Mechanics and Mechanical Engineering
Vol. 14, No. 1 (2010) 5–30

Generalized Electro–Magneto–Thermoelasticity in Case of Thermal Shock Plane Waves for a Finite Conducting Half–Space with Two Relaxation Times

Mohamed I. A. Othman
Faculty of Science, Department of Mathematics
Zagazig University
P.O. Box 44519, Zagazig Egypt

Received (10 October 2009)
Revised (5 January 2010)
Accepted (18 March 2010)

The propagation of electromagneto–thermoelastic disturbances produced by a thermal shock in a finitely conducting elastic half–space is investigated. The formulation is applied to two–dimensional equations of generalized thermoelasticity Green and Lindsay's theory with two relaxation times. There acts an initial magnetic field parallel to the plane boundary of the half–space. The medium deformed because of thermal shock and due to the application of the magnetic field, there result an induced magnetic and an induced electric field in the medium. The Maxwell's equations are formulated and the electromagneto–thermoelastic coupled governing equations are established. The normal mode analysis is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented graphically for different values of times. From the distributions, it can be found the wave type heat propagation in the medium. This indicates that the generalized heat conduction mechanism is completely different from the classic Fourier's in essence. In generalized thermoelasticity theory heat propagates as a wave with finite velocity instead of infinite velocity in medium.

Keywords: Generalized electro–magneto–thermoelasticity, thermal shock problem, finite conductivity, normal mode analysis, two relaxation times

1. Introduction

Investigation of the interaction between magnetic field and stress and strain in a thermoelastic solids is very important due to its many applications in the field of geophysics, plasma physics and related topics. Especially in nuclear fields, the extremely high temperature gradients, as well as the magnetic fields originating inside nuclear reactors, influence their design and operations.

In recent years considerable interest has been shown in the study of plane thermo–elastic and magneto–thermoelastic wave propagation in a medium. The classical theory of thermoelasticity is base on Fourier's law of heat conduction which predicts an infinite speed of propagation of heat. This is physically absurd and many new theories have been proposed to eliminate this absurdity. Lord and Shulman [1] employed a modified version of the Fourier's law and deduced a theory of thermoelasticity known as the generalized theory of thermoelasticity. The Lord–Shulman's theory with a thermal relaxation time has been used by several authors including Puri [2] and Nayfeh and Nemat–Nasser [3] to study plane thermoelastic waves in an infinite media. Othman [4] construct the model of generalized thermoelasticity in an isotropic elastic medium under the dependence of the modulus of elasticity on the reference temperature with one relaxation time. Surface waves have also studied by Agarwal [5] in the generalized thermoelasticity. Electro–magneto–thermoelastic plane waves have also been studied by Nayfeh and Nemat–Nasser [6].

Green and Lindsay [7] have been presented a theory of thermoelasticity with certain special features that contrast with the previous theory having a thermal relaxation time. In this theory Fourier's law of heat conduction is unchanged whereas the classical energy equation and the stress–strain–temperature relations are modified. Two constitutive constants having the dimensions of time appear in the governing equations in place of one relaxation time in the Lord–Shulman' theory. Agarwal [8], [9] studied respectively thermoelastic and magneto–thermoelastic plane wave propagation in an infinite medium. Othman [10] considered a problem of plane wave propagation in a rotating medium in generalized thermoelasticity with two relaxation times.

A comprehensive review of the earlier contributions to the subject can be found in [11]. Among the authors who considered the generalized magneto–thermoelastic equations are Roy Choudhuri [12] extended these results to rotating media. Ezzat and Othman [13] applied the normal mode analysis to a problem of two–dimensional electro–magneto–thermoelastic plane waves with two relaxation times in a medium of perfect conductivity, and they surveyed a electro–magneto–thermoelastic problem by state–space approach in [14]. Dhaliwal and Rokne have solved a thermal shock problem in [15]. Recently, Othman [16] studied the propagation of electro–magneto–thermoelastic disturbances produced by thermal shock problem based on three theories in a perfectly conducting half–space.

In the present work we shall formulate the normal mode analysis to electro–magneto–thermoelastic coupled two-dimensional problem of a thermally and finite conducting half-space solid with two relaxation times subjected to a thermal shock on its surface. The electro–magneto–thermoelastic coupled governing equations are established, the normal mode analysis is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented graphically. From the distributions, it can be found the wave type heat propagation in the medium.

2. Development of the method

We consider the problem of a thermoelastic half–space ($x \geq 0$). A magnetic field with constant intensity $\mathbf{H} = (0, 0, H_o)$ acts parallel to the bounding plane (take as the direction of the z–axis). The surface of the half–space is subjected at time $t = 0$ to a thermal shock that is a function of y and t. Thus, all quantities considered will be functions of the time variable t and of the coordinates x and y.

The displacement equation of motion is

$$\rho\ddot{\mathbf{u}} = (\lambda+\mu)\nabla(\nabla\mathbf{u}) + \mu\nabla^2\mathbf{u} + \mu_o(\mathbf{J}\times\mathbf{H}) - \gamma\nabla(T+\nu_o\dot{T}) \tag{1}$$

Due to the application of initial magnetic field $\mathbf{H}$, there results an induced magnetic field $\mathbf{h}$ and an induced electric field $\mathbf{E}$. The simplified linear equations of electrodynamics of slowly moving medium for a homogeneous, thermally and electrically conducting elastic solid are [17],

$$\text{curl }\mathbf{h} = J+\dot{D} \tag{2}$$

$$\text{curl }\mathbf{E} = -\dot{B} \tag{3}$$

$$\text{div }\mathbf{B} = 0 \qquad \text{div }\mathbf{D} = \rho_e \tag{4}$$

$$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{h}) \qquad \mathbf{D} = \varepsilon_0\mathbf{E} \tag{5}$$

where, $\mathbf{B}$ and $\mathbf{D}$ are the magnetic and electric induction vectors [13]. According to Ohm's law, we can obtain

$$\mathbf{J} = \sigma_o(\mathbf{E} + \mu_o\dot{\mathbf{u}}\times\mathbf{H}) \tag{6}$$

where $\dot{\mathbf{u}}$ is the partial velocity of the medium, and the small effect of temperature gradient on $\mathbf{J}$ is ignored. In the absence of the body force and inner heat source, the generalized electromagneto–thermoelastic governing differential equations in the context of Green and Lindsay's theory are

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e_{kk}\delta_{ij} - \gamma(T - T_o + \nu_o\dot{T}) \tag{7}$$

the heat conduction equation

$$kT_{,ii} = \rho C_E(\dot{T} + \tau_o\ddot{T}) + \gamma T_o u_{i,i} \tag{8}$$

and strain-displacement relations

$$e_{ij} = \frac{1}{2}(u_{i,j}+u_{j,i}) \tag{9}$$

$$e = u_{i,i}+u_{j,j} \tag{10}$$

In the above equations, a comma followed by a suffix denotes material derivative and a superposed dot denotes the derivative with respect to time, $i, j = x, y$.

The displacement components have the following form

$$u_x = u(x,y,t), \qquad u_y = v(x,y,t), \qquad u_z = 0 \tag{11}$$

From Eqs (9) and (11), we obtain the strain components

$$e_{xx} = \frac{\partial u}{\partial x}, \quad e_{yy} = \frac{\partial v}{\partial y}, \quad e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad e_{xz} = e_{yz} = e_{zz} = 0 \tag{12}$$

From Eqs (7) and (12), the stress components are given by

$$\sigma_{xx} = (\lambda + 2\mu)u_{,x} + \lambda v_{,y} - \gamma(T - T_o + \nu_o\dot{T}) \tag{13}$$

$$\sigma_{yy} = (\lambda + 2\mu) v_{,y} + \lambda u_{,x} - \gamma (T - T_o + \nu_o \dot{T}) \tag{14}$$
$$\sigma_{xy} = \mu (u_{,y} + v_{,x}) \tag{15}$$

The components of the magnetic intensity vector in the medium are

$$H_x = 0, \qquad H_y = 0, \qquad H_z = H_o + h(x, y, t) \tag{16}$$

The electric intensity vector is normal to both the magnetic intensity and the displacement vectors. Thus, it has the components

$$E_x = E_1, \qquad E_y = E_2, \qquad E_z = 0 \tag{17}$$

The current density vector **J** be parallel to **E**, thus

$$J_x = J_1, \qquad J_y = J_2, \qquad J_z = 0 \tag{18}$$

Ohm's law (6) after linearization gives

$$J_1 = \sigma_o (E_1 + \mu_o H_o \frac{\partial v}{\partial t}), \qquad J_2 = \sigma_o (E_2 - \mu_o H_o \frac{\partial u}{\partial t}) \tag{19}$$

Eqs (2), (5) and (19) give the two equations

$$\frac{\partial h}{\partial y} = \sigma_o (E_1 + \mu_o H_o \frac{\partial v}{\partial t}) + \varepsilon_o \frac{\partial E_1}{\partial t} \tag{20}$$
$$\frac{\partial h}{\partial x} = -\sigma_o (E_2 - \mu_o H_o \frac{\partial u}{\partial t}) - \varepsilon_o \frac{\partial E_2}{\partial t} \tag{21}$$

From Eqs (3), (5) we can get one–vanishing component, namely

$$\frac{\partial E_1}{\partial y} - \frac{\partial E_2}{\partial x} = \mu_o \frac{\partial h}{\partial t} \tag{22}$$

From Eqs (16) and (19), we obtain

$$(\mathbf{J} \times \mathbf{H})_x = \mu_o \sigma_o H_o \left(E_2 - \mu_o H_o \frac{\partial u}{\partial t} \right) \tag{23}$$
$$(\mathbf{J} \times \mathbf{H})_y = -\mu_o \sigma_o H_o \left(E_1 + \mu_o H_o \frac{\partial v}{\partial t} \right) \tag{24}$$
$$(\mathbf{J} \times \mathbf{H})_z = 0 \tag{25}$$

From Eqs (1) and (23)–(25), we get

$$(\lambda + \mu) \frac{\partial e}{\partial x} + \mu \nabla^2 u - \gamma \left(1 + \nu_o \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial x} + \mu_o \sigma_o H_o \left(E_2 - \mu_o H_o \frac{\partial u}{\partial t} \right) = \rho \frac{\partial^2 u}{\partial t^2} \tag{26}$$

$$(\lambda + \mu) \frac{\partial e}{\partial y} + \mu \nabla^2 v - \gamma \left(1 + \nu_o \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial y} - \mu_o \sigma_o H_o \left(E_1 + \mu_o H_o \frac{\partial v}{\partial t} \right) = \rho \frac{\partial^2 v}{\partial t^2} \tag{27}$$

For convenience, the following non–dimensional variables are used:

$$\begin{aligned} &x' = c_1\eta\, x, && y' = c_1\eta\, y, && u' = c_1\eta\, u, && v' = c_1\eta\, v, \\ &t' = c_1^2\eta\, t, && \nu'_o = c_1^2\eta\,\nu_o, && \tau'_o = c_1^2\eta\,\tau_o, && \theta = \frac{\gamma\,(T - T_o)}{\lambda + 2\,\mu} \\ &\sigma'_{i\,j} = \frac{\sigma_{i\,j}}{\mu}, && h' = \frac{\eta\, h}{\sigma_o\mu_o H_o}, && E'_i = \frac{\eta\, E_i}{\sigma_o\mu_o^2 H_o c_1}\,, && i = 1, 2 \end{aligned} \tag{28}$$

In terms of the non–dimensional quantities defined in Eq. (28), the above governing equations reduce to (dropping the dashed for convenience)

$$\begin{aligned} &(\,\beta^2 - 1\,)\; e_{,x} + \nabla^2 u - \;\beta^2 \left(1 + \nu_o \frac{\partial}{\partial\, t}\right)\,\theta_{,x} \\ &+ \beta^2 \alpha\,\beta_1(\,\beta_1 E_2 - \;\; u_{,t}) = \beta^2 u_{,tt} \end{aligned} \tag{29}$$

$$\begin{aligned} &(\,\beta^2 - 1\,)\; e_{,y} + \nabla^2 v - \;\beta^2 \left(1 + \nu_o \frac{\partial}{\partial\, t}\right)\,\theta_{,y} \\ &- \beta^2 \alpha\,\beta_1(\,\beta_1 E_1 + \;\; v_{,t}) = \beta^2 v_{,tt} \end{aligned} \tag{30}$$

$$\nabla^2\theta = (\,\theta_{,t} + \tau_o\,\theta_{,tt}\,)\; + \;\varepsilon_1\, e_{,t} \tag{31}$$

$$\frac{\partial\, h}{\partial\, y} = \;\beta_1 E_1 + \;\varepsilon_2 \frac{\partial\, E_1}{\partial\, t} + \frac{\partial\, v}{\partial\, t} \tag{32}$$

$$\frac{\partial\, h}{\partial\, x} = \;-\;\beta_1 E_2 - \;\varepsilon_2 \frac{\partial\, E_2}{\partial\, t} + \frac{\partial\, u}{\partial\, t} \tag{33}$$

$$\frac{\partial\, E_1}{\partial\, y} - \;\frac{\partial\, E_2}{\partial\, x} = \;\frac{\partial\, h}{\partial\, t} \tag{34}$$

The constitutive equations reduce to

$$\sigma_{x\,x} = (\,\beta^2 - 2\,)\, e \;+ 2\, u_{,x} - \;\beta^2 \left(1 + \nu_o \frac{\partial}{\partial\, t}\right)\,\theta \tag{35}$$

$$\sigma_{y\,y} = (\,\beta^2 - 2\,)\, e \;+ 2\, v_{,y} - \;\beta^2 \left(1 + \nu_o \frac{\partial}{\partial\, t}\right)\,\theta \tag{36}$$

$$\sigma_{x\,y} = \;u_{,y} + \;\; v_{,x} \tag{37}$$

Differentiating Eq.(29) with respect to x, and Eq. (30) with respect to y, then adding, we obtain

$$\left(\nabla^2 - \;\beta_1\alpha \frac{\partial}{\partial\, t} \;-\; \frac{\partial^2}{\partial\, t^2}\right)\; e \;-\left(1 + \nu_o \frac{\partial}{\partial\, t}\right)\nabla^2\theta \;-\; \beta_1^2\,\alpha \frac{\partial\, h}{\partial\, t} \;= 0 \tag{38}$$

Differentiating Eq.(32) with respect to y, and Eq. (33) with respect to x, then adding, we obtain

$$\left(\nabla^2 - \beta_1 \frac{\partial}{\partial\, t} \;-\; \varepsilon_2 \frac{\partial^2}{\partial\, t^2}\right)\; h \;= \frac{\partial\, e}{\partial\, t} \tag{39}$$

3. Normal Mode Analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$\begin{aligned}&[u, v, e, \theta, h, E_i, \sigma_{ij}]\,(x, y, t)\\&= [u^*(x), v^*(x), e^*(x), \theta^*(x), h^*(x), E_i^*(x), \sigma_{ij}^*(x)]\,\exp(\omega\, t + i\, a\, y)\end{aligned} \tag{40}$$

where ω is the complex time constant and a is the wave number in the y–direction.

Using Eq. (40), Eqs. (31), (38) and (39) take the form

$$(D^2 - a^2 - \omega - \tau_o\omega^2)\theta^*(x) = \varepsilon_1\omega e^*(x) \tag{41}$$

$$\begin{aligned}&(D^2 - a^2 - \beta_1\alpha\omega - \omega^2)e^*(x) - \left(1 + \nu_o\frac{\partial}{\partial t}\right)(D^2 - a^2)\theta^*(x)\\&-\beta_1^2\alpha\omega h^*(x) = 0\end{aligned} \tag{42}$$

$$(D^2 - a^2 - \beta_1\omega - \varepsilon_2\omega^2)h^*(x) = \omega\, e^*(x) \tag{43}$$

where $D = \frac{\partial}{\partial\, x}$

Eliminating $\theta^*(x)$ and $h^*(x)$ between Eqs. (41), (42) and (43), we obtain the following sixth-order partial differential equation satisfied by $e^*(x)$

$$(\, D^6 - A\, D^4 + B\, D^2 - C\,)\, e^*(x) = 0 \tag{44}$$

where

$$\begin{aligned}
A &= 3\, a^2 + b_1 && (45)\\
B &= 3\, a^4 + 2\, a^2 b_1 + b_2 && (46)\\
C &= a^6 + \; a^4 b_1 + \; a^2 b_2 + b_3 && (47)\\
b_1 &= \omega\; [\omega\; (\varepsilon_1\nu_o + \tau_o + \varepsilon_2 + 1\,)\; + \varepsilon_1 + \; \alpha\; \beta_1 + \beta_1 + 1\,] && (48)\\
b_2 &= \omega^2\, \big\{ \omega^2\, [\,\tau_o\; (\varepsilon_2 + 1\,)\; + \varepsilon_2 + \varepsilon_1\varepsilon_2\nu_o]\\
&\quad +\omega\, [\,\varepsilon_1\varepsilon_2 + \alpha\,\beta_1(\,\tau_o + \varepsilon_2) + \beta_1(\,\tau_o + 1\,) + \varepsilon_2{+}1 + \varepsilon_1\beta_1\nu_o] && (49)\\
&\quad + \; \beta_1(\,\varepsilon_1 + \; \alpha + 2\,\beta_1\alpha + 1\,)\big\}\\
b_3 &= \omega^4(\,1 + \tau_o\,\omega\,)\; [\,\beta_1 + \; \varepsilon_2(\,\omega \; + \; \alpha\,\beta_1)\,] && (50)
\end{aligned}$$

In a similar manner we arrive at

$$(\, D^6 - A\, D^4 + B\, D^2 - C\,)\, \theta^*(x) = 0 \tag{51}$$

$$(\, D^6 - A\, D^4 + B\, D^2 - C\,)\, h^*(x) = 0 \tag{52}$$

Eq. (44) can be factorized as

$$(\, D^2 - k_1^2\,)\,(\, D^2 - k_2^2\,)\,(\, D^2 - k_3^2\,)\, e^*(x) = 0 \tag{53}$$

where k_i^2 $(\, i = 1, 2, 3)$ is the root of the following characteristic equation

$$k^6 - A\, k^4 + B\, k^2 - C\; = 0 \tag{54}$$

The solution of Eq. (53) has the form

$$e^*(x) = \sum_{i=1}^{3} e_i^*(x). \tag{55}$$

where $e_i^*(x)$ is the solution of the equation

$$(\, D^2 - k_i^2 \,)\; e_i^*(x) = 0 \;, \qquad i = 1, 2, 3 \tag{56}$$

The solution of Eq. (56) which is bounded as $x \to \infty$, is given by

$$e_i^*(x) = R_i(a, \omega)\, e^{-k_i x} \tag{57}$$

Thus, $e^*(x)$ has the form

$$e^*(x) = \sum_{i=1}^{3} R_i(a, \omega)\, e^{-\,k_i\, x} \tag{58}$$

In a similar manner, we get

$$\theta^*(x) = \sum_{i=1}^{3} R'_i(a, \omega)\, e^{-\,k_i\, x} \tag{59}$$

$$h^*(x) = \sum_{i=1}^{3} R''_i(a, \omega)\, e^{-\,k_i\, x} \tag{60}$$

where $R_i(a, \omega)$, $R'_i(a, \omega)$ and $R''_i(a, \omega)$ are some parameters depending on $aand\omega$.

Substituting from Eqs. (58)-(60) into Eqs. (41) and (43) we get the following relations

$$R'_i(a, \omega) = \frac{\varepsilon_1 \omega}{[k_i^2 - a^2 - \omega\,(\,1 + \tau_o \omega\,)]}\; R_i, \qquad i = 1, 2, 3 \tag{61}$$

$$R''_i(a, \omega) = \frac{\omega}{[k_i^2 - a^2 - \;\omega\,(\,\beta_1 + \varepsilon_2\,\omega\,)]}\; R_i, \qquad i = 1, 2, 3 \tag{62}$$

Substituting from Eqs. (61) and (62) into Eqs. (59) and (60) respectively, we obtain

$$\theta^*(x) = \sum_{i=1}^{3} \frac{\varepsilon_1 \omega}{[k_i^2 - a^2 - \;\omega\,(\,1 + \tau_o \omega\,)]}\; R_i\, e^{-\,k_i\, x} \tag{63}$$

$$h^*(x) = \sum_{i=1}^{3} \frac{\omega}{[k_i^2 - a^2 - \omega(\,\beta_1 + \varepsilon_2\,\omega\,)]}\; R_i\, e^{-\,k_i\, x} \tag{64}$$

In order to obtain the displacement u, in terms of Eq. (40), from Eqs. (29) and (33), we can obtain the following relations

$$\begin{aligned}(\, D^2 - a^2 - \beta^2\omega^2 - \;\beta^2\alpha\,\beta_1\,\omega\,)\, u^*(x) \;&=\; \beta^2(1 + \nu_o\omega) D\,\theta^* \\ -\;\beta^2\,\alpha\,\beta_1^2\, E_2^*(x) - (\,\beta^2 - 1\,)\; D\, e^*\end{aligned} \tag{65}$$

$$E_2^*(x) = \frac{1}{(\,\beta_1 + \varepsilon_2 \omega\,)}\;[\,\omega\; u^* - D\, h^*] \tag{66}$$

Eliminating $E_2^*(x)$ between Eqs. (65) and (66) and using Eqs (58), (63) and (64) we

get the following partial differential equation satisfied by $u^*(x)$

$$(D^2 - m^2)\,u^*(x) = -\sum_{i=1}^{3}\left\{1 + \beta^2\left[\frac{\varepsilon_1\,\omega\,(1+\nu_o\omega)}{[k_i^2 - a^2 - \omega\,(1+\tau_o\omega)]}\right.\right.$$
$$\left.\left. + \frac{\beta_1^2\,\alpha\,\omega}{(\beta_1 + \varepsilon_2\,\omega)\,[\,k_i^2 - a^2 - \,\omega\,(\beta_1 + \varepsilon_2\,\omega\,)]} - 1\right]\right\} k_i\,R_i\,e^{-k_i\,x} \tag{67}$$

where

$$m^2 = a^2 + \beta^2\omega^2 + \frac{\beta^2\,\omega^2\,\beta_1\,\varepsilon_2\alpha}{\beta_1 + \varepsilon_2\,\omega} \tag{68}$$

In order to simplify the right–hand side of Eq. (67) we substitute from Eqs. (58), (63) and (64) into Eq. (42), to get

$$\frac{\varepsilon_1\,\omega\,(1+\nu_o\omega)}{[k_i^2 - a^2 - \omega\,(1+\tau_o\omega)]} = \frac{k_i^2 - a^2 - \omega(\,\alpha\,\beta_1 + \,\omega)}{(k_i^2 - a^2\,)}$$
$$-\frac{\beta_1^2\,\alpha\,\omega^2}{(\,k_i^2 - a^2\,)\,[\,k_i^2 - a^2 - \,\omega\,(\,\beta_1 + \varepsilon_2\,\omega\,)\,]} \tag{69}$$

Using Eq. (69), Eq. (67) reduce to

$$(D^2 - m^2)\,u^*(x) = -\sum_{i=1}^{3}\left[1 - \frac{\beta^2\,\omega^2(\,\beta_1 + \varepsilon_2\,\omega + \varepsilon_2\,\beta_1\alpha\,)}{(k_i^2 - a^2)\,(\,\beta_1 + \varepsilon_2\,\omega\,)}\right] k_i R_i e^{-k_i x} \tag{70}$$

The solution of Eq. (70), bounded as $x \to \infty$, is given by

$$u^*(x) = G\,a^2 e^{-m\,x} - \sum_{i=1}^{3}\left[\frac{k_i R_i}{(k_i^2 - a^2)}\right] e^{-k_i x} \tag{71}$$

where $G = G(a,\omega)$ is some parameter depending on a and ω.

In terms of Eq. (40), from Eq. (10) we can obtain

$$v^*(x) = \frac{-i}{a}\,(\,e^* - \frac{\partial\,u^*}{\partial\,x}) \tag{72}$$

Substituting from Eqs. (58) and (71) into the right–hand side of Eq. (72), we get

$$v^*(x) = -i\,a\left[m\,G\,e^{-m\,x} - \sum_{i=1}^{3}\frac{R_i}{(k_i^2 - a^2)}\,e^{-k_i x}\right] \tag{73}$$

Substituting from Eqs. (64) and (71) into Eq. (66), we get

$$E_2^*(x) = \frac{\omega}{\beta_1 + \varepsilon_2\omega}\,\left[\,G\,a^2\,e^{-m\,x}\right.$$
$$\left. + \omega\,(\,\beta_1 + \varepsilon_2\omega\,)\sum_{i=1}^{3}\frac{k_i R_i}{(\,k_i^2 - a^2)\,[k_i^2 - a^2 - \omega\,(\,\beta_1 + \varepsilon_2\omega\,)\,]}\,e^{-k_i x}\right] \tag{74}$$

In terms of Eq. (40), from Eq. (32), we obtain

$$E_1^*(x) = \frac{1}{\beta_1 + \varepsilon_2 \omega}\,(\,i\,a\,h^* - \,\omega\,v^*) \tag{75}$$

Substituting from Eqs. (64) and (73) into Eq. (75), we get

$$\begin{aligned} E_1^*(x) = {} & \frac{i\,a\,\omega}{(\beta_1 + \varepsilon_2\omega)}\,\Big[\,m\,G\,e^{-m\,x} \\ & +\omega\,(\,\beta_1 + \varepsilon_2\omega\,)\sum_{i=1}^{3} \frac{R_i}{(\,k_i^2 - a^2)\,[k_i^2 - a^2 - \omega\,(\,\beta_1 + \varepsilon_2\omega\,)\,]}\,e^{-k_i x}\Big] \end{aligned} \tag{76}$$

In terms of Eq. (40), substituting from Eqs. (58), (63), (71) and (73) into Eqs. (35)–(57) respectively, we obtain the stress components in the form

$$\begin{aligned} \sigma_{xx}^*(x) = {} & -\,2\,m\,G\,a^2\,e^{-m\,x} + \sum_{i=1}^{3}\Bigg\{\frac{2\,a^2}{(k_i^2 - a^2)} \\ & + \frac{\beta^2[\,k_i^2 - a^2 - \omega\,(\,1 + \tau_o\,\omega\,) - \varepsilon_1\omega\,(\,1 + \nu_o\,\omega\,)\,]}{[k_i^2 - a^2 - \omega\,(\,1 + \tau_o\,\omega\,)\,]}\Bigg\}\,R_i\,e^{-k_i x} \end{aligned} \tag{77}$$

$$\begin{aligned} \sigma_{yy}^*(x) = {} & \,2\,m\,G\,a^2\,e^{-m\,x} - \sum_{i=1}^{3}\Bigg\{\frac{2\,k_i^2}{(k_i^2 - a^2)} \\ & +\,\beta^2\left[1 - \frac{\varepsilon_1\omega\,(\,1 + \nu_o\omega\,)}{[k_i^2 - a^2 - \omega\,(\,1 + \tau_o\omega\,)\,]}\right]\Bigg\}\,R_i\,e^{-k_i x} \end{aligned} \tag{78}$$

$$\sigma_{xy}^*(x) = \,i\,a\,\left[(\,m^2 + a^2\,)\,G\,e^{-m\,x} - 2\sum_{i=1}^{3}\frac{k_i R_i}{(k_i^2 - a^2)}\,e^{-k_i x}\right] \tag{79}$$

We consider the magnetic and electric field intensities in free space. We denote these by h_o, E_{10} and E_{20}, respectively. These variables satisfy the non–dimensional field equations

$$\frac{\partial\,h_o}{\partial\,y} = \varepsilon_2\frac{\partial\,E_{10}}{\partial\,t} \tag{80}$$

$$\frac{\partial\,h_o}{\partial\,x} = -\varepsilon_2\frac{\partial\,E_{20}}{\partial\,t} \tag{81}$$

$$\frac{\partial\,h_o}{\partial\,t} = \frac{\partial\,E_{10}}{\partial\,y} - \frac{\partial\,E_{20}}{\partial\,x} \tag{82}$$

Similarly these variables can be decomposed as the following

$$[\,h_o, E_{10}, E_{20}\,] = \left[\,h_o^*(x), E_{10}^*(x), E_{20}^*(x)\,\right]\;exp(\,\omega\,t + i\,a\,y\,) \tag{83}$$

Applying Eq. (83) to Eqs (80)–(82), and solving the resulting equations, we obtain the solutions bounded for $x < 0$ as

$$h_o^* = F(a,\omega)\, e^{n\,x} \tag{84}$$

$$E_{10}^*(x) = \frac{i\,a}{\varepsilon_2\omega}\, F(a,\omega) e^{n\,x} \tag{85}$$

$$E_{20}^*(x) = \frac{-n}{\varepsilon_2\omega}\, F(a,\omega) e^{n\,x} \tag{86}$$

$$n = \sqrt{a^2 + \varepsilon_2\,\omega^2} \tag{87}$$

where $F(a,\omega)$ is some parameter depending on a and ω.

The normal mode analysis is, in fact, to look for the solution in the Fourier transformed domain. Assuming that all the relations are sufficiently smooth on the real line such that the normal mode analysis of these functions exist.

In order to determine the parameters $R_i(i = 12,3), GandF$, we need to consider the boundary conditions at $x = 0$.We consider two kinds of boundary conditions respectively, and the details are described as following.

Case 1

1. Thermal boundary condition that the surface of the half-space subjected to a thermal shock
$$\theta\,(0,y,t) = f(y,t) \tag{88}$$
2. Mechanical boundary condition that the surface of the half-space is traction free
$$\sigma_{x\,x}\,(0,y,t) = \sigma_{x\,y}\,(0,y,t) = 0 \tag{89}$$
3. The transverse components of the electric field intensity are continuous across the surface of the half–space
$$E_2\,(0,y,t) = E_{20}\,(0,y,t) \tag{90}$$
4. The transverse components of the magnetic field intensity are continuous across the surface of the half-space
$$h\,(0,y,t) = h_0\,(0,y,t) \tag{91}$$

Substituting from the expressions of considered variables into the above boundary conditions, we can obtain the following equations satisfied by the parameters

$$\sum_{i=1}^{3} \frac{\varepsilon_1\omega}{[k_i^2 - a^2 - \omega\,(1+\tau_o\omega)]}\; R_i = f^*(a,\omega) \tag{92}$$

$$-2\,m\,G\,a^2 + \sum_{i=1}^{3} \left\{ \frac{2\,a^2}{(\,k_i^2 - a^2)} + \beta^2[\,1 - \frac{\varepsilon_1\omega\,(1+\nu_o\omega)}{[k_i^2 - a^2 - \omega\,1 + \tau_o\omega)]}\;]\right\} R_i$$
$$= 0 \tag{93}$$

$$2\sum_{i=1}^{3}\frac{k_iR_i}{(k_i^2-a^2)}\;-\;(\,m^2+a^2)\;G=0 \tag{94}$$

$$\frac{G\,a^2}{\omega(\beta_1+\varepsilon_2\omega)}+\frac{n\,F}{\varepsilon_2\,\omega^3}+\sum_{i=1}^{3}\frac{k_iR_i}{(\,k_i^2-a^2)\,[k_i^2-a^2-\omega\,(\,\beta_1+\varepsilon_2\omega\,)\,]}=0 \tag{95}$$

$$\sum_{i=1}^{3}\frac{\omega\,R_i}{[k_i^2-a^2-\omega\,(\,\beta_1+\varepsilon_2\omega)]}\;-F=0 \tag{96}$$

Solving Eqs. (92)–(96), we get the parameters R_i ($i=1,\,2,\,3$), G and F with the following form respectively

$$G=\frac{2}{(m^2+a^2)}\;\sum_{i=1}^{3}\frac{k_i}{(k_i^2-a^2)}\,R_i \tag{97}$$

$$F=\sum_{i=1}^{3}\frac{\omega}{[k_i^2-a^2-\omega\,(\,\beta_1+\varepsilon_2\omega)]}\,R_i \tag{98}$$

$$R_1=\;\frac{f^*(a,\omega\,)\;[\,N_2M_3-N_3M_2-\;\beta^2(1+\nu_o\omega)\,(\,N_2S_3-N_3S_2)\,]}{S_1(\,N_2M_3-N_3M_2)+S_2(\,N_3M_1-N_1M_3)+S_3(\,N_1M_2-N_2M_1)} \tag{99}$$

$$R_2=\;\frac{f^*(a,\omega\,)\;[\,N_3M_1-N_1M_3-\;\beta^2(1+\nu_o\omega)\,(\,N_3S_1-N_1S_3)\,]}{S_1(\,N_2M_3-N_3M_2)+S_2(\,N_3M_1-N_1M_3)+S_3(\,N_1M_2-N_2M_1)} \tag{100}$$

$$R_3=\;\frac{f^*(a,\omega\,)\;[\,N_1M_2-N_2M_1-\;\beta^2(1+\nu_o\omega)\,(\,N_1S_2-N_2S_1)\,]}{S_1(\,N_2M_3-N_3M_2)+S_2(\,N_3M_1-N_1M_3)+S_3(\,N_1M_2-N_2M_1)} \tag{101}$$

where

$$S_i=\frac{\varepsilon_1\omega}{[k_i^2-a^2-\omega\,(1+\tau_o\omega)]} \tag{102}$$

$$\begin{aligned}M_i=&\frac{-4\,m\,a^2k_i}{(\,m^2+a^2)\,(\,k_i^2-a^2)}\;+\;\frac{2a^2}{(\,k_i^2-a^2)}\\&+\beta^2\left[1-\frac{\varepsilon_1\omega\,(\,1+\nu_o\omega\,)}{[\,k_i^2-a^2-\omega\,(\,1+\nu_o\omega\,)\,]}\right]\end{aligned} \tag{103}$$

$$\begin{aligned}N_i=&\;\frac{1}{(\,k_i^2-a^2)}\;\left\{\frac{2\,a^2k_i}{\omega\,(\,m^2+a^2)(\,\beta_1+\varepsilon_2\,\omega)}\right.\\&\left.+\;\frac{n\,(\,k_i^2-a^2)+k_i\varepsilon_2\,\omega^2}{\varepsilon_2\,\omega^2[\,k_i^2-a^2-\omega\,(\,\beta_1+\varepsilon_2\,\omega)\,]}\right\}\end{aligned} \tag{104}$$

Case 2

- Thermal boundary condition that the surface of the half–space subjected to a thermal shock

$$\theta\,(0,y,t)=f(y,t) \tag{105}$$

- Mechanical boundary condition that the surface of the half–space is rigidly fixed

$$u(0,y,t)=v\,(0,y,t)=0 \tag{106}$$

- The transverse components of the electric field intensity are continuous across the surface of the half–space

$$E_2\,(0, y, t) = E_{20}\,(0, y, t) \tag{107}$$

- The transverse components of the magnetic field intensity are continuous across the surface of the half–space

$$h\,(0, y, t) = h_0\,(0, y, t) \tag{108}$$

Substituting from the expressions of considered variables into the above boundary conditions, we can obtain the following equations satisfied by the parameters

$$\sum_{i=1}^{3} \frac{\varepsilon_1 \omega}{[k_i^2 - a^2 - \omega\,(1 + \tau_o \omega)]}\; R_i = f^*(a, \omega) \tag{109}$$

$$G\,a^2 - \sum_{i=1}^{3} \frac{k_i R_i}{(\,k_i^2 - a^2)} = 0 \tag{110}$$

$$G\,m - \sum_{i=1}^{3} \frac{R_i}{(\,k_i^2 - a^2)} = 0 \tag{111}$$

$$\frac{G\,a^2}{\omega\,(\beta_1 + \varepsilon_2 \omega)} + \frac{n\,F}{\varepsilon_2\,\omega^3} + \sum_{i=1}^{3} \frac{k_i R_i}{(\,k_i^2 - a^2)\,[k_i^2 - a^2 - \omega\,(\,\beta_1 + \varepsilon_2 \omega\,)\,]} = 0 \tag{112}$$

$$\sum_{i=1}^{3} \frac{\omega\,R_i}{[k_i^2 - a^2 - \omega\,(\,\beta_1 + \varepsilon_2 \omega\,)]} \quad - \quad F = 0 \tag{113}$$

From Eqs (109)–(113), we get

$$G = \frac{1}{m} \sum_{i=1}^{3} \frac{1}{(k_i^2 - a^2)}\, R_i \tag{114}$$

$$F = \sum_{i=1}^{3} \frac{\omega}{[k_i^2 - a^2 - \omega\,(\,\beta_1 + \varepsilon_2 \omega)]}\; R_i \tag{115}$$

$$R_1 = \frac{f^*(a, \omega\,)\;[\,g_2\,L_3 - g_3\,L_2\,]}{S_1(\,g_2\,L_3 - g_3\,L_2) + S_2(\,g_3\,L_1 - g_1\,L_3) + S_3(\,g_1\,L_2 - g_2\,L_1)} \tag{116}$$

$$R_2 = \frac{f^*(a, \omega\,)\;[\,g_3\,L_1 - g_1\,L_3\,]}{S_1(\,g_2\,L_3 - g_2\,L_3) + S_2(\,g_3\,L_1 - g_1\,L_3) + S_3(\,g_1\,L_2 - g_2\,L_1)} \tag{117}$$

$$R_3 = \frac{f^*(a, \omega\,)\;[\,g_1\,L_2 - g_2\,L_1\,]}{S_1(\,g_2\,L_3 - g_2\,L_3) + S_2(\,g_3\,L_1 - g_1\,L_3) + S_3(\,g_1\,L_2 - g_2\,L_1)} \tag{118}$$

where,

$$L_i = \frac{a^2 - m\,k_i}{(\,k_i^2 - a^2)} \tag{119}$$

$$g_i = \frac{1}{(\,k_i^2 - a^2)} \left\{ \frac{a^2}{m\,\omega\,(\,\beta_1 + \varepsilon_2\,\omega)} \right.$$

$$\left. + \frac{n\,(\,k_i^2 - a^2) + k_i \varepsilon_2\,\omega^2}{\varepsilon_2\,\omega^2 [\,k_i^2 - a^2 - \omega\,(\,\beta_1 + \varepsilon_2\,\omega)]} \right\} \tag{120}$$

4. Numerical Results

The copper material is chosen for numerical evaluations. In the calculation process, the material constants necessary to be known can be found in [18].

The thermal shock $f\,(y,t)$ applied on the surface, is taken of the form

$$f\,(y,t) = \theta_o H\,(\,L - |\,y\,|\,)\,\mathbf{exp}(\,-\,b\,t\,) \tag{121}$$

where H is the Heaviside unit step function and $\theta_o$is a constant. This means that heat is applied on the surface of the half-space on a narrow band of width 2L surrounding the y-axis to keep it at temperature θ_o,while the rest of the surface is kept at zero temperature.

Since we have$\omega = \omega_o + i\,\zeta$, where i is an imaginary unit, $e^{\omega\,t} = e^{\omega_o t}(\,cos\,\zeta\,t + i\,sin\,\zeta\,t)$ and for small values of time, we can let $\omega = \omega_o$. The other constant of the problem are taken as $L = 4, \theta_o = 1$, $b = 1$, $\nu_o = 0.05$, $\tau_o = 0.03$, $\omega_o = 1$, $a = 1.2$. Considering the distributions of temperature, displacement, stress, induced magnetic field and induced electric field for $y = 0$ at $t = 0.01$ and $t = 0.3$ respectively. Calculated results of the real part of the non–dimensional temperature, displacement, stress, induced magnetic field and induced electric field are shown in Figs 1–10 respectively. The graph shows the four curves predicted by the different theories of thermo–elasticity. In these figures the solid lines represent the solution for Green–Lindsay' theory and the dashed lines represent the solution corresponding to using the coupled equation of heat conduction $(\,\nu_o = 0, \tau_o = 0\,)$. The phenomenon of finite speeds of propagation is manifested in all these figures. The medium deforms because of thermal shock, and due to the application of the magnetic field, there result an induced magnetic field in the medium. This indicates the electromagneto–thermoelastic coupled effects. Due to the symmetries of geometrical shape and thermal boundary condition, the displacement component v, the component of stress $\sigma_{x\,y}$ and the induced electric field component E_1 are zero when $y = 0$.

5. Concluding remarks

In all figures, it is clear that all the distributions considered have a non-zero value only a bounded region of space. Outside this region the values vanish identically and this means that the region has not felt thermal disturbance yet. From the distributions of temperature, it can be found the wave type heat propagation in the medium. The heat wave front moves forward with a finite speed in the medium with

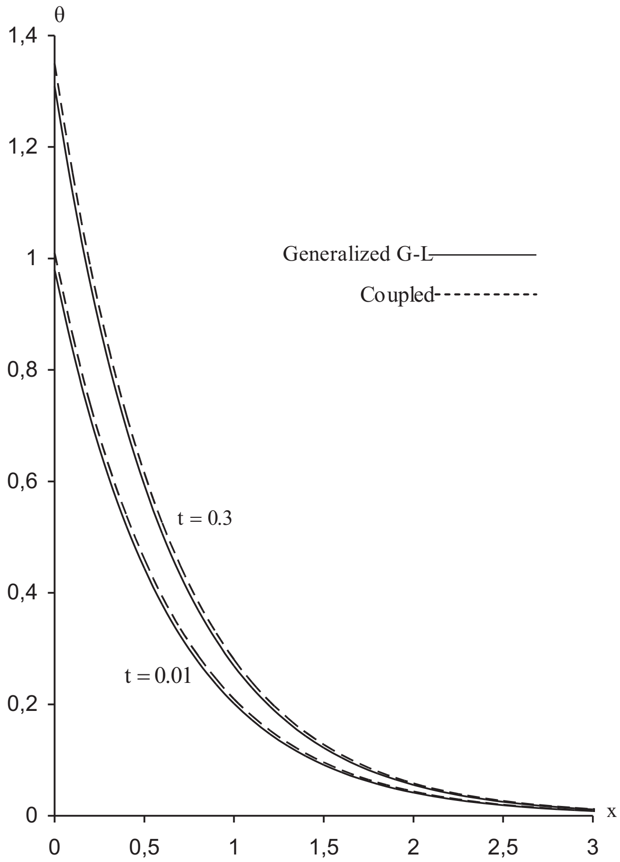

Figure 1 Temperature distribution for $y = 0$ of case 1

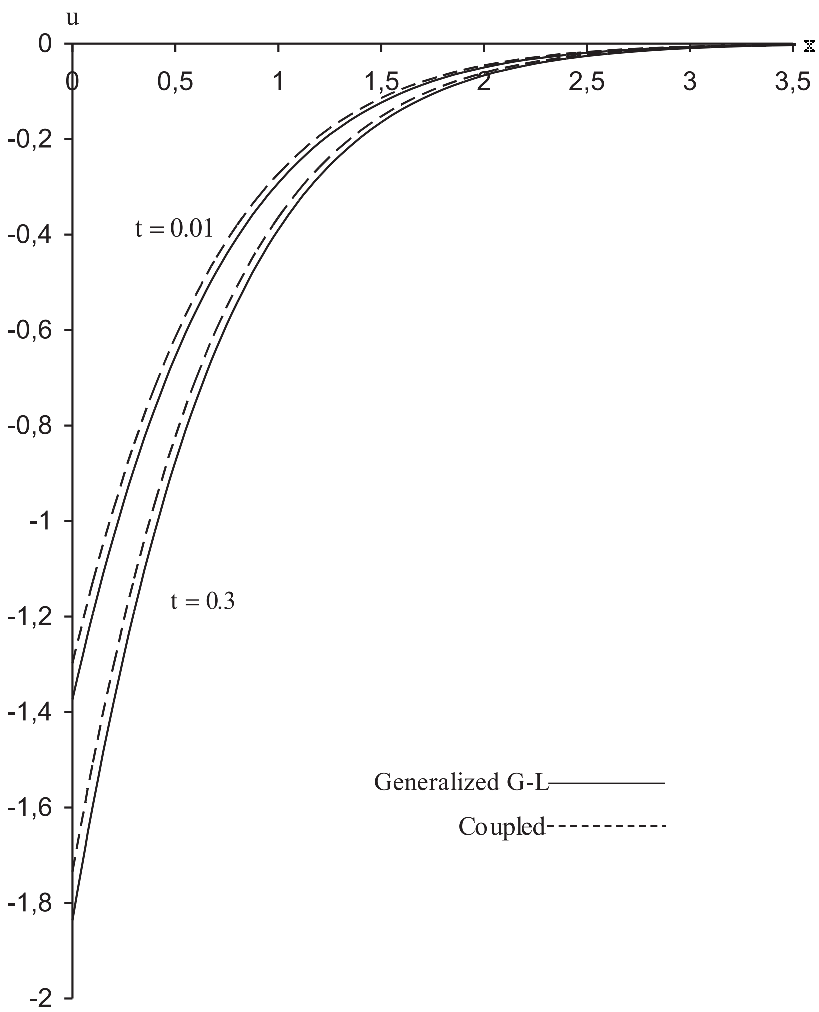

Figure 2 Horizontal displacement distribution for $y = 0$ of case 1

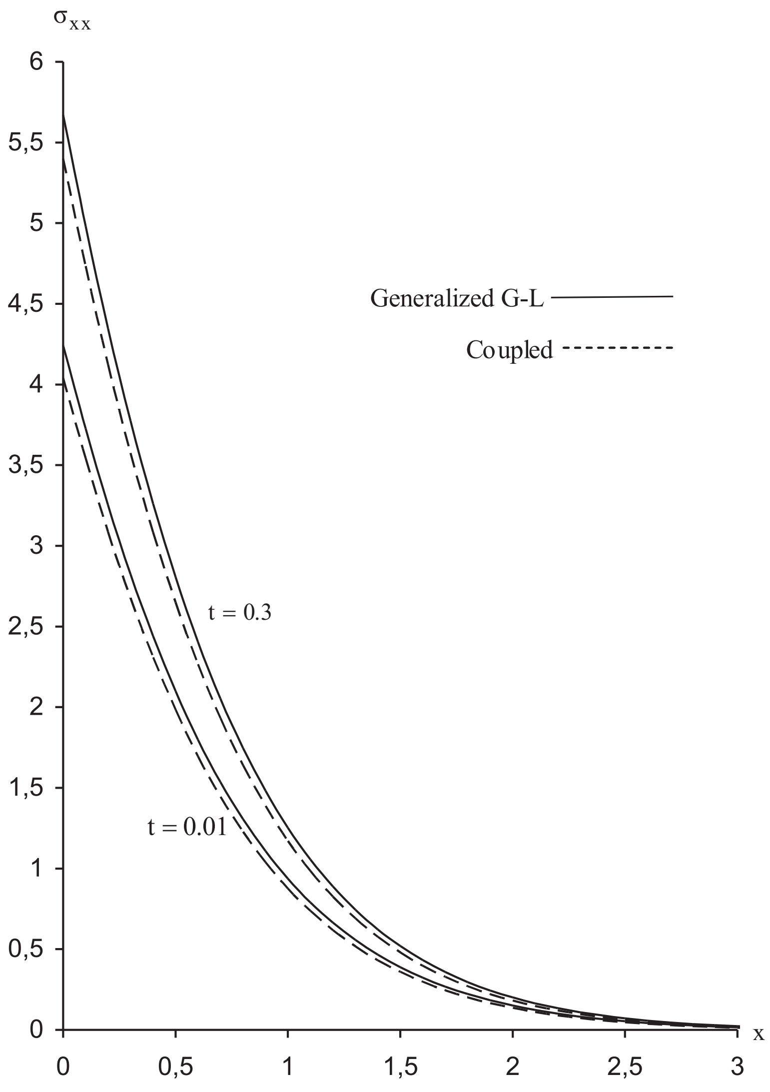

Figure 3 The distribution of stress component σ_{xx} for $y = 0$ of case 1

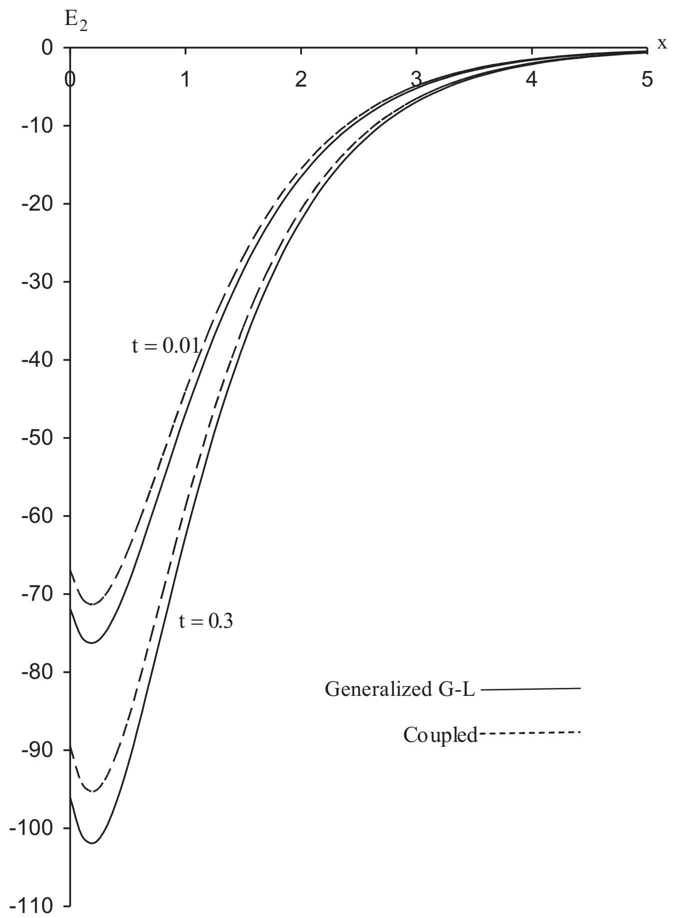

Figure 4 The distribution of induced electric component E_2 for $y = 0$ of case 1

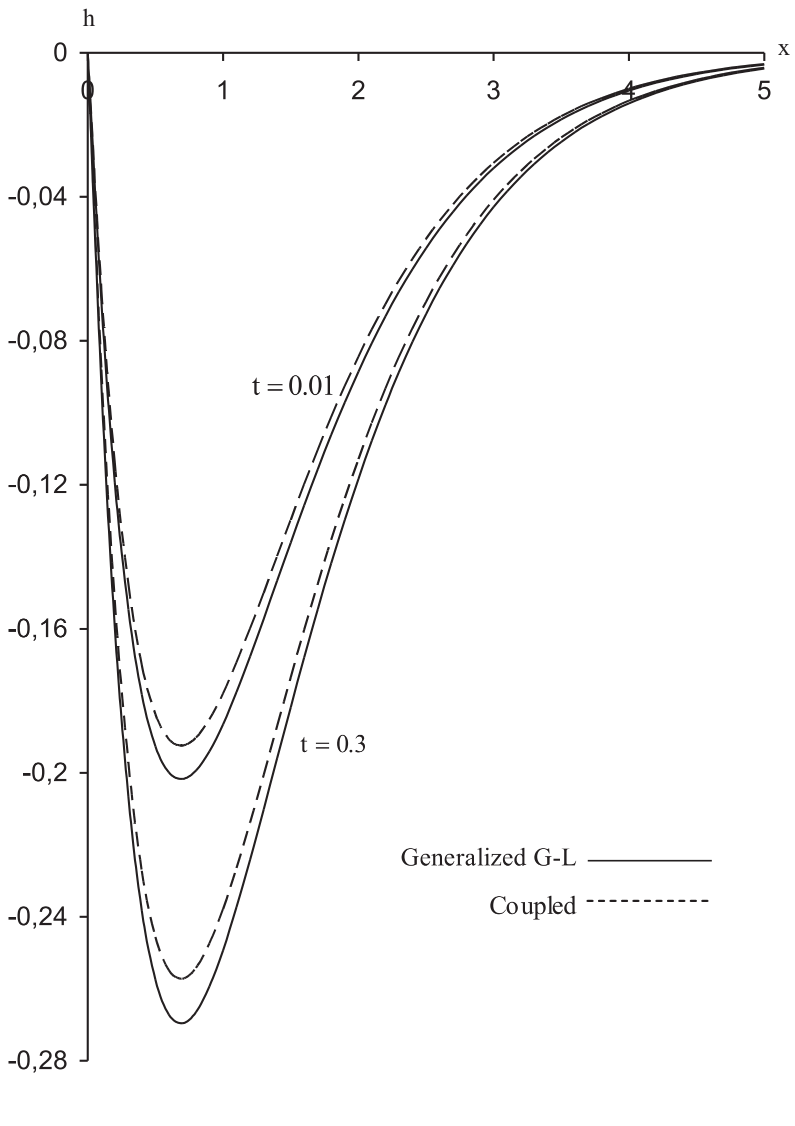

Figure 5 Induced magnetic filed distribution h for $y = 0$ of case 1

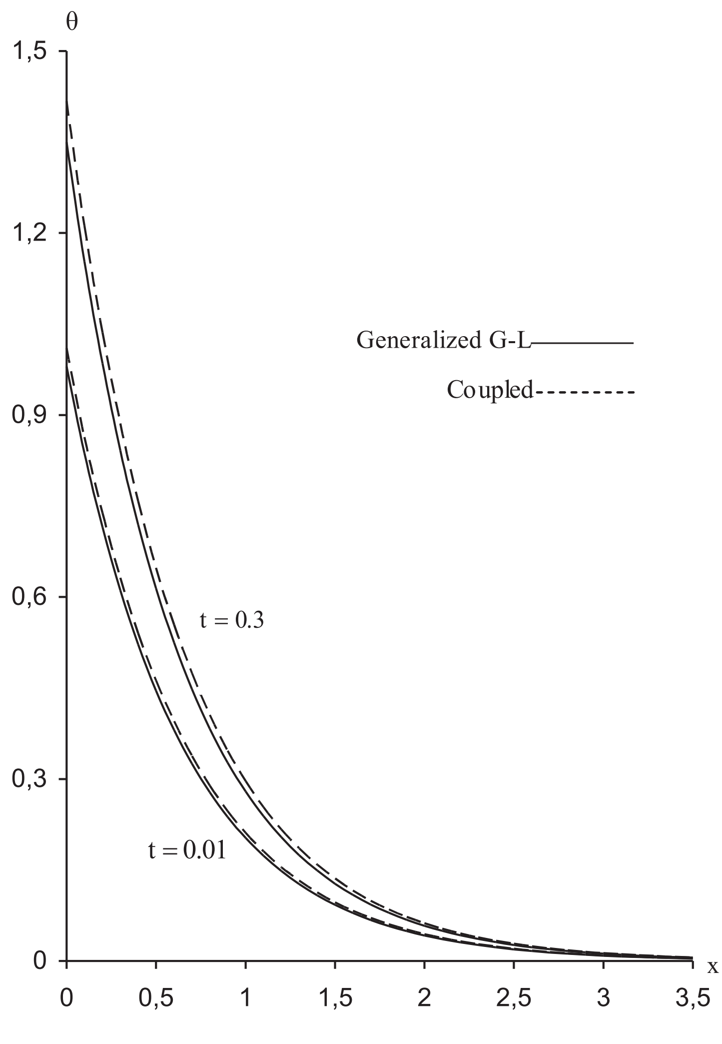

Figure 6 Temperature distribution for $y = 0$ of case 2

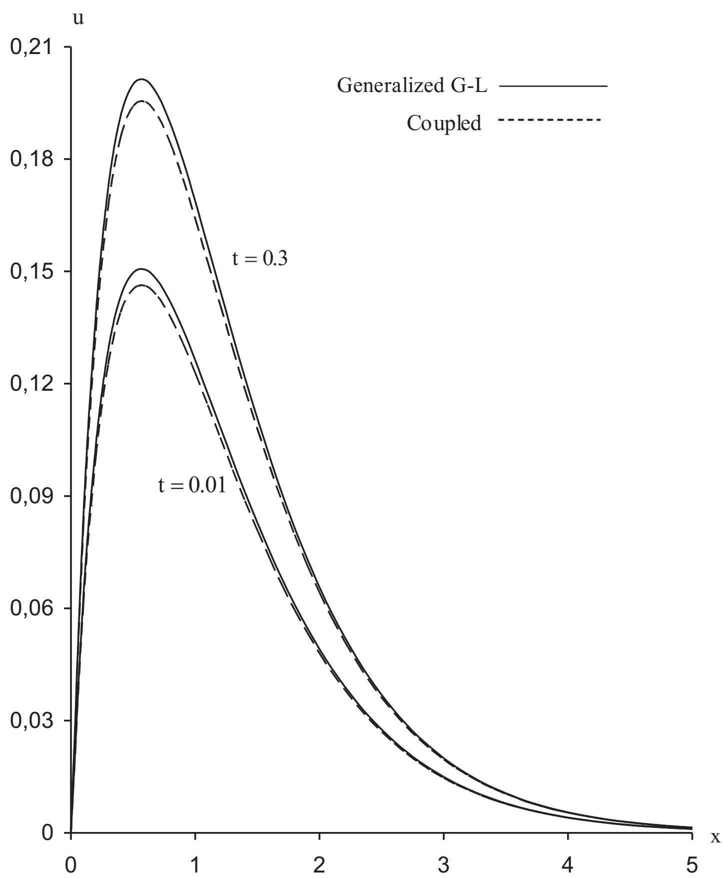

Figure 7 Horizontal displacement distribution for $y = 0$ of case 2

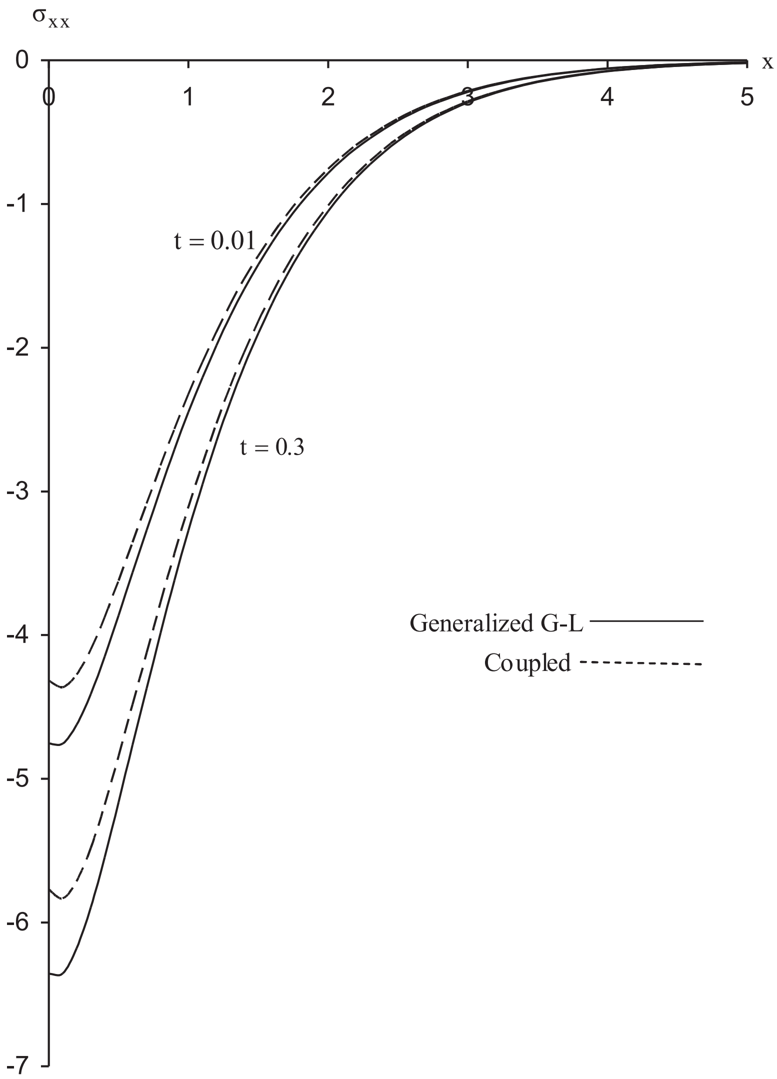

Figure 8 The distribution of stress component σ_{xx} for $y = 0$ of case 2

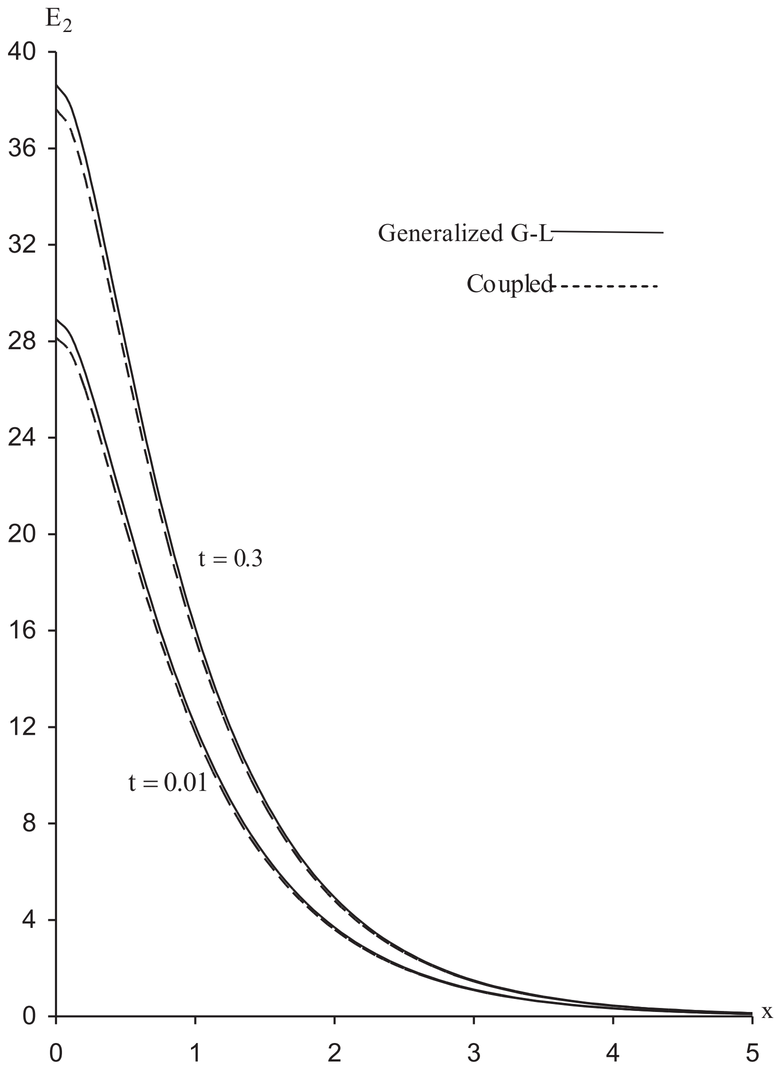

Figure 9 The distribution of induced electric component E_2 for $y = 0$ of case 2

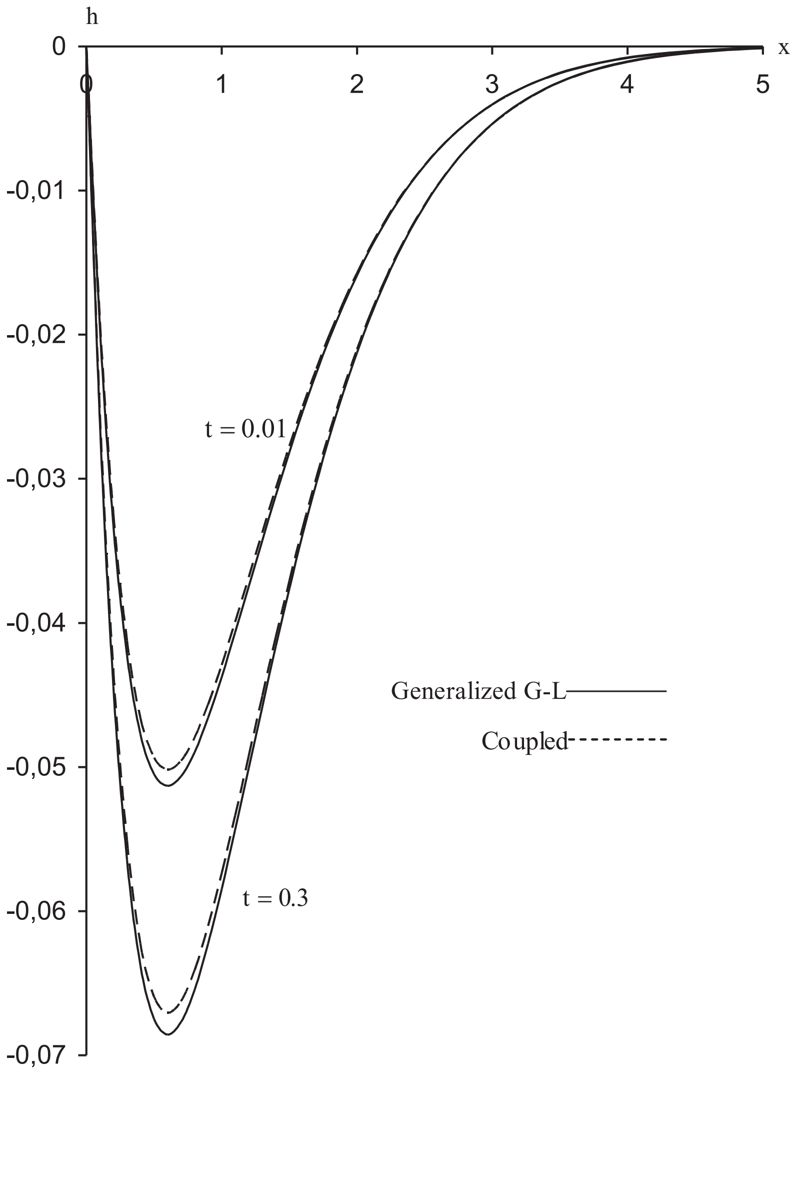

Figure 10 Induced magnetic filed distribution h for $y = 0$ of case 2

the passage of time. This is not the case for the classical theories of thermo–elasticity where an infinite speed of propagation is inherent and hence all the considered functions have a non–zero (although may be very small) value for any point in the medium. This indicates that the generalized heat conduction mechanism is completely different from the classic Fourier's in essence. In generalized thermo–elasticity theory heat propagates as a wave with finite velocity instead on infinite velocity in medium.

References

[1] **Lord, H. and Shulman, Y.**: A Generalized dynamical theory of thermoelasticity, *J. Mech. Phys. Solid*, 15, 299–309, **1967**.

[2] **Puri, P.**: Plane waves in thermoelasticity and magneto–elasticity, *Int. J. Engng Sci.*, 10, 467–476, **1972**.

[3] **Nayfeh, A.H. and Nasser, S.N.**: Thermoelastic waves in solids with thermal relaxation, *Acta Mech.*, 12, 53–69, **1971**.

[4] **Othman, M.I.A.**: Lord–Shulman theory under the dependence of the modulus of elasticity on the reference temperature in two–dimensional generalized thermo–elasticity, *J. Thermal Stresses*, 25, 1027–1045, **2002**.

[5] **Agarwal, V.K.**: On surface waves in generalized thermoelasticity, *J. Elasticity*, 8, 171–177, **1978**.

[6] **Nayfeh, A.H. and Nasser, S.N.**: Electromagneto–thermoelastic plane waves in solids with thermal relaxation, *J. Appl. Mech.*, 113, 108–113, **1972**.

[7] **Green, A.E. and Lindsay, K.A.**: Thermoelasticity, *J. Elast.*, 2, 1–7, **1972**.

[8] **Agarwal, V.K.**: On plane waves in generalized thermoelasticity, *Acta Mech.*, 31, 185–198, **1971**.

[9] **Agarwal, V.K.**: On electromagneto–thermoelastic plane waves, *Acta Mech.*, 10, 181–191, **1967**.

[10] **Othman, M.I.A.**: Effect of rotation on plane waves in generalized thermo–elasticity with two relaxation times, *Int. J. Solids and Structures*, 41, 2939–2956, **2004**.

[11] **Paria, G.**: Magneto–elasticity and magneto–thermoelasticity, *Adv. Appl. Mech.*, 10, 73, **1967**.

[12] **Roy Choudhuri, S.K.**: Temperature-rate dependent magneto–thermoelastic waves in a finitely conducting elastic half–space, *Computers Math. Applic.*, 19, 85–93, **1990**.

[13] **Ezzat, M.A. and Othman, M.I.A.**: Electromagneto–thermoelastic plane waves with two relaxation times in a medium of perfect conductivity, *Int. J. Engng. Sci.*, 38, 107–120, **2000**.

[14] **Ezzat, M.A., Othman, M.I.A. and Samaan, A.A.**: State space approach to two–dimensional electromagneto–thermoelastic problem with two relaxation times, *J. Thermal Stresses*, 39, 1383–1404, **2001**.

[15] **Dhaliwal, R. and Rokne, J.**: Thermal shock problem in generalized thermoelastic, *J. Thermal Stresses*, 12, 259–279, **1989**.

[16] **Othman, M.I.A.**: Relaxation effects on thermal shock problems in an elastic half–space of generalized magneto–thermoelastic waves, *Mechs. Mech. Engng*, 7, 156-278, **2004**.

[17] **Nowacki, W.**: Dynamic problem of thermoelasticity, *Noordhoff International the Netherlands*, p. 399, **1975**.

[18] **Sherief, H.H.**: Problem in electromagneto thermoelasticity for an infinitely long solid conducting circular cylinder with thermal relaxation, *Int. J. Engng. Sci.*, 32, 1137–1149, **1994**.

Nomenclature

λ, μ	Lamé's constants
ρ	density
C_E	specific heat at constant strain
t	time
T	absolute temperature
T_o	reference temperature chosen so that $\left\lvert \frac{T-T_o}{T_o} \right\rvert << 1$
σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
u_i	components of displacement vector
k	thermal conductivity
J	current density vector
μ_o	magnetic permeability
ε_o	electric permeability
σ_o	electric conductivity
$c_1^2=(\lambda+2\mu)/\rho$	
$c_2 = \sqrt{\mu/\rho}$,	velocity of transverse waves
$c^2 = 1/\mu_o\varepsilon_o$	sound speed
ν_o, τ_o	two relaxation times
e	cubical dilatation
α_t	coefficient of linear thermal expansion
$\gamma=(3\lambda+2\mu)\alpha_t$	
$\varepsilon_1 = \frac{\gamma^2 T_o}{\rho C_E(\lambda+2\mu)}$	
$\alpha = \frac{\mu_o H_o^2}{\lambda+2\mu}$	
$\beta_1 = \sigma_o\mu_o/\eta$	
$\varepsilon_2 = c_1^2/c^2$	
$\eta = \rho C_E/k$	
$\beta^2=(\lambda+2\mu)/\mu$	
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$	