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Effect of Rotation in Case of 2–D Problem of the Generalized Thermo–viscoelasticity with Two Relaxation Times

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In this paper, a two–dimensional problem of thermoviscoelasticity with two relaxation times when the entire medium rotates with a uniform angular velocity is studied. The normal mode analysis is used to obtain the exact expressions for the temperature, displacement and thermal stress components. The resulting formulation is applied to the case of a thick plate subjected to a time–dependent heat source on each face. Numerical results are given and illustrated graphically. Comparisons are made with the results predicted by the coupled theory and with the theory of generalized thermoelasticity with two relaxation times in the absence of rotation and for different values of time.

Keywords: Thermo–viscoelasticity, rotation, two relaxation times, normal mode analysis

1. Introduction

The theory of thermo–elasticity with thermal relaxation times was proposed by Lord and Shulman [1] and Green and Lindsay [2]. These theories have been developed by introducing one or two relaxation times in the thermoelastic theory, with an aim to eliminate the paradox of infinite speed for the propagation of thermal signals. The Lord and Shulman model itself is based on a modified Fourier's law, but the Green and Lindsay model admits second sound even without violating classical Fourier' law. The two theories are structurally different from one another, and one cannot be obtained as a particular case of the other. Various problems characterizing these theories have been investigated, and reveal some interesting phenomena. Brief reviews of this topic have been reported by Chandrasekharaiah [3], [4]. The theory of generalized thermoelasticity with two relaxation times was first introduced by Müller [5]. Green and Laws [6] then introduced a more explicit version. Dhaliwal and Rokne [7] solved the thermal shock problem.

The theory of thermovisco–elasticity and the solutions of some boundary value problems of thermovisco–elasticity were solved by Ilioushin and Pobedria [8]. Drozdov [9] derived a constitutive model in thermovisco–elasticity which accounts for changes in elastic moduli and relaxation times. Existence of a solution for a non-linear system in thermo–viscoelasticity is proved by Blanchard [10]. Results of important experiments determining the mechanical properties of viscoelastic materials were involved in Koltunov [11]. Ezzat and Othman [12] have established the model of two–dimensional equations of generalized magneto–thermoelasticity with two relaxation times in a medium of perfect conductivity.

Using the Green and Lindsay theory, Agarwal [13] studied thermo–elastic plane wave propagation in an infinite non–rotating medium. In a paper by Schoenberg and Censor [14] the propagation of plane harmonic waves in a rotating elastic medium without a thermal field has been studied. Roy Choudhuri [15] has studied the propagation of harmonically time–dependent thermo–elastic plane waves of assigned frequency in finite rotating media. Othman [16] has studied the effect of rotation on plane waves in generalized thermo–elasticity with two relaxation times. Ezzat et al. [17] introduced the state space approach to two–dimensional problems of generalized thermo—viscoelasticity with two relaxation times. Othman et al. [18] applied the normal mode analysis to a two–dimensional generalized thermo–viscoelastic plane wave with two relaxation times without rotation. Recently Othman [19] studied the problem of two–dimensional electro–magneto–thermo–viscoelasticity based on the Lord–Shulman theory for a thermally and electrically conducting half–space solid whose surface is subjected to a thermal shock.

In this paper, the normal mode analysis is used to study two–dimensional problem of thermo–viscoelasticity with two relaxation times under the effect of rotation in the context of linearized theory of Green and Lindsay. The exact expressions for temperature, displacement and stress components are obtained.

2. Formulation of the problem

Consider a slowly moving isotropic and homogeneous visco–elastic medium. The medium is rotating uniformly with an angular velocity $\mathbf{\Omega} = \Omega\,\mathbf{n}$, where $\mathbf{n}$ is a unit vector representing the direction of the axis of rotation. We shall study only the simplest case of the two–dimensional problem. We assume that all quantities will be functions of the time variable t and of the coordinates x and y. Thus the waves are propagated only in the xy–plane. Thus, the displacement vector will have the components $u = u(x, y, t)$, $v = v(x, y, t)$ and $w = 0$.

The generalized equation of heat conduction in the context of Green and Lindsay's theory in the absence of heat sources is given by

$$kT_{,ii} = \rho C_E(\dot{T} + \tau_o\ddot{T}) + \gamma T_o\dot{e} \tag{1}$$

The constitutive equations are given by Othman et al. [18]

$$S_{ij} = \int_0^t R(t-\tau)\frac{\partial e_{ij}(x,\tau)}{\partial\tau}d\tau = \hat{R}(e_{ij}) \tag{2}$$

with the assumptions

$$\begin{aligned} &\sigma_{ij}(x,\, y,\, t) = \frac{\partial \sigma_{ij}(x,\, y,\, t)}{\partial t} = 0 \\ &\varepsilon_{ij}(x,\, y,\, t) = \frac{\partial \varepsilon_{ij}(x,\, y,\, t)}{\partial t} = 0 \\ &-\infty < t < 0 \end{aligned} \tag{3}$$

where,

$$S_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3}\delta_{ij} \qquad e_{ij} = \varepsilon_{ij} - \frac{e}{3}\delta_{ij} \qquad e = \varepsilon_{kk} \qquad \sigma_{ij} = \sigma_{ji} \tag{4}$$

and $R(t)$ is the relaxation function given by

$$R(t) = 2\mu[1 - A\int_0^t e^{-\beta t} t^{\alpha^* - 1} dt] \qquad R(0) = 2\mu \tag{5}$$

where,

$$0 < \alpha^* < 1 \qquad A > 0 \qquad \beta > 0$$

Assuming that the relaxation effects of the volume properties of the material are ignored, one can write for the generalized theory of thermo–viscoelasticity with two relaxation times

$$\sigma = K[e - 3\,\alpha_T(T - T_o + \nu\dot{T})] \tag{6}$$

where,

$$\sigma = \frac{\sigma_{kk}}{3}$$

Substituting from (6) into (4) we obtain

$$\sigma_{ij} = \hat{R}(\varepsilon_{ij} - \frac{e}{3}\delta_{ij}) + Ke\delta_{ij} - \gamma(T - T_o + \nu\dot{T})\delta_{ij} \tag{7}$$

The equation of motion, in the absence of body forces, is

$$\sigma_{ij,j} = \rho\,[\ddot{u}_i + \{\mathbf{\Omega} \wedge (\mathbf{\Omega} \wedge \mathbf{u})\}_i + (2\mathbf{\Omega} \wedge \dot{\mathbf{u}})_i] \tag{8}$$

Combining Eqs. (7) and (8), we obtain the displacement equation of motion in the rotating frame of reference

$$\begin{aligned} &\rho\,[\ddot{\mathbf{u}} + \{\mathbf{\Omega} \wedge (\mathbf{\Omega} \wedge \mathbf{u})\} + (2\mathbf{\Omega} \wedge \dot{\mathbf{u}})] = \\ &\hat{R}(\frac{1}{2}\nabla^2 u_i + \frac{1}{6}e_{,i}) + Ke_{,i} - \gamma(T - T_o + \nu\dot{T})_{,i}\delta_{ij} \end{aligned} \tag{9}$$

The governing equation can be put into a more convenient form by using the following non–dimensional variables

$$\begin{aligned} &x' = c_o\eta_o x \quad y' = c_o\eta_o y \quad u' = c_o\eta_o u \quad v' = c_o\eta_o v \quad t' = c_o^2\eta_o t \\ &\nu' = c_o^2\eta_o\nu \quad {\tau'}_o = c_o^2\eta_o\tau_o \quad \theta = \frac{\gamma(T - T_o)}{\rho c_o^2} \quad {\sigma'}_{ij} = \frac{\sigma_{ij}}{K}\,\hat{R}' = \frac{2\quad \hat{R}}{3K} \end{aligned} \tag{10}$$

where

$$c_o^2 = \frac{K}{\rho} \qquad \eta_o = \frac{\rho C_E}{k} \qquad i = 1,\, 2$$

In terms of these non–dimensional variables, Eqs (9), (1) and (7) taking the following form (dropping the dashed for convenience).

$$\begin{aligned}&\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t} = \\ &\hat{R}\left(\frac{\partial^2 u}{\partial x^2} + \frac{3}{4}\frac{\partial^2 u}{\partial y^2} + \frac{1}{4}\frac{\partial^2 v}{\partial x\,\partial y}\right) + \frac{\partial e}{\partial x} - \left(1 + \nu\frac{\partial}{\partial\,t}\right)\frac{\partial\theta}{\partial x}\end{aligned} \tag{11}$$

$$\begin{aligned}&\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega\frac{\partial u}{\partial t} = \\ &\hat{R}\left(\frac{\partial^2 v}{\partial y^2} + \frac{3}{4}\frac{\partial^2 v}{\partial x^2} + \frac{1}{4}\frac{\partial^2 u}{\partial x\,\partial y}\right) + \frac{\partial\, e}{\partial y} - \left(1 + \nu\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial y}\end{aligned} \tag{12}$$

$$\nabla^2\theta = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\theta + \varepsilon_1\frac{\partial e}{\partial t} \tag{13}$$

$$\sigma_{x\,x} = \hat{R}\left(\frac{\partial u}{\partial x} - \frac{1}{2}\frac{\partial v}{\partial y}) + e - (1 + \nu\frac{\partial}{\partial t}\right)\theta \tag{14}$$

$$\sigma_{y\,y} = \hat{R}\left(\frac{\partial v}{\partial y} - \frac{1}{2}\frac{\partial u}{\partial x}\right) + e - \left(1 + \nu\frac{\partial}{\partial t}\right)\theta \tag{15}$$

$$\sigma_{x\,y} = \frac{3}{4}\hat{R}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{16}$$

$$\sigma_{z\,z} = -\frac{1}{2}\hat{R}(e) + e - \left(1 + \nu\frac{\partial}{\partial t}\right)\theta \tag{17}$$

Differentiating Eq. (11) with respect to x and Eq. (12) with respect to y, then adding, we arrive at

$$\left[(1+\hat{R})\nabla^2 - \frac{\partial^2}{\partial t^2} + \Omega^2\right]e = (1 + \nu\frac{\partial}{\partial t})\nabla^2\theta + 2\Omega\frac{\partial\zeta}{\partial t} \tag{18}$$

Differentiating Eq. (11) with respect to y and Eq. (12) with respect to x, then subtracting, we arrive at

$$\left(\frac{3}{4}\hat{R}\nabla^2 - \frac{\partial^2}{\partial t^2} + \Omega^2\right)\zeta = -2\Omega\frac{\partial e}{\partial t} \tag{19}$$

where

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \qquad \zeta = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

is the Laplace's operator in a two–dimensional space.

3. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$\begin{aligned}&[u,v,\, e,\, \zeta,\, \theta,\sigma_{i\,j}](x,\, y,\, t) = \\ &[u^*(y), v^*(y), e^*(y),\, \zeta^*(y),\, \theta^*(y), \sigma^*_{i\,j}(y)]\exp(\omega\, t + i\, a\, x)\end{aligned} \tag{20}$$

where ω is the (complex) time constant, i is an imaginary unit and a is the wave number in the x–direction and $u^*(y), v^*(y)$, $e^*(y)$, $\zeta^*(y)$, $\theta^*(y)$ and $\sigma^*_{ij}(y)$ are the amplitude of the functions.

$$\hat{R}(f(x,\, y,\, t)) = \int_0^t R(t-\tau)\frac{\partial f(x,\, y,\, t)}{\partial \tau}d\tau = \omega \bar{R}(\omega) f^*(y)\exp(\omega t + i\, a\, x) \tag{21}$$

for any function $f(x, y, t)$ of class $C^{(1)}$, which satisfies the conditions:

$$f(x,\, y,\, t) = \frac{\partial f(x,\, y,\, t)}{\partial t} = 0 \quad (-\infty < t < 0) \tag{22}$$

where,

$$\bar{R}(\omega) = \int_0^\infty e^{-\,\omega\, t} R(t)dt \tag{23}$$

Using Eqs. (20) we can obtain the following equations from Eqs (18), (19) and (13) Respectively

$$\left[D^2 - a^2 - \alpha(\omega^2 + \Omega^2)\right] e^*(y) = \alpha(1+\nu\omega)(D^2 - a^2)\theta^*(y) + 2\Omega\omega\alpha\zeta^*(y) \tag{24}$$

$$[D^2 - a^2 - \alpha_o(\omega^2 - \Omega^2)]\zeta^*(y) = -2\alpha_o \omega \Omega e^*(y) \tag{25}$$

$$\left[D^2 - a^2 - \omega(1+\tau_o\omega)\right] \theta^*(y) = \varepsilon_o \omega e^*(y) \tag{26}$$

where

$$D = \frac{\partial}{\partial y} \qquad \alpha = \frac{1}{1+\omega\bar{R}} \qquad \alpha_o = \frac{4}{3\omega\bar{R}} \tag{27}$$

Eliminating $\theta^*(y)$ and $\zeta^*(y)$ between Eqs (24)–(26) we get the following sixth–order partial differential equation satisfied by $e^*(y)$

$$(D^6 - a_1 D^4 + a_2 D^2 - a_3)e^*(y) = 0 \tag{28}$$

where

$$a_1 = 3a^2 + b_1 \tag{29}$$

$$a_2 = 3a^4 + 2b_1 a^2 + b_2 \tag{30}$$

$$a_3 = a^6 + b_1 a^4 + b_2 a^2 + b_3 \tag{31}$$

$$b_1 = \omega_2 + \omega_3(\alpha + \alpha_o) + \alpha\varepsilon_1\omega_1 \tag{32}$$

$$b_2 = \omega_2\omega_3(\alpha + \alpha_o) + \alpha\alpha_o(\omega_3^2 + \varepsilon_1\omega_1\omega_3 + 4\Omega^2\omega^2) \tag{33}$$

$$b_3 = \alpha\alpha_o\omega_2(\omega_3^2 + 4\Omega^2\omega^2) \tag{34}$$

$$\omega_1 = \omega(1+\nu\omega), \quad \omega_2 = \omega(1+\tau_o\omega), \quad \omega_3 = \omega^2 - \Omega^2 \tag{35}$$

In a similar manner we arrive at

$$(D^6 - a_1 D^4 + a_2 D^2 - a_3)\zeta^*(y) = 0 \tag{36}$$

$$(D^6 - a_1 D^4 + a_2 D^2 - a_3)\theta^*(y) = 0 \tag{37}$$

Eq. (28) can be factorized as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)e^*(y) = 0 \tag{38}$$

Where, $k_j^2(j = 1, 2, 3)$ are the roots of the following characteristic equation

$$k^6 - a_1 k^4 + a_2 k^2 - a_3 = 0 \tag{39}$$

The solution of Eq. (38) has the form

$$e^*(y) = \sum_{j=1}^{3} e_j^*(y) \tag{40}$$

Where $e_j^*(y)$ is the solution of the following equation

$$(D^2 - k_j^2)e_j^*(y) = 0, \qquad j = 1, 2, 3 \tag{41}$$

The solution of Eq. (41) which is bounded as $y \to \infty$, is given by

$$e_j^*(y) = M_j(a, \omega)e^{-k_j y} \tag{42}$$

Substituting from Eq. (42) into Eq. (40), $e^*(x)$ has the form

$$e^*(y) = \sum_{j=1}^{3} M_j(a, \omega)e^{-k_j y} \tag{43}$$

In a similar manner, we get

$$\theta^*(y) = \sum_{j=1}^{3} M'_j(a, \omega)e^{-k_j y} \tag{44}$$

$$\zeta^*(y) = \sum_{j=1}^{3} M''{}_j(a, \omega)e^{-k_j y} \tag{45}$$

where $M_j(a,\, \omega)$, $M'{}_j(a,\, \omega)$ and $M''{}_j(a,\, \omega)$ are some parameters depending on a and ω.

Substituting from Eqs. (43)–(45) into Eqs. (25) and (26) we get the following relations

$$M'_j(a,\, \omega) = \frac{\varepsilon_1 \omega}{[k_j^2 - a^2 - \omega_2]} M_j, \qquad j = 1,\, 2,\, 3. \tag{46}$$

$$M''{}_j(a, \omega) = \frac{-2\Omega\alpha_o\omega}{[k_j^2 - a^2 - \alpha_o\omega_3]} M_j, \qquad j = 1, 2, 3. \tag{47}$$

Substituting from Eqs. (46) and (47) into Eqs. (44) and (45) respectively, we obtain

$$\theta^*(y) = \sum_{j=1}^{3} \frac{\varepsilon_1 \omega}{[k_j^2 - a^2 - \omega_2]} M_j e^{-k_j y} \tag{48}$$

$$\zeta^*(y) = \sum_{j=1}^{3} \frac{-2\Omega\alpha_o\omega}{[k_j^2 - a^2 - \alpha_o\omega_3]} M_j e^{-k_j y} \tag{49}$$

since

$$e^* = iau^* + Dv^* \tag{50}$$

$$\zeta^* = Du^* - iav^* \tag{51}$$

In order to obtain the displacement components, from Eqs. (43), (45), (48) and (49) we can obtain

$$u^*(y) = Ce^{ay} + \sum_{j=1}^{3} \frac{1}{(k_j^2 - a^2)} \left[\frac{2\Omega\alpha_o\omega k_j}{(k_j^2 - a^2 - \alpha_o\omega_3)} + ia \right] M_j e^{-k_j y} \tag{52}$$

$$v^*(y) = -iCe^{ay} - \sum_{j=1}^{3} \frac{1}{(k_j^2 - a^2)} \left[k_j - \frac{2ia\Omega\alpha_o\omega}{(k_j^2 - a^2 - \alpha_o\omega_3)} \right] M_j e^{-k_j y} \tag{53}$$

where $C = 0$ to make Eqs. (52) and (53) are bounded as $y \to \infty$.

In terms of Eq. (20), substituting from Eqs. (43), (48), (49), (52) and (53) into Eqs. (14)–(17), respectively, we obtain the stress components in the form

$$\sigma_{xx}^*(y) = \sum_{j=1}^{3} \{\alpha_{j3} + i\beta_{j1}\} M_j e^{-k_j y} \tag{54}$$

$$\sigma_{yy}^*(y) = \sum_{j=1}^{3} \{\alpha_{j1} - i\beta_{j1}\} M_j e^{-k_j y} \tag{55}$$

$$\sigma_{xy}^*(y) = -\frac{3\omega\bar{R}}{2} \sum_{j=1}^{3} \{\alpha_{j2} + i\beta_{j2}\} M_j e^{-k_j y} \tag{56}$$

$$\sigma_{zz}^*(y) = \sum_{j=1}^{3} \beta_{j3} M_j e^{-k_j y} \tag{57}$$

$$\alpha_{j3} = 1 - \frac{\varepsilon_1\omega_1}{(k_j^2 - a^2 - \omega_2)} - \frac{(k_j^2 + 2a^2)\omega\bar{R}}{2(k_j^2 - a^2)}, \qquad j = 1,\, 2,\, 3 \tag{58}$$

$$\alpha_{j1} = 1 - \frac{\varepsilon_1\omega_1}{(k_j^2 - a^2 - \omega_2)} + \frac{(2k_j^2 + a^2)\omega\bar{R}}{2(k_j^2 - a^2)}, \qquad j = 1,\, 2,\, 3 \tag{59}$$

$$\alpha_{j2} = \frac{\Omega\alpha_o\omega(k_j^2 + a^2)}{(k_j^2 - a^2)(k_j^2 - a^2 - \alpha_o\omega_3)}, \qquad j = 1, 2, 3 \tag{60}$$

$$\beta_{j1} = \frac{3a\Omega\alpha_o k_j \omega^2 \bar{R}}{(k_j^2 - a^2)(k_j^2 - a^2 - \alpha_o\omega_3)}, \qquad j = 1,\, 2,\, 3 \tag{61}$$

$$\beta_{j2} = \frac{ak_j}{(k_j^2 - a^2)}, \qquad j = 1,\, 2,\, 3 \tag{62}$$

$$\beta_{j3} = (1 - \frac{\omega\bar{R}}{2}) - \frac{\varepsilon_1\omega_1}{[k_j^2 - a^2 - \omega_2]}, \qquad j = 1,\, 2,\, 3 \tag{63}$$

The normal mode analysis is, in fact, to look for the solution in Fourier transformed domain. This assumes that all the field quantities are sufficiently smooth on the real line provided that the normal mode analysis of these functions exists.

In order to determine the parameters $M_j(a,\,\omega), j=1,2,3$ we need to consider the following case:

A plate subjected to time–dependent heat sources on both sides Othman et al. [18].

We shall consider a homogeneous isotropic thermo–viscoelastic infinite conductive thick flat plate of a finite thickness $2L$ occupying the region G^* given by:

$$G^* = \{(x,\,y,\,z) \quad x,\,y,\,z \in R, \quad -L \le y \le L\}$$

with the middle surface of the plate coinciding with the plane $y=0$.

The boundary conditions of the problem are taken as:

- The normal and tangential stress components are zero on both surfaces of the plate,

  thus

$$\sigma_{yy}(x,\,y,\,t) = 0 \qquad \text{on} \quad y = \pm L \tag{64}$$

$$\sigma_{xy}(x,\,y,\,t) = 0 \qquad \text{on} \quad y = \pm L \tag{65}$$

- The thermal boundary condition

$$q_n + h_o\theta = r(x,\,t) \qquad \text{on} \quad y = \pm L \tag{66}$$

  where q_n denotes the normal component of the heat flux vector h_o is the Biot's number and $r\,(x,\,t)$ represents the intensity of the applied heat sources.

Now we make use of the generalized Fourier' law of heat conduction in the non–dimensional form, namely,

$$q_n + \tau_o \frac{\partial q_n}{\partial t} = -\frac{\partial \theta}{\partial y} \tag{67}$$

by using the normal mode we get

$$q_n^* = -\frac{1}{(1+\tau_o\omega)}\frac{\partial \theta^*}{\partial y} \tag{68}$$

Using Eqs. (66) and (68) we arrive at

$$\omega_2 h_o \theta^*(y) - \omega D\theta^*(y) = \omega_2 r^*(a,\,\omega) \tag{69}$$

Substituting from Eq. (48) into Eq. (69), one obtains

$$\begin{aligned} &g_1[\omega_2 h_o \cosh(k_1 L) - \omega k_1 \sinh(k_1 L)]M_1 + g_2[\omega_2 h_o \cosh(k_2 L) - \omega k_2 \sinh(k_2 L)]M_2 \\ &+g_3[\omega_2 h_o \cosh(k_3 L) - \omega k_3 \sinh(k_3 L)]M_3 = \omega_2 r^*(a,\omega) \end{aligned} \tag{70}$$

Due to symmetry with respect to y–axis, Eqs. (55) and (56) together with Eqs. (64) and (65) respectively, we get

$$L_1 M_1 \cosh(k_1 L) + L_2 M_2 \cosh(k_2 L) + L_3 M_3 \cosh(k_3 L) = 0 \tag{71}$$

$$S_1M_1\sinh(k_1L)+S_2M_2\sinh(k_2L)+S_3M_3\sinh(k_3L)=0 \tag{72}$$

where

$$g_j=\frac{\varepsilon_1\omega}{[k_j^2-a^2-\omega_2]},\qquad j=1,\,2,\,3 \tag{73}$$

$$L_j=\alpha_{j1}-i\beta_{j1},\qquad j=1,2,3 \tag{74}$$

$$S_j=\alpha_{j\,2}+i\beta_{j\,2},\qquad j=1,2,3 \tag{75}$$

Eqs (70)–(72) can be solved for the three unknowns M_1, M_2 and M_3 one obtains

$$M_1=\frac{\omega_2r^*(a,\,\omega)[(\lambda_1\Delta_1+\lambda_2\Delta_2)+i(\,\lambda_2\Delta_1-\lambda_1\Delta_2)]}{(\Delta_1^2+\Delta_2^2)\cosh\,(k_1L)} \tag{76}$$

$$M_2=\frac{\omega_2r^*(a,\,\omega)[(\lambda_3\Delta_1+\lambda_4\Delta_2)+i(\lambda_4\Delta_1-\lambda_3\Delta_2)]}{(\Delta_1^2+\Delta_2^2)\cosh\,(k_2L)} \tag{77}$$

$$M_3=\frac{\omega_2r^*(a,\,\omega)[(\lambda_5\Delta_1+\lambda_6\Delta_2)+i(\lambda_6\Delta_1-\lambda_5\Delta_2)]}{(\Delta_1^2+\Delta_2^2)\cosh\,(k_3L)} \tag{78}$$

where

$$\lambda_1=(\alpha_{21}\alpha_{32}+\beta_{21}\beta_{32})\tanh(k_3L)-(\alpha_{31}\alpha_{22}+\beta_{31}\beta_{22})\tanh(k_2L) \tag{79}$$

$$\lambda_2=(\alpha_{21}\beta_{32}-\alpha_{32}\beta_{21})\tanh(k_3L)-(\alpha_{31}\beta_{22}-\alpha_{22}\beta_{31})\tanh(k_2L) \tag{80}$$

$$\lambda_3=(\alpha_{31}\alpha_{12}+\beta_{31}\beta_{12})\tanh(k_1L)-(\alpha_{11}\alpha_{32}+\beta_{11}\beta_{32})\tanh(k_3L) \tag{81}$$

$$\lambda_4=(\alpha_{31}\beta_{12}-\alpha_{12}\beta_{31})\tanh(k_1L)-(\alpha_{11}\beta_{32}-\alpha_{32}\beta_{11})\tanh(k_3L) \tag{82}$$

$$\lambda_5=(\alpha_{11}\alpha_{22}+\beta_{11}\beta_{22})\tanh(k_2L)-(\alpha_{21}\alpha_{12}+\beta_{21}\beta_{12})\tanh(k_1L) \tag{83}$$

$$\lambda_6=(\alpha_{11}\beta_{22}-\alpha_{22}\beta_{11})\tanh(k_2L)-(\alpha_{21}\beta_{12}-\alpha_{12}\beta_{21})\tanh(k_1L) \tag{84}$$

$$\begin{aligned}\Delta_1=&\,g_1\lambda_1[\omega_2h_o-\omega k_1\tanh(k_1L)]+g_2\lambda_3[\omega_2h_o-\omega k_2\tanh(k_2L)]\\&+g_3\lambda_5[\omega_2h_o-\omega k_3\tanh(k_3L)]\end{aligned} \tag{85}$$

$$\begin{aligned}\Delta_2=&\,g_1\lambda_2[\omega_2h_o-\omega k_1\tanh(k_1L)]+g_2\lambda_4[\omega_2h_o-\omega k_2\tanh(k_2L)]\\&+g_3\lambda_6[\omega_2h_o-\omega k_3\tanh(k_3L)].\end{aligned} \tag{86}$$

4. Numerical results

As a numerical example we have considered acrylic plastic material which has a wide application in industry and medicine. Taking $\alpha^*=0.5$ in Eq. (5) and using Eq. (23), we get

$$\bar{R}(\omega)=\frac{4\mu}{3K}\left[\frac{1}{\omega_o}-\frac{A\sqrt{\pi}}{\omega_o\sqrt{\omega_o+\beta}}\right] \tag{87}$$

For this plastic the Poisson's ratio can be taken equal to 0.25 which leads to $\frac{4\mu}{3K}=0.8$. The numerical constants of the problem were taken as $a=1.2$, $\beta=0.005$,

$$A=0.106,\quad \varepsilon_1=0.0145,\quad \tau_o=0.01,\quad \nu=0.03,\quad h_o=1000\quad\text{and}\quad r^*=1000$$

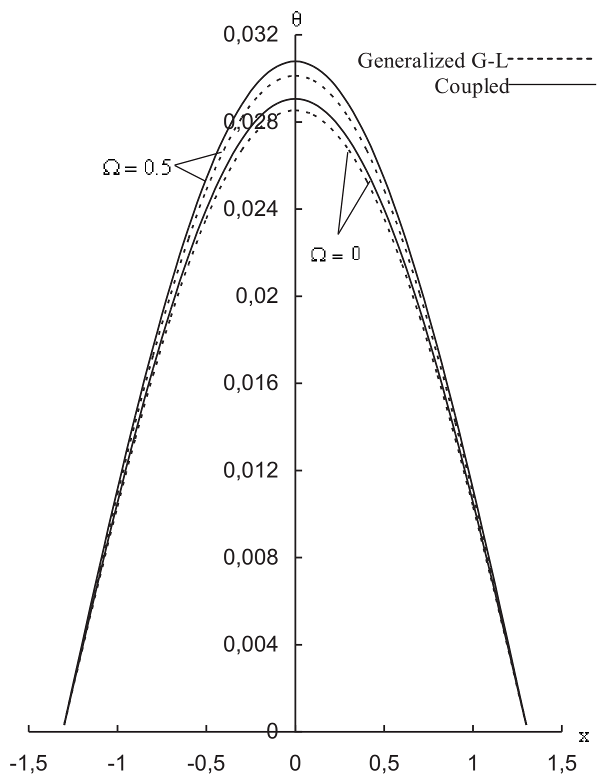

Figure 1 Temperature distribution θ on the surface at $t = 0.1$.

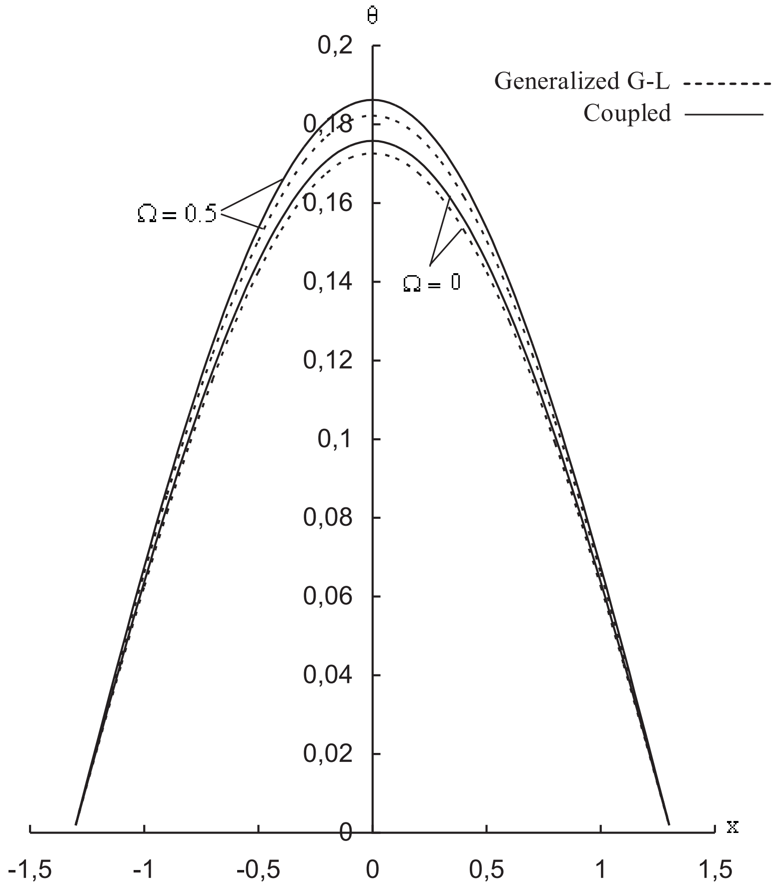

Figure 2 Temperature distribution θ on the surface at $t = 1$.

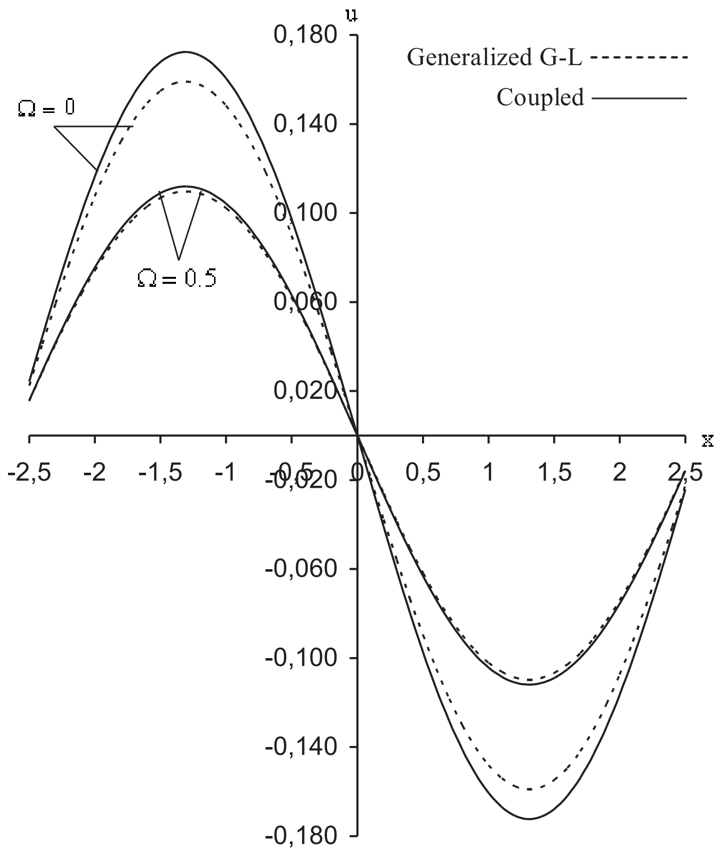

Figure 3 Horizontal displacement distribution u on the surface at $t = 0.1$.

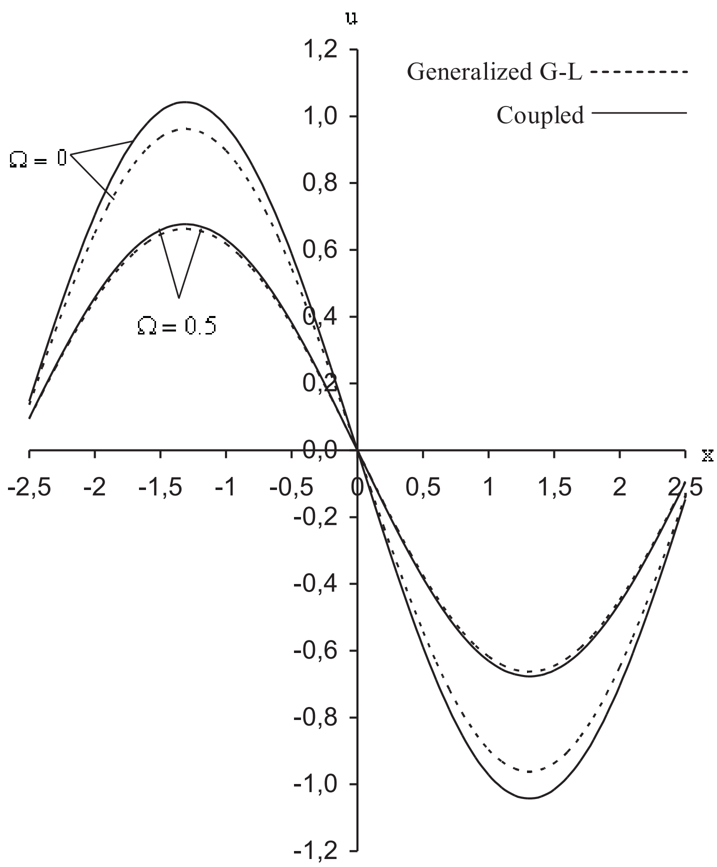

Figure 4 Horizontal displacement distribution u on the surface at $t = 1$.

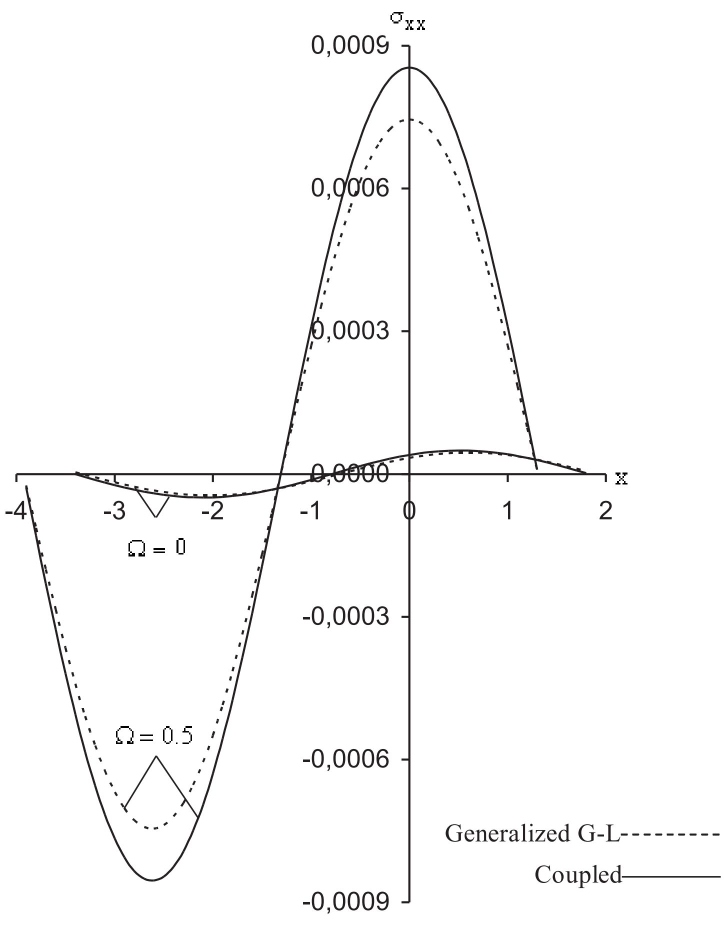

Figure 5 The distribution of stress component σ_{xx} on the surface at $t = 0.1$.

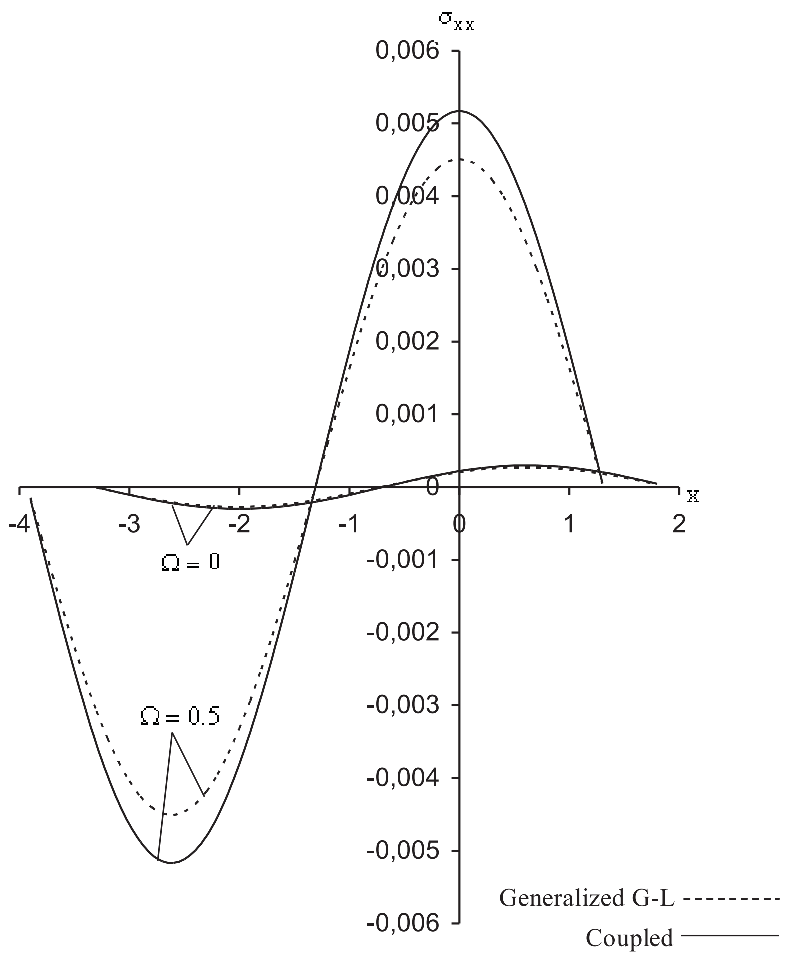

Figure 6 The distribution of stress component σ_{xx} on the surface at $t = 1$.

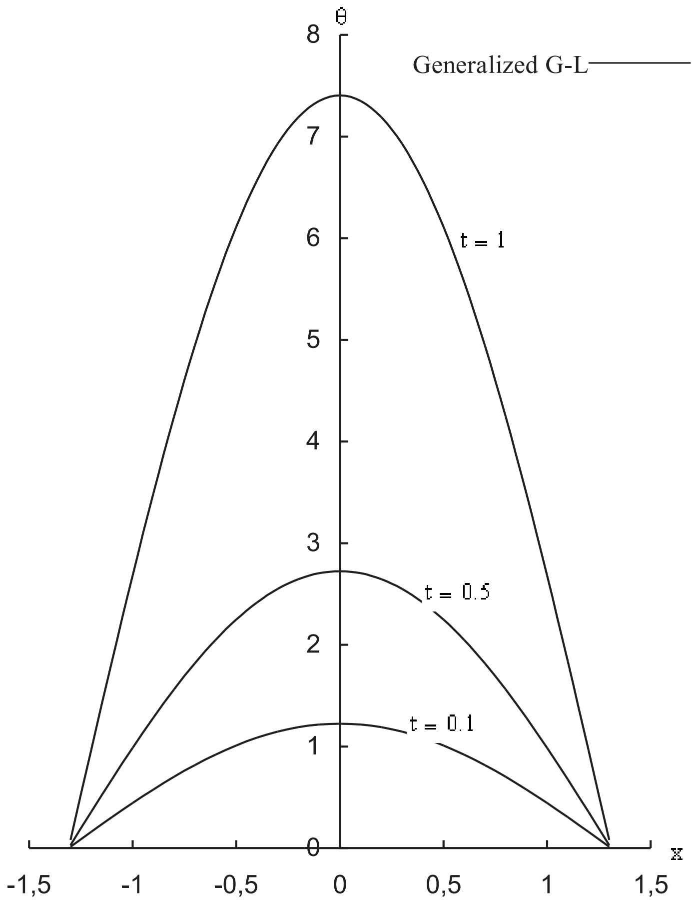

Figure 7 Temperature distribution θ on the middle plane at $\Omega = 0 and 0.5$.

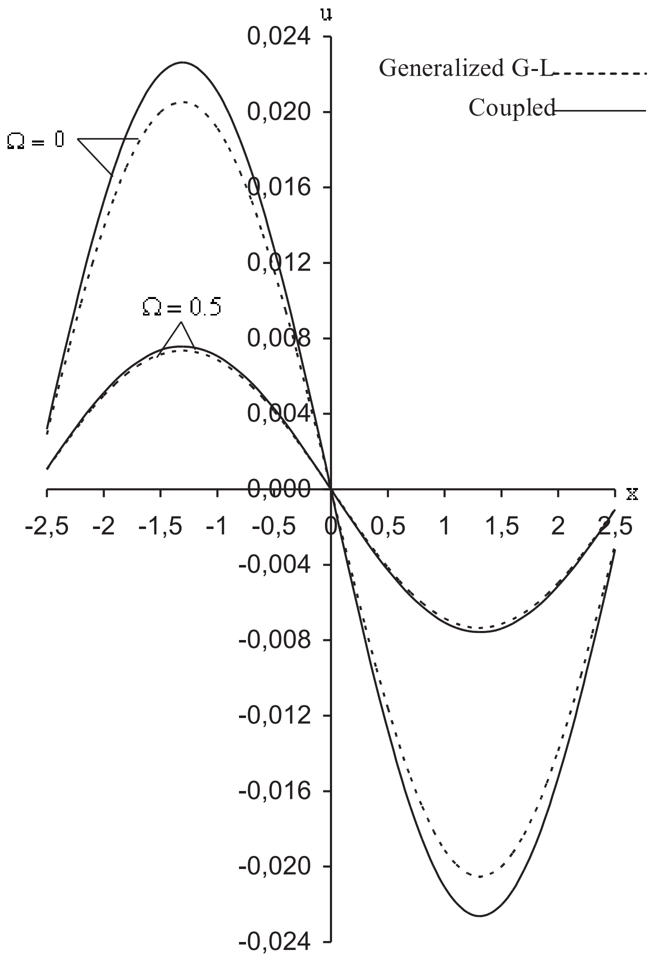

Figure 8 Horizontal displacement distribution u on the middle plane at $t = 0.1$.

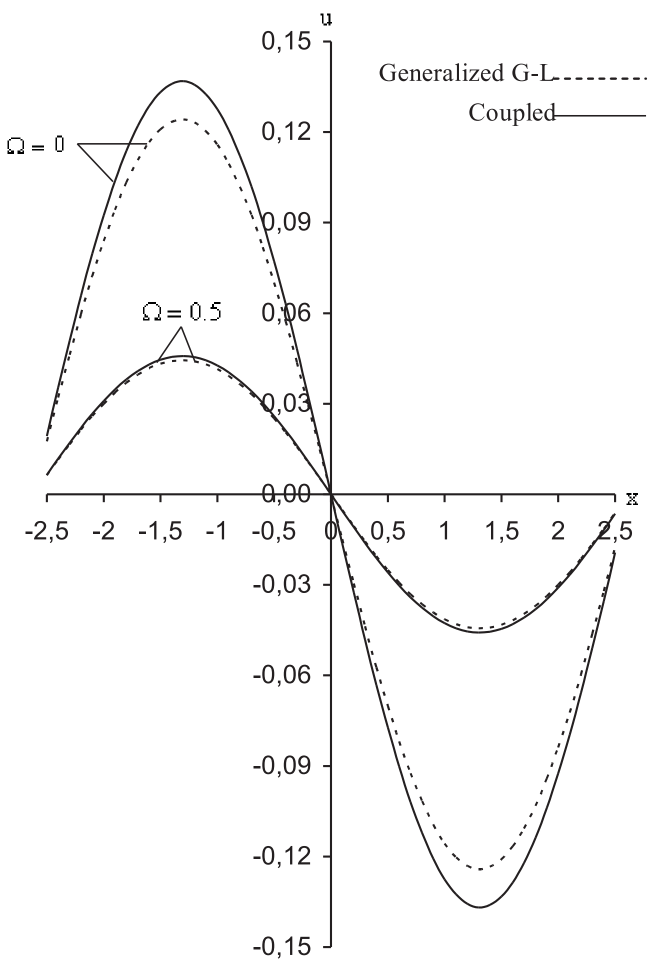

Figure 9 Horizontal displacement distribution u on the middle plane at $t = 1$.

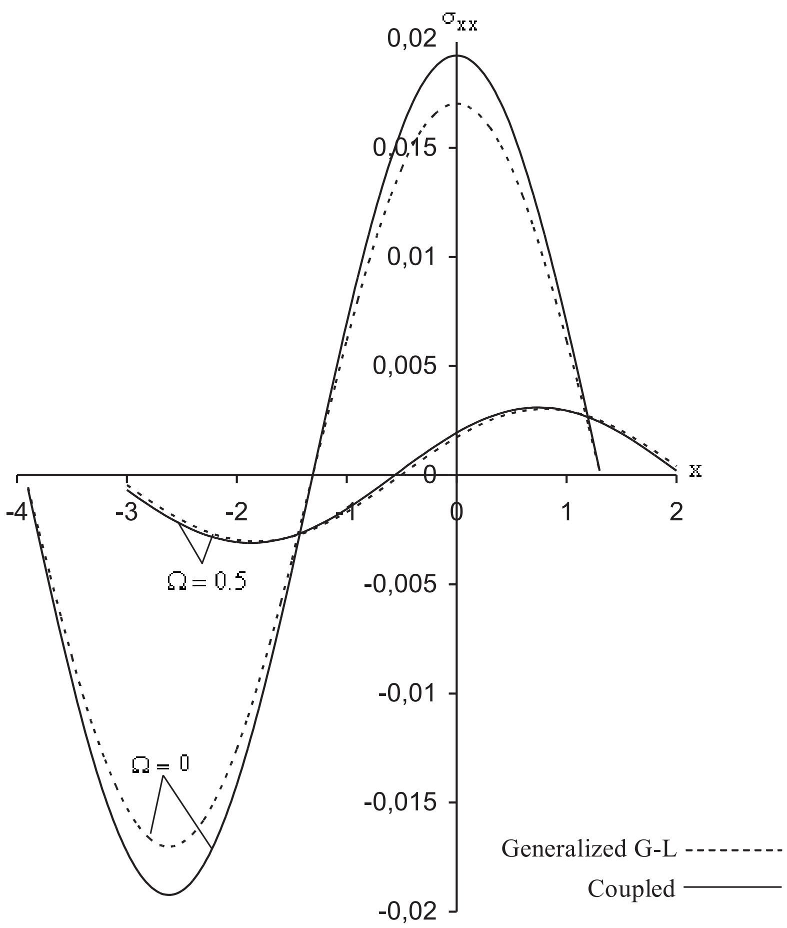

Figure 10 The distribution of stress component σ_{xx} on the middle plane at $t = 0.1$.

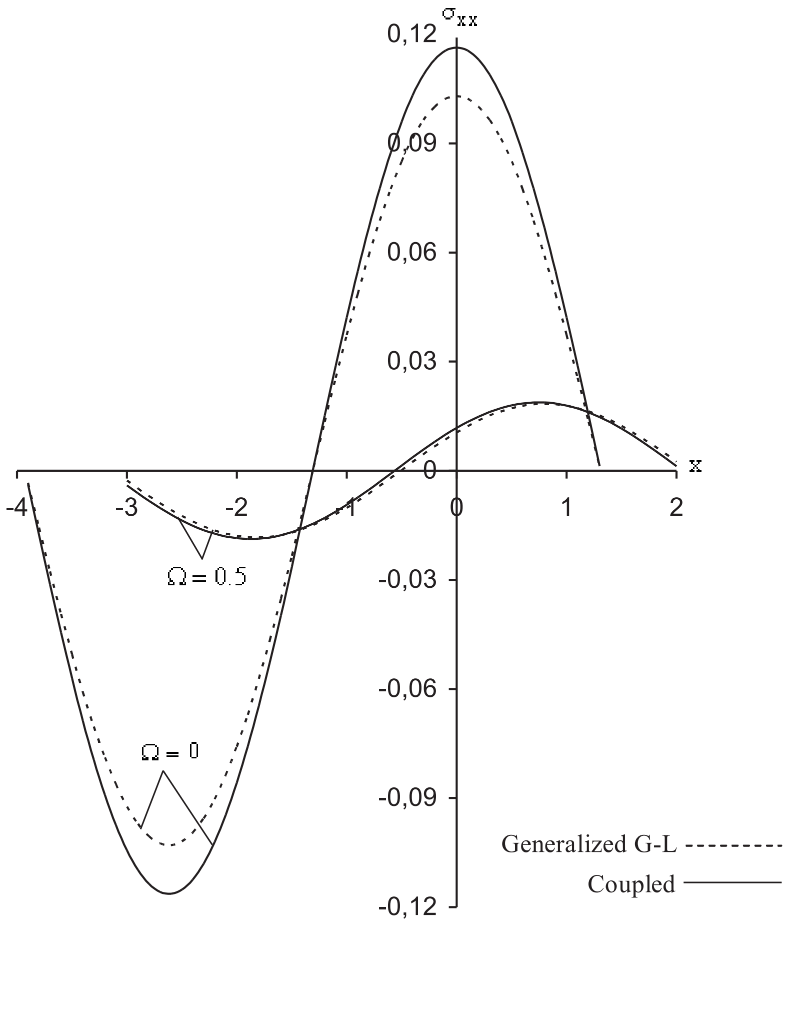

Figure 11 The distribution of stress component σ_{xx} on the middle plane at $t = 1$.

The real part of the functions $\theta\,(x,\,y,\,t)$, $u\,(x,\,y,\,t)$ and $\sigma_{xx}\,(x,\,y,\,t)$ on the plane $y\,=\,2$ and $y\,=\,0$ respectively at $t\,=\,0.1$ and $t\,=\,1$ for two different values of $\Omega = 0$ and $\Omega = 0.5$, where $L = 4$. The results are shown in Figs 1–11. Figs 1–11 show the four curves predicted by different theories of thermoelasticity. In all these figures, the solid lines represent the solution obtained by using the coupled theory ($\nu = \tau_o = 0$)and the dashed lines represent the solution obtained by using the Green–Lindsay theory ($\tau_o = 0.01, \nu = 0.03$). We notice that the representation of the field quantities when the relaxation time appears in the equation of motion and heat equation are distinctly different from those when the relaxation times are not mentioned in that equation. This is due to the fact that thermal waves in the Fourier theory of heat equation travel with an infinite speed of propagation as opposed to finite speed in the non–Fourier case. This shows the difference between the coupled and generalized theories of thermoelasticity.

Figs 1–6 show the effect of different rise time and rotation on the temperature θ, horizontal component of displacement u and stress component σ_{xx} on the surface. But Figs 7–11 show the effect of rise time and rotation for these functions on the middle plane. Due to the symmetries of geometrical shape and thermal boundary condition, the displacement component $v\,(x,\,y,\,t)$ and the component of stress $\sigma_{xy}\,(x,\,y,\,t)$ are zero when $y = 0$. On the surface Figs 1 and 2 show that the amplitude of the temperature increases with increasing time and rotation, but on the middle plane as in Fig. 7 for different values of time the rotation has no effect. Figs 3–6 and 8–11 show that the amplitude of the horizontal component of displacement and the component of stress increases with increasing time and decreasing with increasing rotation. Also, on the middle plane the amplitude of the horizontal component of displacement and the component of stress are much less than that on the surface.

5. Concluding remarks

We obtain the following conclusions from the above analysis:

1. On the surface and the middle plane the time has an increasing effect on all the field quantities studied.

2. On the surface the rotation has an increasing effect on the thermal propagation but on the middle plane it has no effect.

3. On the surface and the middle plane the rotation has a decreasing effect on the horizontal component of displacement and the stress component.

4. The amplitude of the horizontal component of displacement and the stress component on the middle plane are much less than that on the surface.

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Nomenclature

λ, μ	Lamé's constants
ρ	density
C_E	specific heat at constant strain
t	time
T	absolute temperature
T_o	reference temperature chosen so that $\left\lvert \frac{T-T_o}{T_o} \right\rvert >> 1$
θ	temperature increment
σ_{ij}	components of stress tensor
ε_{ij}	components of strain tensor
S_{ij}	components of stress deviator tensor
e	dilatation
u_i	components of displacement vector
$R(t)$	relaxation function
k	thermal conductivity
A, β, α^*	experimental constants
$\mathrm{K} = \lambda + \frac{2}{3}\mu$	bulk modulus
τ_o, ν	two relaxation times
α_T	coefficient of linear thermal expansion
$\gamma \quad = 3K\alpha_T$	
$\varepsilon \quad = \gamma/\rho C_E$	coupling parameter
$\delta_o \quad = 3T_o \alpha_T$	non–dimensional number