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On the Effectiveness of Heat Generation/Absorption on Heat Transfer in a Stagnation Point Flow of a Micropolar Fluid over a Stretching Surface

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The heat transfer in a steady laminar stagnation point flow of an incompressible non–Newtonian micropolar fluid impinging on a permeable stretching surface with heat generation or absorption is investigated. Numerical solution for the governing nonlinear momentum and energy equations is obtained. The effect of the characteristics of the non–Newtonian fluid, the surface stretching velocity, and the heat generation/absorption coefficient on the heat transfer is presented.

Keywords: Sstagnation point flow, stretching sheet, non–Newtonian fluid, heat transfer

1. Introduction

The two dimensional flow of a fluid near a stagnation point is a classical problem in fluid mechanics. It was first examined by Hiemenz who demonstrated that the Navier–Stokes equations governing the flow can be reduced to an ordinary differential equation of third order using similarity transformation [1]. Owing to the nonlinearities in the reduced differential equation, no analytical solution is available and the nonlinear equation is usually solved numerically subject to two–point boundary conditions, one of which is prescribed at infinity.

Flow of an incompressible viscous fluid over stretching surface has important applications in polymer industry. For instance, a number of technical processes concerning polymers involves the cooling of continuous strips (or filaments) extruded from a die by drawing them through a stagnant fluid with controlled cooling system and in the process of drawing these strips are sometimes stretched. The quality of the final product depends on the rate of heat transfer at the stretching surface. Crane [2] gave a similarity solution in closed analytical form for steady two–dimensional incompressible boundary layer flow caused by the stretching of a sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point. Chiam [3] has combined the steady boundary layer problems of flow over a stretching sheet, originally studied by Crane [2], and the Hiemenz two–dimensional stagnation–point flow into a new boundary layer problem of steady two–dimensional point Newtonian fluid flow over a stretching sheet. Subsequently, Chiam [4] extended his earlier analysis to include heat transfer with variable conductivity. Temperature distribution in the steady plane stagnation–point flow of a viscous fluid towards a stretching surface was investigated by Mahapatra and Gupta [5]. Steady flow of a non–Newtonian micropolar fluid past a stretching sheet was investigated by Nasar et al. [6] with zero vertical velocity at the surface.

The purpose of the present paper is to study the heat transfer in a steady laminar stagnation point flow of an incompressible non–Newtonian micropolar fluid impinging on a permeable stretching surface with heat generation or absorption. The wall and stream temperatures are assumed to be constants. A numerical solution is obtained for the governing momentum and energy equations using finite difference approximations which takes into account the asymptotic boundary conditions. The numerical solution computes the flow and heat characteristics for the whole range of the non–Newtonian fluid characteristics, the stretching velocity, the heat generation/absorption coefficient and the Prandtl number.

2. Formulation of the Problem

Consider the two–dimensional stagnation point flow of an incompressible non–Newtonian micropolar fluid near a stagnation point at a surface coinciding with the plane $y = 0$, the flow being in a region $y > 0$. Two equal and opposing forces are applied along the x–axis so that the surface is stretched in the x–axis direction keeping the origin fixed such that the x–component of the velocity varies linearly along it. Considering the case of vanishing of anti–symmetric part of the stress tensor, which denotes weak concentration of microelements [6], the final form of the equations governing the flow in the boundary layer of a steady, laminar and incompressible micropolar fluid are given by [6],

$$\left(1 + \frac{K}{2}\right) f''' + ff'' - f'^2 + C^2 = 0 \tag{1}$$

subject to the boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = C \tag{2}$$

where K is the material parameter, C is the stretching parameter, f is a non–dimensional scaled variable related to velocity, and primes denote differentiation with respect to dimensionless vertical distance η. If we now take

$$f(\bar{\eta}) = (1 + K/2)^{1/2} h(\bar{\eta})$$

$$\bar{\eta} = (1 + K/2)^{-1/2} \eta$$

Eq. (1) reduces, respectively, to (the bar will be dropped for convenience)

$$f''' + ff'' - f'^2 + C^2 = 0 \tag{3}$$

Using the boundary layer approximations and neglecting the dissipation, the final form of the energy equation and the appropriate boundary conditions including the heat generation/absorption term is given by [5],

$$\theta'' + \Pr f\theta' + \Pr B\theta = 0 \tag{4}$$

$$\theta(0) = 1, \quad \theta(\infty) = 0 \tag{5}$$

where

$\Pr = \mu c_p/k$	is the Prandtl number,
$B = Q/b\rho c_p$	is the dimensionless heat generation/absorption coefficient,
θ	is the dimensionless temperature,
ρ	is the density of the fluid,
c_p	is the specific heat capacity at constant pressure of the fluid,
k	is the thermal conductivity of the fluid,
Q	is the volumetric rate of heat generation/absorption,
b	is a constant related to the stretching velocity of the wall.

It should be pointed out that in our work we have added heat generation and absorption in contrast to the work of Chiam [4] and Mahapatra and Gupta [5] where this term was not taken into consideration. Also, we choose constant wall temperature condition as given by Mahapatra, A.S. Gupta [5], while constant and variable wall temperature distributions are considered.

The flow Eqs (2) and (3) are decoupled from the energy Eqs (4) and (4), and need to be solved before the latter can be solved. The flow Eq. (3) constitutes a non–linear, non–homogeneous boundary value problem (BVP). In the absence of an analytical solution of a problem, a numerical solution is indeed an obvious and natural choice. The flow Eqs (2) and (3) are solved numerically using finite difference approximations.

The energy Eq. (4) is a linear second order ordinary differential equation with variable coefficient, $f(\eta)$, which is known from the solution of the flow Eqs (2) and (3) and the Prandtl number Pr is assumed constant. Equation (4) is solved numerically under the boundary condition (5) using central differences for the derivatives and Thomas' algorithm for the solution of the set of discritized equations. The resulting system of equations has to be solved in the infinite domain $0 < \eta < \infty$. A finite domain in the η–direction can be used instead with η chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. Grid–independence studies show that the computational domain $0 < \eta < \eta_\infty$ can be divided into intervals each is of uniform step size which equals 0.02. The value $\eta_\infty = 10$ was found to be adequate for all the ranges of parameters studied here.

3. Results and Discussion

Fig. 1. presents the profile of temperature θ for various values of C and K and for $\Pr = 0.7$ and $B = 0.1$. It is clear that increasing C decreases θ and its effect on θ becomes more apparent for smaller values of K. The figure indicates that

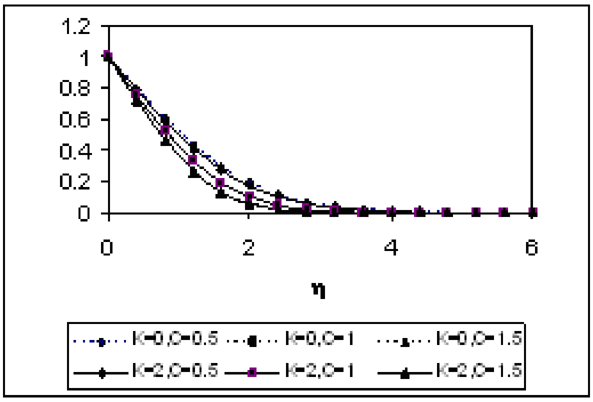

Figure 1 Effect of the parameters C and K on the profile of $\mathrm{Pr} = 0.7$, $B = 0.1$

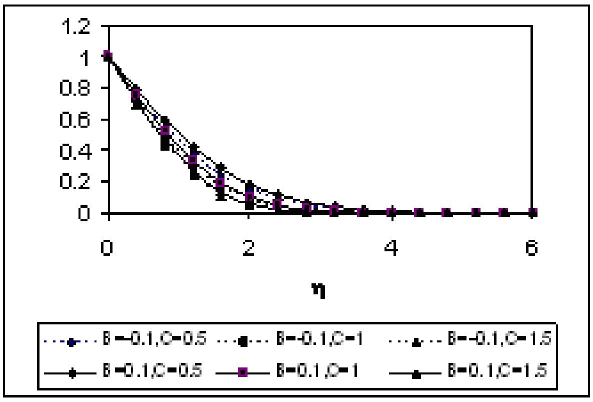

Figure 2 Effect of the parameters C and B on the profile of $K = 1$, $\mathrm{Pr} = 0.7$

the thermal boundary layer thickness decreases when C increases. Increasing K decreases θ for all C and its effect is more clear for smaller C. Fig. 2. presents the temperature profiles for various values of C and B and for $K = 1$ and $\mathrm{Pr} = 0.7$. Increasing B increases the temperature θ and the boundary layer thickness. The effect of B on θ is more pronounced for smaller C. Fig. 3. presents the effect of the parameter K on the profile of the temperature θ for C=1.5, Pr=0.7 and $B = 0.1$. It is shown in Fig. 3 that increasing K slightly increases θ for all distances.

Table 1 present the variation of the dimensionless heat transfer rate at the wall $\theta'(0)$ for various values of C and K and for $\mathrm{Pr} = 0.7$ and B=0.1. It is shown that, increasing C or K increases $-\theta'(0)$. Table 2 presents the effect of the parameters C and B on $-\theta'(0)$ for K=1 and $\mathrm{Pr} = 0.7$. Increasing C increases $-\theta'(0)$ for all B. But, increasing B decreases $-\theta'(0)$ for all C.

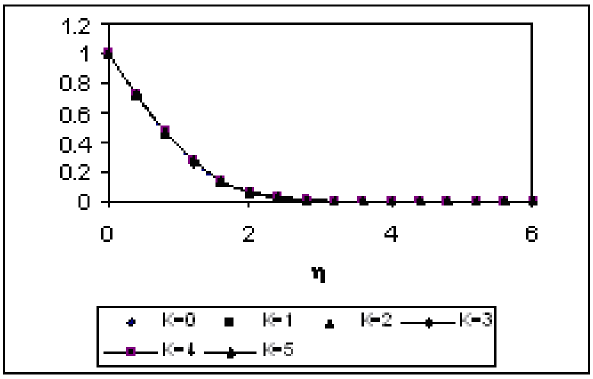

Figure 3 Effect of the parameter K on the profile of $C = 1.5$, $\Pr = 0.7$, $B = 0.1$

Table 1 Variation of the rate of heat transfer at the wall $-\theta'(0)$ with C and K $(\Pr = 0.7, B = 0.1)$

K	$C = 0.1$	$C = 0.2$	$C = 0.5$	$C = 1$	$C = 1.1$	$C = 1.2$	$C = 1.5$
0	0.3913	0.4254	0.5089	0.6201	0.6399	0.6589	0.7136
1	0.4295	0.4540	0.5221	0.6201	0.6379	0.6553	0.7049
2	0.4538	0.4731	0.5313	0.6201	0.6366	0.6527	0.6989

4. Conclusions

The heat transfer in a steady laminar stagnation point flow of an incompressible non–Newtonian micropolar fluid impinging on a permeable stretching surface with heat generation/ absorption was investigated. A numerical solution for the governing nonlinear momentum and energy equations was obtained which allows the computation of the flow and heat transfer characteristics for various values of the non–Newtonian parameter K, the stretching velocity C, and the heat generation/absorption coefficient B. The results indicate that increasing the stretching velocity decreases the temperature as well as the thermal boundary layer thickness. The effect of the stretching parameter on temperature is more apparent for smaller values of the non–Newtonian parameter. The effect of the heat generation/absorption parameter B on the rate of heat transfer at the wall becomes more apparent for smaller C.

Table 2 Variation of the rate of heat transfer at the wall $-\theta'(0)$ with C and B $(K = 1, \Pr = 0.7)$

B	$C = 0.1$	$C = 0.2$	$C = 0.5$	$C = 1$	$C = 1.1$	$C = 1.2$	$C = 1.5$
-0.1	0.5745	0.5885	0.6351	0.7127	0.7277	0.7424	0.7853
0	0.5072	0.5252	0.5807	0.6676	0.6839	0.6998	0.7459
0.1	0.4295	0.4540	0.5221	0.6201	0.6379	0.6553	0.7049

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