uniform velocity U at the plane surface $y=0$, we have

$$F\left(x,t\right)=H\left(t\right)\delta\left(x-Ut\right),$$

where

$$\tilde{F}\left(\xi,s\right)=\frac{1}{s-i\xi U}. \tag{43}$$

7. Numerical results

With a view to illustrating the analytical procedure presented earlier, we now consider a numerical example for which computational results are given. The results depict the variations of temperature, displacement and stress fields in the context of G–L theory. For this purpose we take the following values of physical constants as

$$\begin{aligned}
E &= 6.9 \times 10^{11}\,[\text{dyne/cm}^2] \\
\sigma &= 0.33 \\
\rho &= 2.7\,[\text{gm/cm}^3] \\
C_E &= 0.236\,[\text{cal/gms}^0\text{C}] \\
K^* &= 0.492\,[\text{cal/cm}\,\text{s}^0\text{C}] \\
\nu &= \frac{\alpha}{\rho K_T} \\
\alpha &= 0.01 \\
K_T &= 0.5 \\
T_0 &= 20\,[^0\text{C}] \\
\mu &= \frac{E}{2\eta\,(1+\sigma)} \\
\lambda &= \frac{E\sigma}{\eta\,(1+\sigma)\,(1-2\sigma)} \\
\eta &= 1
\end{aligned}$$

corresponds to isotropic elastic medium.

The computations are carried out on the surface $y = 1.0$ at $t = 1.0$. The graphical results for normal displacementu_2, normal force stress t_{22} and temperature distribution $T$are shown in Figs 1–12. with $\Omega = 0.5$ and $p = 2.0$ for a

- Generalized thermoelastic medium with hydrostatic initial stress and rotation (GTESHR) by solid line (——).

- Generalized thermoelastic medium with hydrostatic initial stress and without rotation (GTESHWR) by dashed line (-----) .

- Generalized thermoelastic medium with rotation and without hydrostatic initial stress (GTESR) by solid line with centered symbol ($*$—$*$—$*$).

- Generalized thermoelastic solid without rotation and without hydrostatic initial stress (GTESWR) by dashed line with centered symbol ($*---*---*$).

  These graphical results represent the solutions obtained by using generalized theory with two relaxation times (GL–theory by taking $\tau_0 = 0.03,\ \vartheta_0 = 0.05$.)

8. Inversion of the transform

The transformed displacements, microrotation and stresses are functions of y, the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $\tilde{f}\,(\xi, y, s)$. To get the function in the physical domain, first we first invert the Fourier transform and then Laplace transform by using the method applied by Sharma and Kumar [33].

9. Special cases of thermoelastic theory

1. The equations of the coupled thermoelasticity (C–T theory) for a rotating media are obtained when

$$n^* = n_1 = 1, \quad t_1 = \tau_0 = \vartheta_0 = 0 \tag{44}$$

2. For Lord–Shulman (L–S theory)

$$n^* = n_1 = n_0 = 1, \quad t_1 = \vartheta_0 = 0 \quad \tau_0 > 0 \tag{45}$$

3. For Green–Lindsay (G–L theory),

$$n^* = n_1 = 1 \quad n^* = 0 \quad t_1 = 0 \quad \vartheta_0 \geq \tau_0 > 0 \tag{46}$$

where ϑ_0, τ_0 are the two relaxation times .

4. The equations of the generalized thermoelasticity for a rotating media, without energy dissipation (the linearlized GN theory of type II) are obtained when

$$n^* > 0 \quad n_1 = 0 \quad n_0 = 1 \quad t_1 = \vartheta_0 = 0 \quad \tau_0 = 1 \tag{47}$$

Equations (1) and (2) is the same and equation (3) takes the form

$$K^* \nabla^2 T = \rho C_E \frac{\partial^2 T}{\partial t^2} + \upsilon T_0 \frac{\partial^2 e}{\partial t^2} \tag{48}$$

where n^* is constant which has the dimension of $\left[\frac{1}{s}\right]$ and

$$n^* k^* = K\prime = C_E (\lambda + 2\,\mu)/4$$

is a characteristic constant of this theory.

10. Discussions

10.1. *Concentrated force*

The variations of normal displacement for a generalized thermoelastic medium in the absence of hydrostatic initial stress and rotation are highly oscillatory in nature. As a result of which, the values of normal displacement for the thermoelastic medium (with hydrostatic initial stress and/or rotation) lie in the range $-0.2 \leq u_2 \leq 0.2$. Also the magnitude of oscillations decrease with increase in horizontal distance. These variations of normal displacement are shown in Fig. 1.

When the medium is not rotating, the values of normal force stress lie in a short range but when the thermoelastic medium is rotating with some angular velocity, the values of normal force stress decrease sharply initially and then oscillates with horizontal distance. The magnitude of sharpness in the initial range is more for a thermoelastic medium without hydrostatic initial stress. These variations of normal force stress are shown in Fig. 2. It is observed form Fig. 3 that the variations of temperature distribution are more oscillatory in nature for a non–rotating (with or without hydrostatic initial stress) medium. The values of temperature distribution are very large for a thermoelastic medium without rotation and without hydrostatic initial stress. These values have been demagnified by a certain quantity to show the comparison with other mediums.

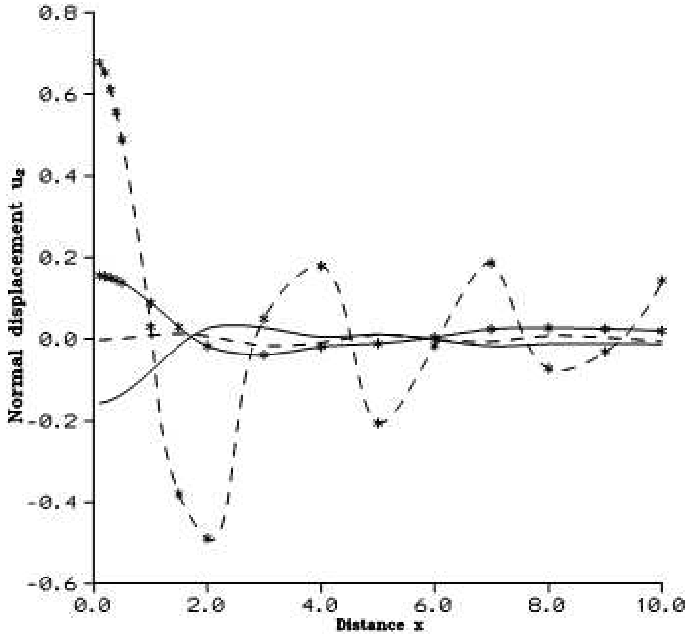

Figure 1 Variation of Normal displacement $u_2$with horizontal distance x due to concentrated force

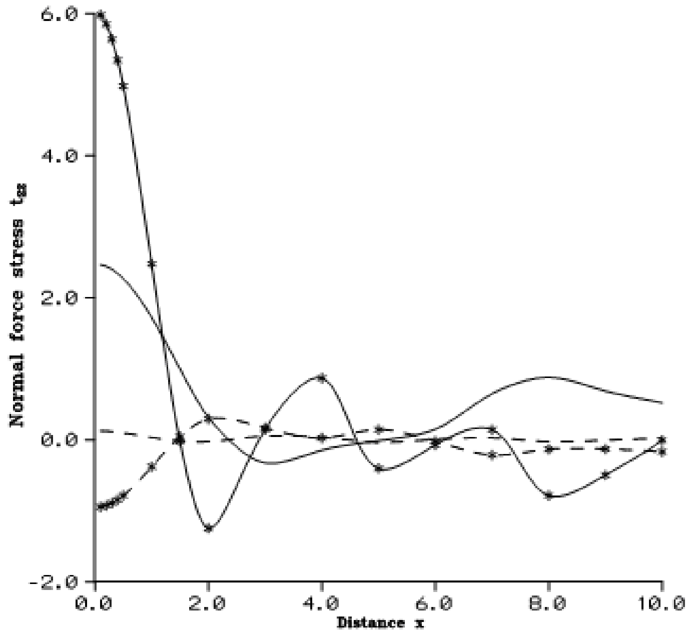

Figure 2 Variation of Normal force stress $t_{22}$with horizontal distance x due to concentrated force

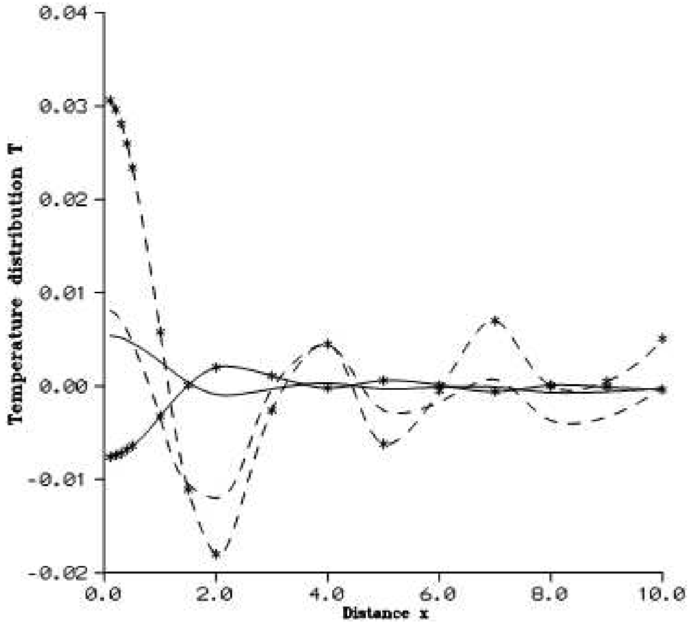

Figure 3 Variation of Temperature distribution T with horizontal distance x due to concentrated force

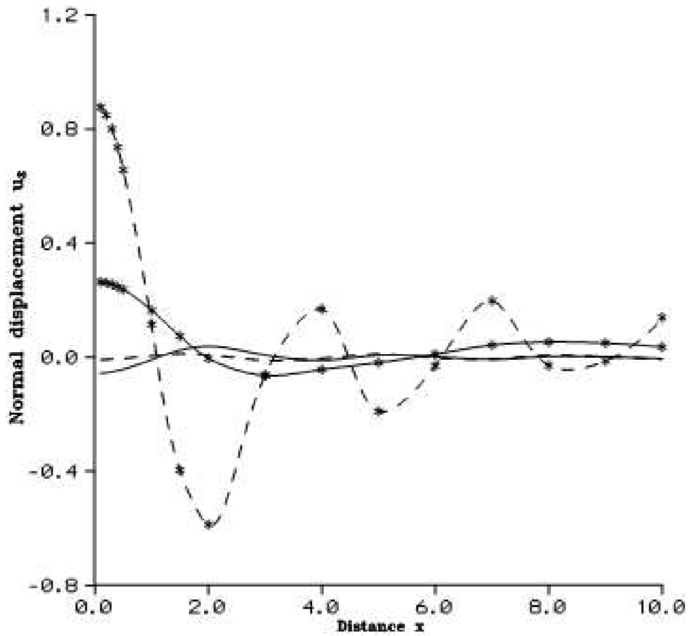

Figure 4 Variation of Normal displacement u_2 with horizontal distance x due to uniformly distributed force

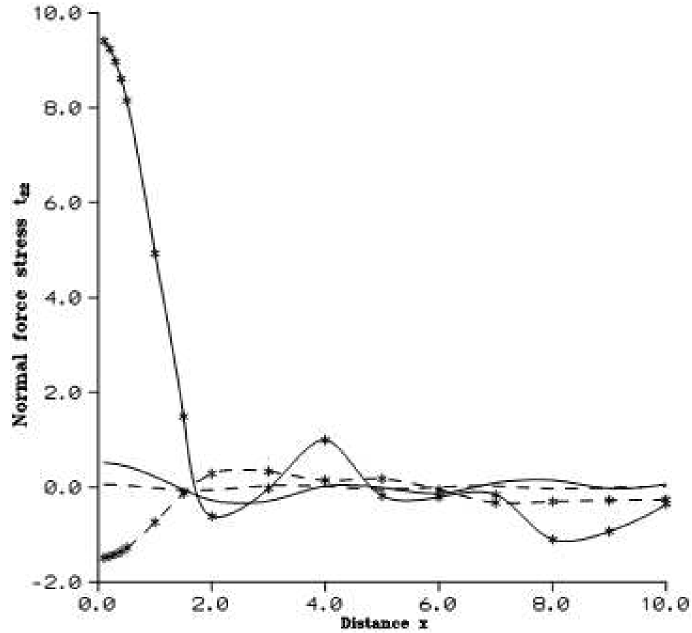

Figure 5 Variation of Normal force stress t_{22} with horizontal distance x due to uniformly distributed force

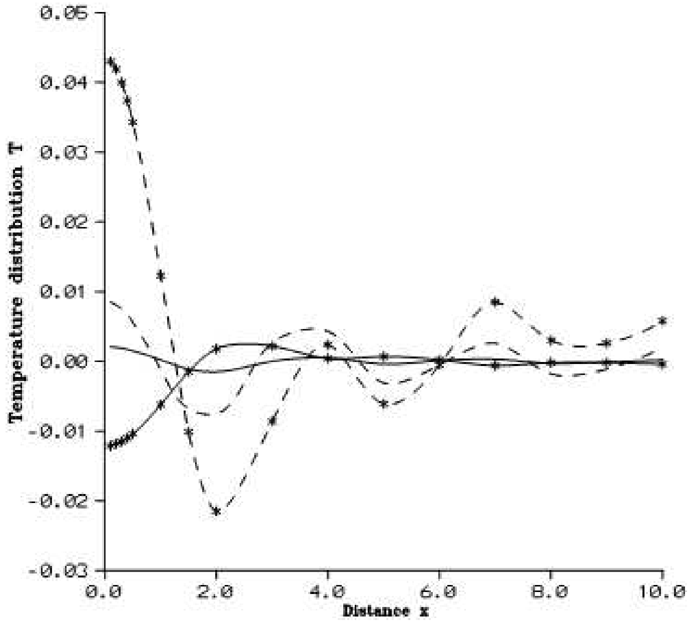

Figure 6 Variation of Temperature distribution T with horizontal distance x due to uniformly distributed force

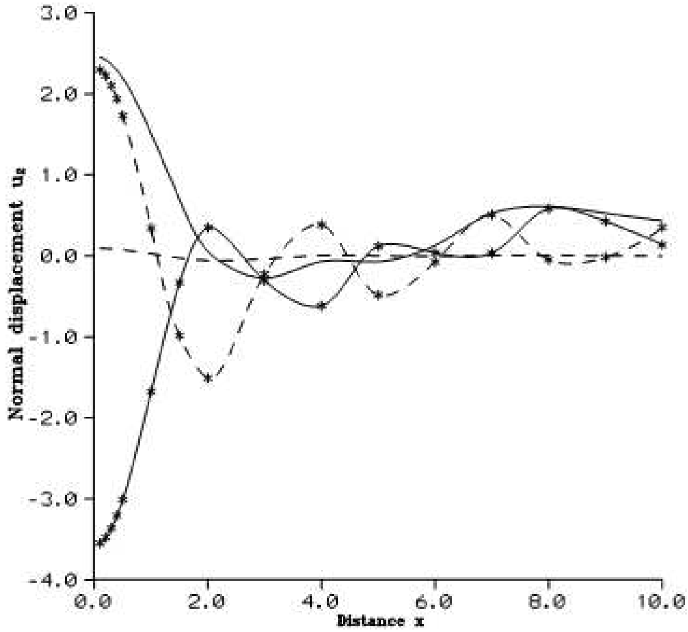

Figure 7 Variation of Normal displacement u_2 with horizontal distance x due to a moving force

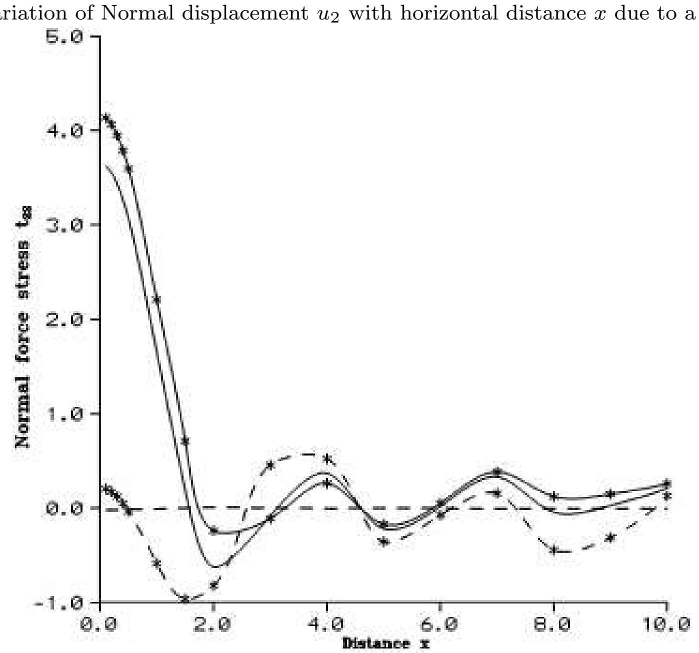

Figure 8 Variation of Normal force stress t_{22} with horizontal distance x due to a moving force

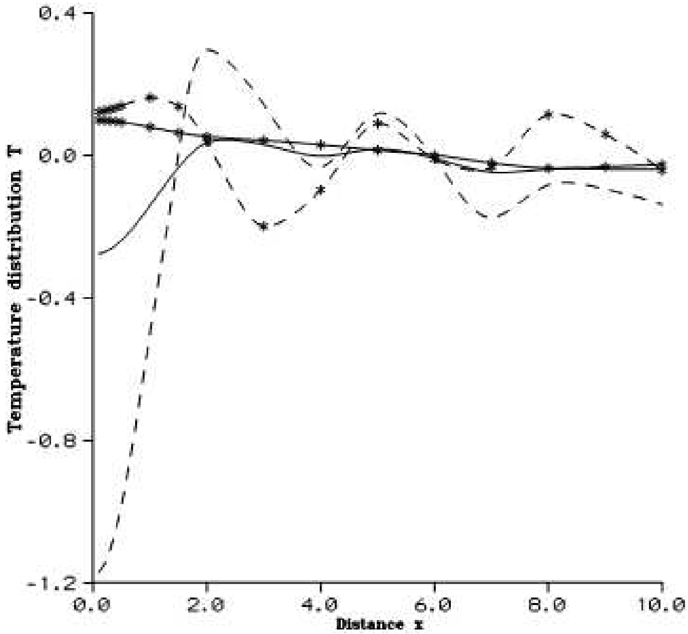

Figure 9 Variation of Temperature distribution T with horizontal distance x due to a moving force

10.2. *Uniformly distributed force*

The variations of all the quantities are similar in nature to the variations obtained in case of concentrated force with difference in magnitude. The variations of normal displacement, normal force stress and temperature distribution on application of uniformly force are shown in Figs 4–6. respectively.

10.3. *Moving force*

It is interesting to see that the variations of all the quantities are more uniform in nature when a force of constant magnitude is moving along the surface of solid. In this case also, the values of temperature distribution have been demagnified for a thermoelastic medium without angular velocity and without hydrostatic initial stress. The values of normal displacement and normal force stress are close to zero for a generalized thermoelastic medium without rotation and with hydrostatic initial stress. The variations of normal displacement, normal force stress and temperature distribution in case of moving force may be observed in Figs 7–9. respectively.

11. Conclusion

The variations of all the quantities are similar in nature when a concentrated force or uniformly distributed force is applied on the surface of solid. However, these variations are more uniform in nature when a force is moving along the surface of solid. The effect of rotation and hydrostatic initial stress are also observed on all the quantities.

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Nomenclature

λ, μ	Lame's constants
ρ	Density
$\vec{u}$	Displacement vector
t_{ij}	Stress tensor
$\tau_0, \ \vartheta_0$	Thermal relaxation times
$\upsilon = (3\lambda + 2\mu)\,\alpha_t$	Linear thermal expansion
$e = \mathrm{div}\vec{u}$	Dilatation
K^*	Coefficient of thermal conductivity
C_E	Specific heat
p	Initial pressure
η	Initial stress parameter
E	Young's modulus
σ	poisson ratio