Mechanics and Mechanical Engineering
Vol. 14, No. 1 (2010) 31–55
© Technical University of Lodz

The Effect of Rotation and Thermal Shock on a Perfect Conducting Elastic Half–Space in Generalized Magneto–thermoelasticity with Two Relaxation Times

Mohamed I. A. Othman
Faculty of Science, Department of Mathematics
Zagazig University
P.O. Box 44519, Zagazig Egypt

Received (18 October 2009)
Revised (15 January 2010)
Accepted (20 March 2010)

The propagation of electromagneto–thermoelastic disturbances produced by a thermal shock in a perfectly conducting elastic half–space when the entire elastic medium is rotating with a uniform angular velocity is investigated. The problem is in the context of the Green and Lindsay's generalized thermoelasticity with two relaxation times. There acts an initial magnetic field parallel to the plane boundary of the half–space. The medium deformed because of thermal shock, and due to the application of the magnetic field. The normal mode analysis is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented graphically. From the distributions, it can be found the wave type heat propagation in the medium. This indicates that the generalized heat conduction mechanism is completely different from the classic Fourier's in essence. In generalized thermoelasticity theory heat propagates as a wave with finite velocity instead of infinite velocity in medium. Comparisons are made with the results predicted by the coupled theory in present and absent rotation.

Keywords: Thermal relaxation times, generalized thermo–elasticity theory, rotation effect, normal mode analysis, electromagneto–thermoelastic couple

1. Introduction

In recent years considerable interest has been shown in the study of plane thermo–elastic and magneto–thermo–elastic wave propagation in a non–rotating medium. The classical theory of thermo–elasticity is based on Fourier's law of heat conduction which predicts an infinite speed of propagation of heat. This is physically absurd and many new theories have been proposed to eliminate this absurdity. Lord and Shulman [1] employed a modified version of the Fourier's law and deduced a theory of thermo–elasticity known as the generalized theory of thermo–elasticity. Lord and Shulman's theory with a thermal relaxation time has been used by several authors including Puri [2] and Nayfeh and Nemat–Nasser [3] to study plane thermo–elastic waves in non–rotating infinite media. Surface waves have also studied by Agarwal [4] in the generalized thermo–elasticity. Ezzat and Othman [5] have studied the generalized magneto–thermo–elasticity plane waves with thermal relaxation in a non–rotating medium of perfect conductivity by using the state space approach. Othman [6], [7] used the normal mode analysis to study two–dimensional problems of generalized thermo–elasticity with one relaxation time with the modulus of elasticity dependent on the reference temperature for non-rotating and rotating medium, respectively. Othman [8] have studied the problem of two–dimensional electro–magneto–thermovisco–elasticity for a thermally and electrically conducting half–space solid whose surface is subjected to a thermal shock with one relaxation time.

Green and Lindsay [9] have presented a theory of thermo–elasticity with certain special features that contrast with the previous theory having a thermal relaxation time. In Green and Lindsay's theory Fourier's law of heat conduction is unchanged whereas the classical energy equation and the stress–strain–temperature relations are modified. Two constitutive constants ν_o, τ_o having the dimensions of time appear in the governing equations in place of one relaxation time τ_o in Lord–Shulman's theory. Using the Green–Lindsay's theory, Agarwal [10,11] studied respectively thermo–elastic and magneto–thermo–elastic plane wave propagation in an infinite non-rotation medium. In a paper by Schoenberg and Censor [12], the propagation of plane harmonic waves in a rotating elastic medium without a thermal field has been studied. It was shown there that the rotation causes the elastic medium to be depressive and anisotropic.

Investigation of the interaction between magnetic field and stress and strain in a thermoelastic solid is very important due to its many applications in the field of geophysics, plasma physics and related topics. Especially in nuclear fields, the extremely high temperatures and temperature gradients, as well as the magnetic fields originating inside nuclear reactors, influence their design and operations.

Great attention has been devoted to the study of electromagneto–thermoelastic coupled problems based on the generalized Thermoelastic theories. In the context of Green and Lindsay's theory, Roy Choudhuri and Chatterjee Roy [13] considered a one–dimensional problem of magneto–thermoelastic waves in a finitely conducting elastic half–space subjected to a thermal shock; Ezzat and Othman [14] applied the normal mode analysis to a problem of two–dimensional electro–magneto–thermoelastic plane waves with two relaxation times in a medium of perfect conductivity, and they surveyed a electro–magneto–thermoelastic problem by state–space approach in [15]. Dhaliwal and Rokne [16] have solved a thermal shock problem. Recently Othman [17] considered a problem of plane wave propagation in a rotating medium in generalized thermo–elasticity with two relaxation times.
In the present work we shall formulate the normal mode analysis to electromagneto–thermoelastic coupled two–dimensional problem of a thermally and perfect conducting half–space solid subjected to a thermal shock on its surface when the entire medium rotates with a uniform angular velocity. The magneto–thermoelastic coupled governing equations under the effect of Centripetal acceleration are established. The normal mode analysis is used to obtain the exact expressions for the considered variables. The distributions of the considered variables in present and absent rotation are represented graphically. From the distributions, it can be found the

wave type heat propagation in the medium.

2. Formulation of the Problem and Basic Equations

We consider the problem of a thermo–elastic half–space ($z \geq 0$). A magnetic field with constant intensity $\mathbf{H} = (0, H_o, 0)$ acts parallel to the bounding plane (take as the direction of the y–axis). The surface of the half–space is subjected at time $t = 0$ to a thermal shock that is a function of z and t. Thus, all quantities considered will be functions of the time variable t and of the coordinates x and z. The elastic medium is rotating uniformly with an angular velocity $\mathbf{\Omega} = \Omega\,\mathbf{n}$, where $\mathbf{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame of reference has two additional terms [17]: Centripetal acceleration, $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})$ due to time–varying motion only and the Corioli's acceleration $2\,\mathbf{\Omega} \times \dot{\mathbf{u}}$ where $\mathbf{u}$ is the dynamic displacement vector. These terms don't appear in non–rotating media.

Due to the application of initial magnetic field $\mathbf{H}$, there results an induced magnetic field $\mathbf{h}$ and an induced electric field $\mathbf{E}$. The simplified linear equations of electrodynamics of slowly moving medium for a homogeneous, thermally and electrically conducting elastic solid are [18],

$$\mathrm{curl}\,\mathbf{h} = \mathbf{J} + \varepsilon_o \dot{\mathbf{E}} \tag{1}$$

$$\mathrm{curl}\;\mathbf{E} = -\,\mu_o \dot{\mathbf{h}} \tag{2}$$

$$\mathrm{div}\;\mathbf{h} = 0 \tag{3}$$

$$\mathbf{E} = -\mu_o(\dot{\mathbf{u}} \times \mathbf{H}) \tag{4}$$

where $\dot{\mathbf{u}}$ is the particle velocity of the medium, and the small effect of temperature gradient on $\mathbf{J}$ is also ignored. The dynamic displacement vector is actually measured from a steady–state deformed position and the deformation is supposed to be small.

The displacement equation of motion in a rotating frame of reference is

$$\begin{aligned}\rho\,[\,\ddot{\mathbf{u}} + \mathbf{\Omega} \times (\,\mathbf{\Omega} \times \mathbf{u}\,) + 2\,\mathbf{\Omega} \times \dot{\mathbf{u}}\,] \; &= (\,\lambda + \mu\,)\,\nabla\,(\,\nabla \cdot \mathbf{u}\,) \\ + \,\mu\,\nabla^2\mathbf{u} \; + \mu_o(\,\mathbf{J} \times \mathbf{H}\,) - \;&\gamma\,(\,1 + \nu_o \frac{\partial}{\partial\,t}\,)\nabla\,T\end{aligned} \tag{5}$$

In the absence of the body force and inner heat source, the generalized electro–magneto–thermoelastic governing differential equations in the context of Green and Lindsay's theory are

$$\sigma_{i\,j} = \;\; 2\;\mu\;\; e_{i\,j} \;+\;\; \lambda\;e\,\delta_{i\,j} - \;\gamma\,(\,T - T_o + \nu_o \dot{T}\,)\;\delta_{i\,j} \tag{6}$$

the heat conduction equation

$$kT_{,i\,i} = \rho\,C_E(\,\dot{T} + \tau_o \ddot{T}\,) + \;\gamma T_o \dot{u}_{i\,,i} \tag{7}$$

and strain–displacement relations

$$e_{i\,j} = \frac{1}{2}\;\;(\;u_{i\,,j} + u_{j,\,i}\;) \tag{8}$$

In the above equations, a comma followed by a suffix denotes material derivative and a superposed dot denotes the derivative with respect to time, $i, j = x, z$.

The displacement components have the following form

$$u_x = u(x, z, t), \qquad u_y = 0, \qquad u_z = w(x, z, t) \tag{9}$$

From Eqs. (8) and (9), we obtain the strain components

$$\begin{aligned} & e_{xx} = \frac{\partial u}{\partial x}, \qquad e_{yy} = 0, \qquad e_{zz} = \frac{\partial w}{\partial z}, \qquad e_{xy} = e_{yz} = e_{yy} = 0 \\ & e_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \qquad e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \end{aligned} \tag{10}$$

From Eqs. (6) and (10), the stress components are given by

$$\sigma_{xx} = (\lambda + 2\mu)u_{,x} + \lambda w_{,z} - \gamma(T - T_o + \nu_o \dot{T}) \tag{11}$$

$$\sigma_{zz} = (\lambda + 2\mu)w_{,z} + \lambda u_{,x} - \gamma(T - T_o + \nu_o \dot{T}) \tag{12}$$

$$\sigma_{xy} = \mu\,(u_{,z} + w_{,x}) \tag{13}$$

The components of the magnetic intensity vector in the medium are

$$H_x = 0, \qquad H_y = H_o + h(x, z, t), \qquad H_z = 0 \tag{14}$$

The electric intensity vector is normal to both the magnetic intensity and the displacement vectors. Thus, it has the components

$$E_x = E_1, \qquad E_y = 0, \qquad E_z = E_3 \tag{15}$$

The current density vector **J** be parallel to **E**, thus

$$J_x = J_1, \qquad J_y = 0, \qquad J_z = J_3 \tag{16}$$

From Eqs. (1)–(4) and (5), we get

$$\begin{aligned} & \rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \dot{w}\right] = (\lambda + \mu)\frac{\partial e}{\partial x} + \mu \nabla^2 u \\ & -\gamma\left(1 + \nu_o \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} - \mu_o H_o \frac{\partial h}{\partial x} - \varepsilon_o \mu_o^2 H_o^2 \frac{\partial^2 u}{\partial t^2} \end{aligned} \tag{17}$$

$$\begin{aligned} & \rho\left[\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \dot{u}\right] = (\lambda + \mu)\frac{\partial e}{\partial z} + \mu \nabla^2 w \\ & -\gamma\left(1 + \nu_o \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} - \mu_o H_o \frac{\partial h}{\partial z} - \varepsilon_o \mu_o^2 H_o^2 \frac{\partial^2 w}{\partial t^2} \end{aligned} \tag{18}$$

We introduce the displacement potentials φ and ψ by the relations

$$u = \varphi_{,x} + \psi_{,z}, \qquad w = \varphi_{f,z} - \psi_{,x} \tag{19}$$

we can obtain from Eqs. (1)–(4)

$$h = -H_o \nabla^2 \varphi \tag{20}$$

For convenience, the following non–dimensional variables are used:

$$\bar{x}_i = \frac{x_i}{C_T\omega^*} \qquad \bar{u}_i = \frac{u_i}{C_T\omega^*} \qquad \bar{\varphi} = \frac{\varphi}{(C_T\omega^*)^2} \qquad \bar{\psi} = \frac{\psi}{(C_T\omega^*)^2}$$

$$\bar{\tau}_o = \frac{\tau_o}{\omega^*} \qquad \bar{\nu}_o = \frac{\nu_o}{\omega^*} \qquad \bar{\Omega} = \omega^*\Omega \qquad \theta = \frac{\gamma\,(T - T_o)}{\lambda + 2\,\mu} \tag{21}$$

$$\bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\mu}, \qquad \bar{h} = \frac{h}{H_o}, \quad i = 1, 2.$$

In terms of the non–dimensional quantities defined in Eq. (21), the above governing equations reduce to (dropping the dashed for convenience)

$$\beta^2[\alpha\,\ddot{u} - \Omega^2 u + 2\Omega\dot{w}] = (\beta^2 - 1)\frac{\partial e}{\partial x} + \nabla^2 u$$

$$-\beta^2\left(1 + \nu_o\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial x} - R_H\frac{\partial h}{\partial x} \tag{22}$$

$$\beta^2[\alpha\,\ddot{w} - \Omega^2 w + 2\Omega\dot{u}] = (\beta^2 - 1)\frac{\partial e}{\partial z} + \nabla^2 w$$

$$-\beta^2\left(1 + \nu_o\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial z} - R_H\frac{\partial h}{\partial z} \tag{23}$$

$$\nabla^2\theta = \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\theta + \bar{\varepsilon}\frac{\partial e}{\partial t} \tag{24}$$

The constitutive equations reduce to

$$\sigma_{xx} = (\beta^2 - 2)\,e + 2\,u_{,x} - \beta^2\left(\theta + \nu_o\frac{\partial\theta}{\partial t}\right) \tag{25}$$

$$\sigma_{zz} = (\beta^2 - 2)\,e + 2\,w_{,z} - \beta^2\left(\theta + \nu_o\frac{\partial\theta}{\partial t}\right) \tag{26}$$

$$\sigma_{xz} = u_{,z} + w_{,x} \tag{27}$$

In the subsequent analysis we are taking into consideration the case of low speed so that centrifugal stiffening effects can be neglected. By differentiating Eq.(22) with respect to x, and Eq. (23) with respect to z, then adding, we obtain

$$\left[\alpha\frac{\partial^2}{\partial t^2} - \Omega^2 - (1 + R_H)\nabla^2\right]\varphi = -2\,\Omega\frac{\partial\psi}{\partial t} - \left(1 + \nu_o\frac{\partial}{\partial t}\right)\theta \tag{28}$$

by differentiating (22) with respect to z and (23) with respect to x and subtracting we obtain

$$[\beta^2\alpha\frac{\partial^2}{\partial t^2} - \beta^2\Omega^2 - \nabla^2]\,\psi = -2\,\beta^2\Omega\frac{\partial\varphi}{\partial t} \tag{29}$$

Equation (24) take the form

$$\left[\nabla^2 - \left(\frac{\partial}{\partial t} + \tau_o\frac{\partial^2}{\partial t^2}\right)\right]\theta = \varepsilon\,\nabla^2\frac{\partial\varphi}{\partial t} \tag{30}$$

Equation (20) has the form

$$h = -\nabla^2\varphi\,. \tag{31}$$

where R_H is the number of magnetic pressure. It is a measure of the relative importance of magnetic effects in comparison with mechanical ones.

3. Normal Mode Analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$\left[u, w, e, \theta, h, \varphi, \psi, \sigma_{ij} \right] (x, z, t) \\ = \left[u^* f, w^*, e^*, \theta^*, h^*, \varphi^*, \psi^*, \sigma_{ij}^* \right] (x) \exp(\omega\, t + i\, a\, z) \tag{32}$$

where ω is the complex time constant and a is the wave number in the y–direction.

Using Eq. (32), Eqs (28), (29) and (30) take the form

$$[(1+R_H)\,(D^2-a^2) - \alpha\,\omega^2 + \Omega^2\,]\,\varphi^*(x) \\ = (\,1+\nu_o\omega)\,\theta^*(x) + 2\,\omega\,\Omega\,\psi^*(x) \tag{33}$$

$$[\,D^2 - a^2 - \beta^2(\,\alpha\,\omega^2 - \Omega^2)\,]\,\psi^*(x) = 2\,\omega\,\Omega\,\beta^2\varphi^*(x) \tag{34}$$

$$[\,D^2 - a^2 - \omega(1+\tau_o\,\omega)\,]\,\theta^*(x) = \varepsilon\,\omega\,(\,D^2 - a^2)\,\varphi^*(x) \tag{35}$$

where $D = \frac{\partial}{\partial x}$

Eliminating $\theta^*(x)$ and $\psi^*(x)$ between Eqs (33), (34) and (35), we obtain the following sixth–order partial differential equation satisfied by $\varphi^*(x)$

$$(\,D^6 - A\,D^4 + B\,D^2 - C\,)\,\varphi^*(x) = 0 \tag{36}$$

where

$$A = 3\,a^2 + b_1 \tag{37}$$

$$B = 3\,a^4 + 2\,a^2 b_1 + b_2 \tag{38}$$

$$C = a^6 + a^4 b_1 + a^2 b_2 + b_3$$

$$b_1 = \frac{1}{(1+R_H)}\left[\,(\,\alpha\,\omega^2 - \Omega^2)\,[\,1+\beta^2(\,1+R_H)\,] \right. \\ \left. + \varepsilon\,\omega(\,1+\nu_o\omega\,) + \omega\,(1+\tau_o\omega\,)\,(1+R_H)\right] \tag{39}$$

$$b_2 = \frac{1}{(\,1+R_H)}\left\{\,(\,\alpha\,\omega^2 - \Omega^2)\,\left[\beta^2(\,\alpha\,\omega^2 - \Omega^2) \right.\right. \\ \left.\left. + \omega\,(\,1+\tau_o\omega\,)\,[1+\beta^2(\,1+R_H)\,] + \varepsilon\,\beta^2\omega(\,1+\nu_o\omega\,)\right] - 4\,\omega^2\Omega^2\beta^2\right\} \tag{40}$$

$$b_3 = \frac{\beta^2\omega\,(\,1+\tau_o\omega\,)}{(\,1+R_H)}\,\left\{\,(\,\alpha\,\omega^2 - \Omega^2)^2 - 4\,\omega^2\Omega^2\,\right\} \tag{41}$$

In a similar manner we arrive at

$$(\,D^6 - A\,D^4 + B\,D^2 - C\,)\,\theta^*(x) = 0 \tag{42}$$

$$(\,D^6 - A\,D^4 + B\,D^2 - C\,)\,\psi^*(x) = 0 \tag{43}$$

Eq. (36) can be factorized as

$$(\,D^2 - k_1^2\,)\,(\,D^2 - k_2^2\,)\,(\,D^2 - k_3^2\,)\,\varphi^*(x) = 0 \tag{44}$$

where k_i^2 $(\,i = 1, 2, 3)$ is the root of the following characteristic equation

$$k^6 - A\,k^4 + B\,k^2 - C\, = 0 \tag{45}$$

The solution of Eq. (44) has the form

$$\varphi^*(x) = \sum_{i=1}^{3} \varphi_i^*(x) \tag{46}$$

where $\varphi_i^*(x)$ is the solution of the equation

$$(\,D^2 - k_i^2\,)\,\varphi_i^*(x) = 0\,, \qquad i = 1, 2, 3 \tag{47}$$

The solution of Eq. (47) which is bounded as $x \to \infty$, is given by

$$\varphi_i^*(x) = M_i(a, \omega)\, e^{-k_i x} \tag{48}$$

Thus, $\varphi^*(x)$ has the form

$$\varphi^*(x) = \sum_{i=1}^{3} M_i(a, \omega)\, e^{-\,k_i\, x} \tag{49}$$

In a similar manner, we get

$$\theta^*(x) = \sum_{i=1}^{3} M'_i(a, \omega)\, e^{-\,k_i\, x} \tag{50}$$

$$\psi^*(x) = \sum_{i=1}^{3} M''_i(a, \omega)\, e^{-\,k_i\, x} \tag{51}$$

where $M_i(a, \omega)$, $M'_i(a, \omega)$ and $M''_i(a, \omega)$ are some parameters depending on a and ω.

Substituting from Eqs (49)–(51) into Eqs (33) and (35) we get the following relations

$$M'_i(a, \omega) = \frac{\varepsilon\, \omega\,(\,k_i^2 - a^2)}{[k_i^2 - a^2 - \omega\,(\,1 + \tau_o \omega\,)]}\, M_i, \qquad i = 1, 2, 3 \tag{52}$$

$$M''_i(a, \omega) = \frac{2\omega\,\Omega\,\beta^2}{[k_i^2 - a^2 -\ \beta^2\,(\alpha\,\omega^2 - \Omega^2\,)]}\, M_i, \qquad i = 1, 2, 3 \tag{53}$$

Substituting from Eqs (52) and (53) into Eqs (50) and (51) respectively, we obtain

$$\theta^*(x) = \sum_{i=1}^{3} \frac{\varepsilon\, \omega\,(\,k_i^2 - a^2)}{[k_i^2 - a^2 - \omega\,(\,1 + \tau_o \omega\,)]}\, M_i\, e^{-\,k_i x} \tag{54}$$

$$\psi^*(x) = \sum_{i=1}^{3} \frac{2\omega\,\Omega\,\beta^2}{[k_i^2 - a^2 -\ \beta^2\,(\alpha\,\omega^2 - \Omega^2\,)]}\, M_i\, e^{-\,k_i\, x} \tag{55}$$

In order to obtain the displacement components u and w in terms of Eq. (19), from

Eqs (49) and (55), we can obtain

$$u(x,z,t) = \sum_{i=1}^{3}\left[-k_i + \frac{2\,i\,a\,\omega\Omega\,\beta^2}{[k_i^2 - a^2 - \beta^2(\alpha\,\omega^2 - \Omega^2)\,]}\right] M_i e^{-k_i x} e^{(\,i\,a\,z+\omega t\,)} \tag{56}$$

$$w(x,z,t) = \sum_{i=1}^{3}\left[\,i\,a - \frac{2\,\omega\Omega\,\beta^2 k_i}{[k_i^2 - a^2 - \beta^2(\alpha\,\omega^2 - \Omega^2)\,]}\right] M_i e^{-k_i x} e^{(\,i\,a\,z+\omega t\,)} \tag{57}$$

From Eq. (31) in terms of Eq. (49), we can obtain

$$h(x,z,t) = -\sum_{i=1}^{3}\left[\,k_i^2 - a^2\,\right] M_i e^{-k_i x} e^{(\,i\,a\,z+\omega t\,)} \tag{58}$$

Substituting from Eqs (49), (54), (55) into Eqs (25)-(27) we obtain

$$\sigma_{x\,x}(x,z,t) = \sum_{i=1}^{3}\left\{k_i^2 - \frac{4\,i\,a\,\omega\Omega\,\beta^2 k_i}{[k_i^2 - a^2 - \beta^2(\alpha\,\omega^2 - \Omega^2)\,]} - a^2(\,\beta^2 - 2\,) - \frac{\beta^2\varepsilon\,\omega\,(\,1+\nu_o\omega\,)\,(\,k_i^2 - a^2\,)}{[k_i^2 - a^2 - \omega\,(\,1+\tau_o\omega\,)\,]}\right\} M_i e^{-k_i x} e^{(\,i\,a\,z+\omega t\,)} \tag{59}$$

$$\sigma_{z\,z}(x,z,t) = \sum_{i=1}^{3}\left\{(\,\beta^2 - 2\,)k_i^2 - \frac{4\,i\,a\,\omega\Omega\,\beta^2 k_i}{[k_i^2 - a^2 - \beta^2(\alpha\,\omega^2 - \Omega^2)\,]} - a^2\beta^2 - \frac{\beta^2\varepsilon\,\omega\,(\,1+\nu_o\omega\,)\,(\,k_i^2 - a^2\,)}{[k_i^2 - a^2 - \omega\,(\,1+\tau_o\omega\,)\,]}\right\} M_i e^{-k_i x} e^{(\,i\,a\,z+\omega t\,)} \tag{60}$$

$$\sigma_{x\,z}(x,z,t) = 2\sum_{i=1}^{3}\left[-i\,a\,k_i + \frac{\omega\Omega\,\beta^2(\,k_i^2 - a^2)}{[k_i^2 - a^2 - \beta^2(\alpha\,\omega^2 - \Omega^2)\,]}\right] M_i e^{-k_i x} e^{(\,i\,a\,z+\omega t\,)} \tag{61}$$

The normal mode analysis is, in fact, to look for the solution in the Fourier transformed domain. Assuming that all the relations are sufficiently smooth on the real line such that the normal mode analysis of these functions exist.

In order to determine the parameters $M_i (i = 1, 2, 3)$, we need to consider the boundary conditions at $x = 0$ as following

1. Thermal boundary condition that the surface of the half–space subjected to a thermal shock

$$\theta\,(0, y, t) = f(y, t) \tag{62}$$

2. Mechanical boundary condition that the surface of the half–space is traction free

$$\sigma_{x\,x}\,(0, y, t) = 0 \tag{63}$$

$$\sigma_{x\,z}\,(0, y, t) = 0 \tag{64}$$

Substituting from the expressions of considered variables into the above boundary conditions, we can obtain the following equations satisfied by the parameters

$$\sum_{i=1}^{3} \frac{\varepsilon\,\omega\,(\,k_i^2 - a^2)}{[k_i^2 - a^2 - \omega\,(\,1 + \tau_o\omega\,)]}\, M_i \;= f^*(\,a, \omega\,) \tag{65}$$

$$\sum_{i=1}^{3} \left\{ k_i^2 - a^2(\beta^2 - 2) - \frac{4\,i\,a\,\omega\Omega\,\beta^2 k_i}{[k_i^2 - a^2 - \beta^2(\alpha\,\omega^2 - \Omega^2)]} \right.$$

$$\left. - \frac{\beta^2\varepsilon\,\omega\,(1 + \nu_o\omega\,)\,\,(k_i^2 - a^2)}{[k_i^2 - a^2 - \omega\,(\,1 + \tau_o\omega\,)]} \right\} M_i = 0 \tag{66}$$

$$\sum_{i=1}^{3} \left[-i\,a\,k_i + \frac{\omega\Omega\,\beta^2(\,k_i^2 - a^2)}{[k_i^2 - a^2 - \beta^2(\alpha\,\omega^2 - \Omega^2)\,]} \right] M_i = 0 \tag{67}$$

Solving Eqs (65)–(67), we get the parameters $M_i (i = 1, 2, 3)$ with the following from respectively

$$M_1 = \frac{f^*(a, \omega\,)\;[\,(c_{11}c_{44} + d_{11}d_{44}) + i\,(\,c_{11}d_{44} - c_{44}d_{11})\,]}{c_{44}^2 - d_{44}^2} \tag{68}$$

$$M_2 = \frac{f^*(a, \omega\,)\;[\,(c_{22}c_{44} + d_{22}d_{44}) + i\,(\,c_{22}d_{44} - c_{44}d_{22})\,]}{c_{44}^2 - d_{44}^2} \tag{69}$$

$$M_3 = \frac{f^*(a, \omega\,)\;[\,(c_{33}c_{44} + d_{33}d_{44}) + i\,(\,c_{33}d_{44} - c_{44}d_{33})\,]}{c_{44}^2 - d_{44}^2} \tag{70}$$

where

$$S_j = \frac{\varepsilon\,\omega\,(\,k_j^2 - a^2)}{[k_j^2 - a^2 - \omega\,(1 + \tau_o\omega)]}\,, \qquad j = 1, 2, 3 \tag{71}$$

$$a_{j1} = k_j^2 - a^2(\,\beta^2 - 2\,) - \,\beta^2(\,1 + \nu_o\omega\,)\,S_j\,, \qquad j = 1, 2, 3 \tag{72}$$

$$b_{j1} = \frac{4\,a\,\omega\,\Omega\,\beta^2 k_j}{[\,k_j^2 - a^2 - \beta^2(\,\alpha\,\omega - \Omega^2)\,]}\,, \qquad j = 1, 2, 3 \tag{73}$$

$$a_{j\,2} = \frac{\omega\,\Omega\,\beta^2(\,k_j^2 - a^2)}{[\,k_j^2 - a^2 - \beta^2(\,\alpha\,\omega - \Omega^2)\,]}\,, \qquad j = 1, 2, 3 \tag{74}$$

$$b_{j\,2} = a\,k_j\,, \qquad j = 1, 2, 3 \tag{75}$$

$$c_{11} = a_{21}a_{32} - \,b_{21}b_{32} - a_{31}a_{22} + \,b_{31}b_{22} \tag{76}$$

$$d_{11} = a_{21}b_{32} + \,a_{32}b_{21} - a_{31}b_{22} - \,a_{22}b_{31} \tag{77}$$

$$c_{22} = a_{31}a_{12} - \,b_{31}b_{12} - a_{11}a_{32} + \,b_{11}b_{32} \tag{78}$$

$$d_{22} = a_{31}b_{12} + \,a_{12}b_{31} - a_{11}b_{32} - \,a_{32}b_{11} \tag{79}$$

$$c_{33} = a_{11}a_{22} - \,b_{11}b_{22} - a_{21}a_{12} + \,b_{21}b_{12} \tag{80}$$

$$d_{33} = a_{11}b_{22} + \,a_{22}b_{11} - a_{21}b_{12} - \,a_{12}b_{21} \tag{81}$$

$$c_{44} = S_1c_{11} - \,S_2c_{22} + \,S_3c_{33} \tag{82}$$

$$d_{44} = S_1d_{11} - \,S_2d_{22} + \,S_3d_{33} \tag{83}$$

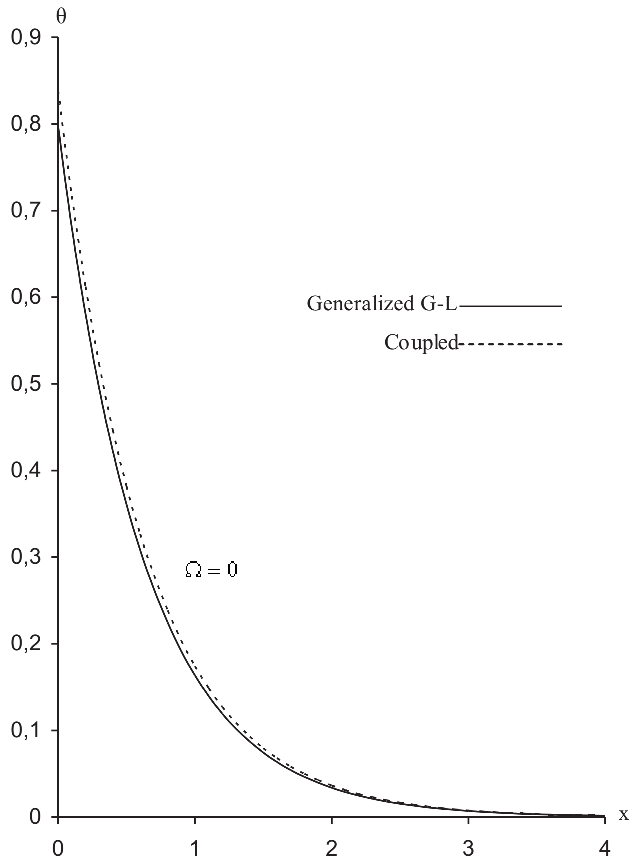

Figure 1 Temperature distribution for $z = 0$ and $\Omega = 0$

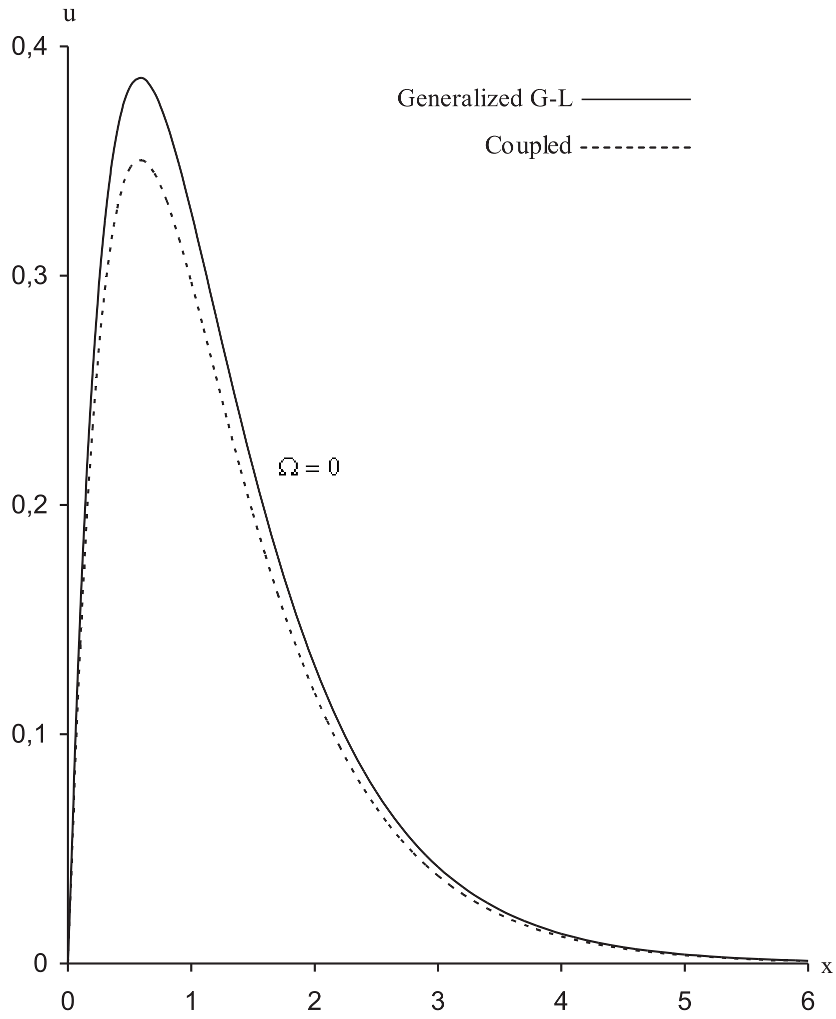

Figure 2 Distribution of displacement component u for $z = 0$ and $\Omega = 0$

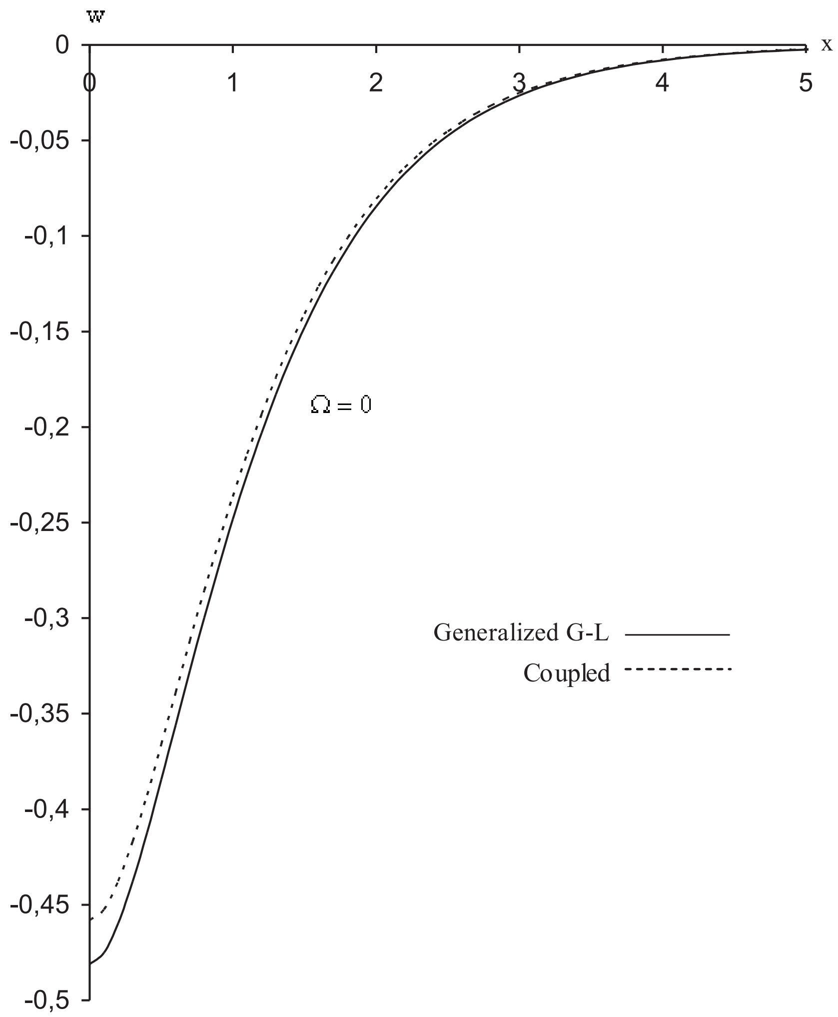

Figure 3 Distribution of displacement component w for $z = 0$ and $\Omega = 0$

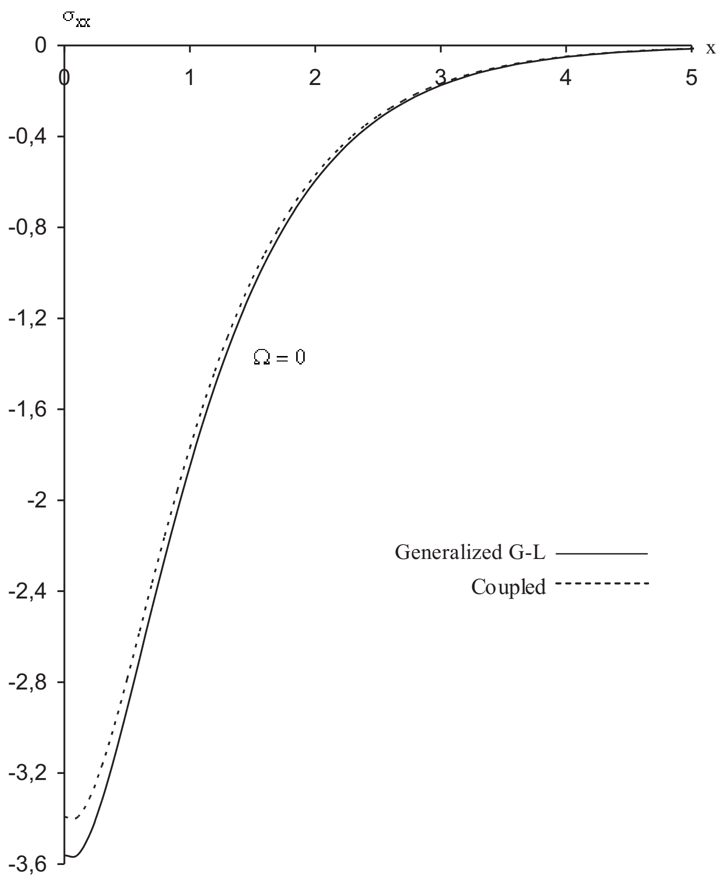

Figure 4 Distribution of stress component σ_{xx} for $z = 0$ and $\Omega = 0$

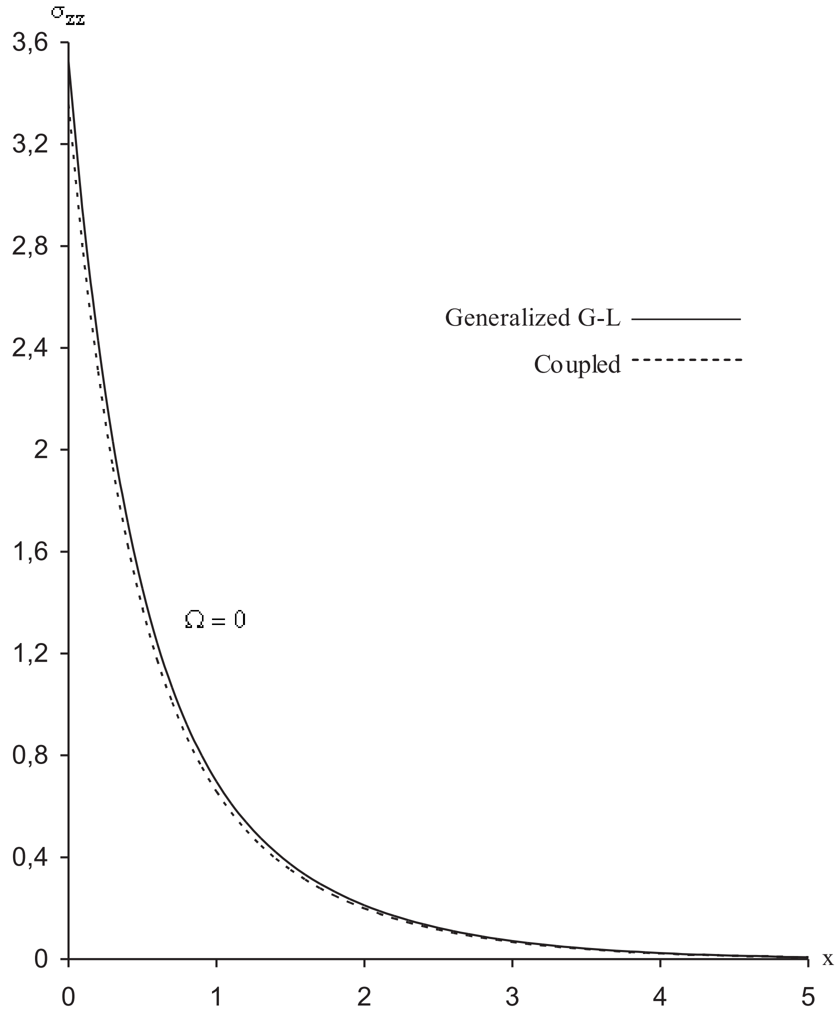

Figure 5 Distribution of stress component σ_{zz} for $z = 0$ and $\Omega = 0$

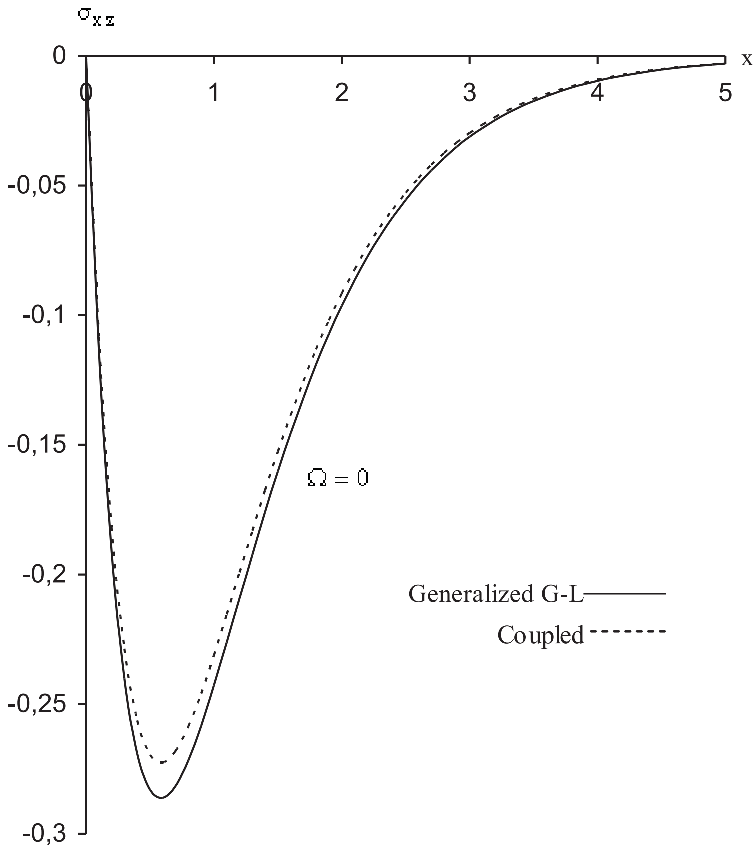

Figure 6 Distribution of stress component σ_{xz} for $z = 0$ and $\Omega = 0$

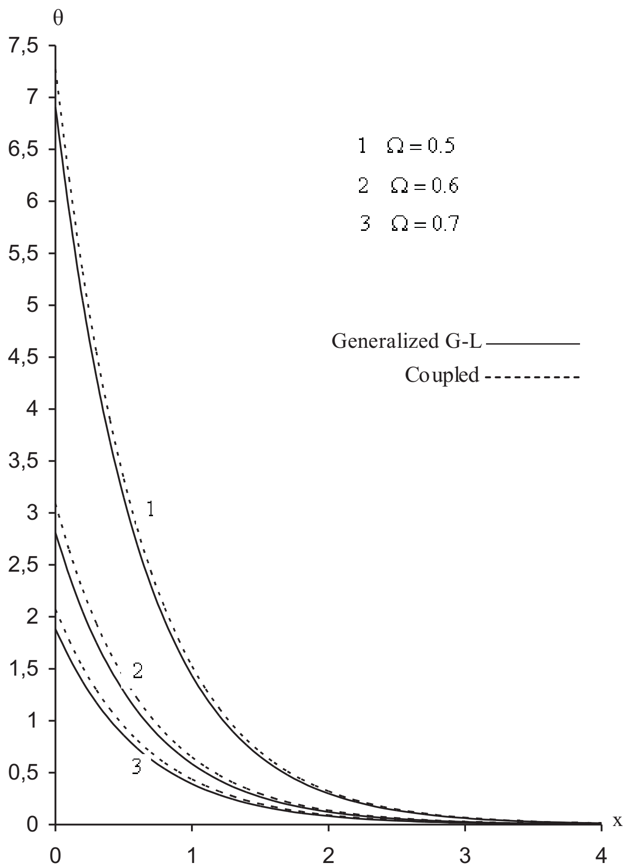

Figure 7 Temperature distribution for $z = 0$ and $\alpha = 1.02$

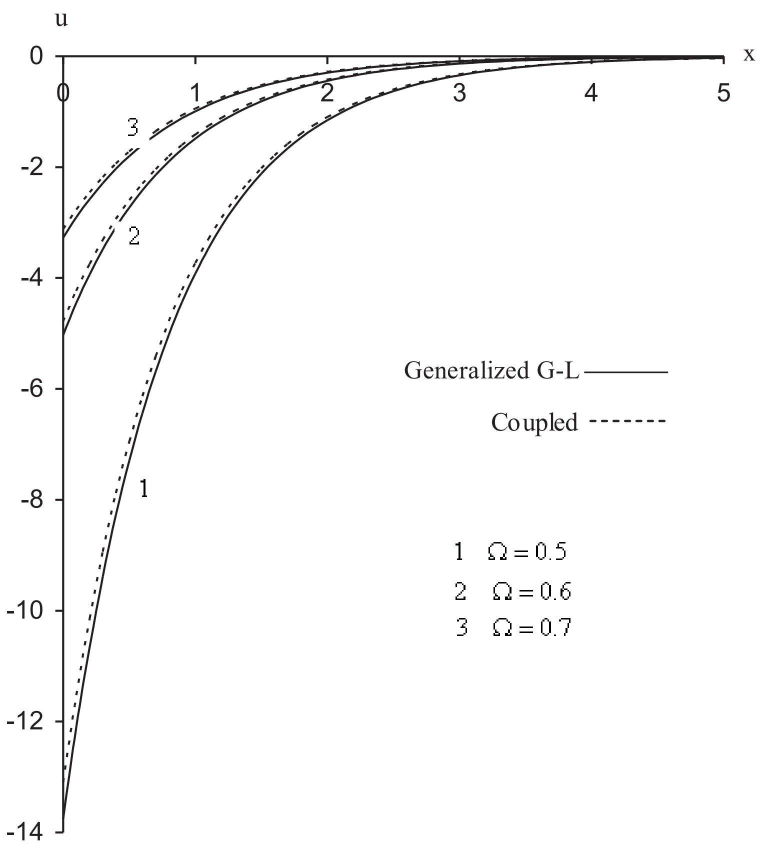

Figure 8 Distribution of displacement component u for $z = 0$ and $\alpha = 1.02$

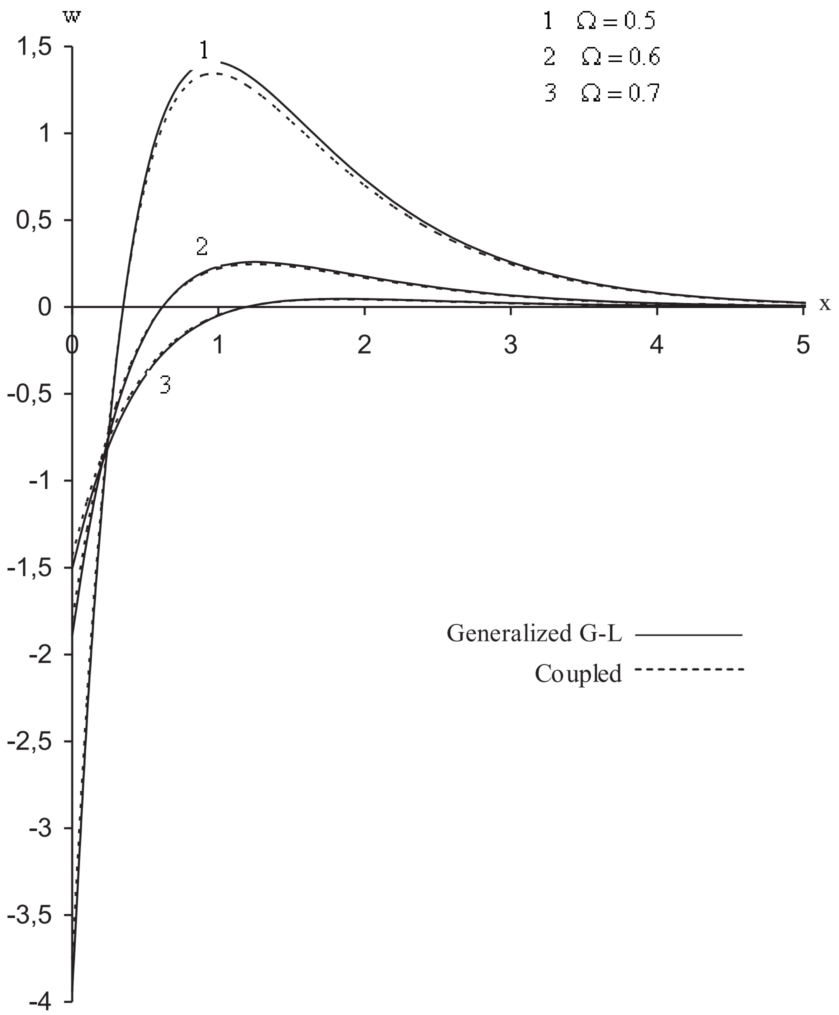

Figure 9 Distribution of displacement component w for $z = 0$ and $\alpha = 1.02$

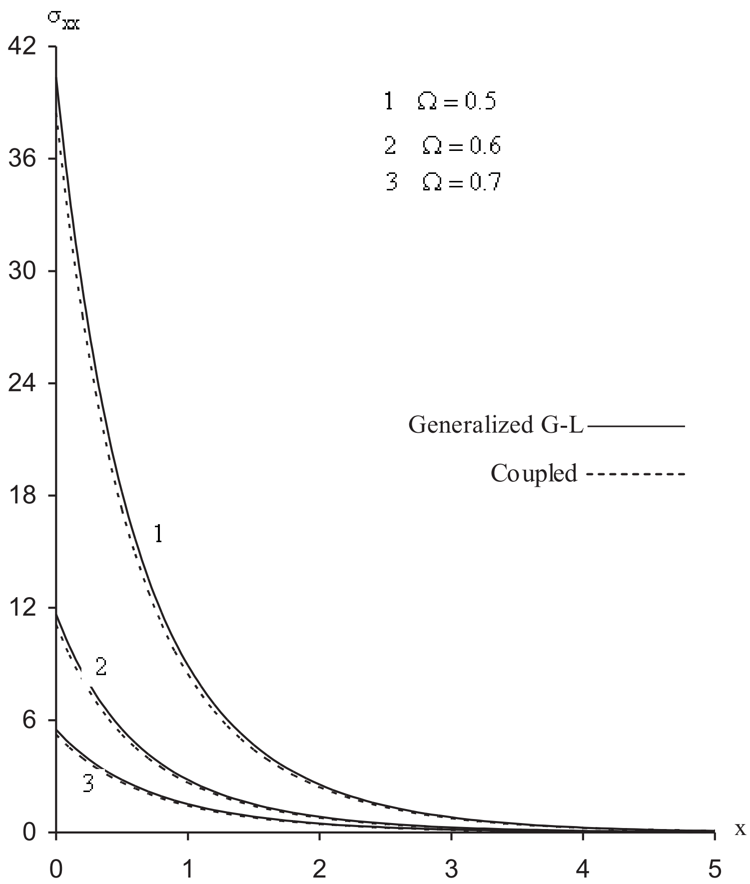

Figure 10 Distribution of stress component σ_{xx} for $z = 0$ and $\alpha = 1.02$

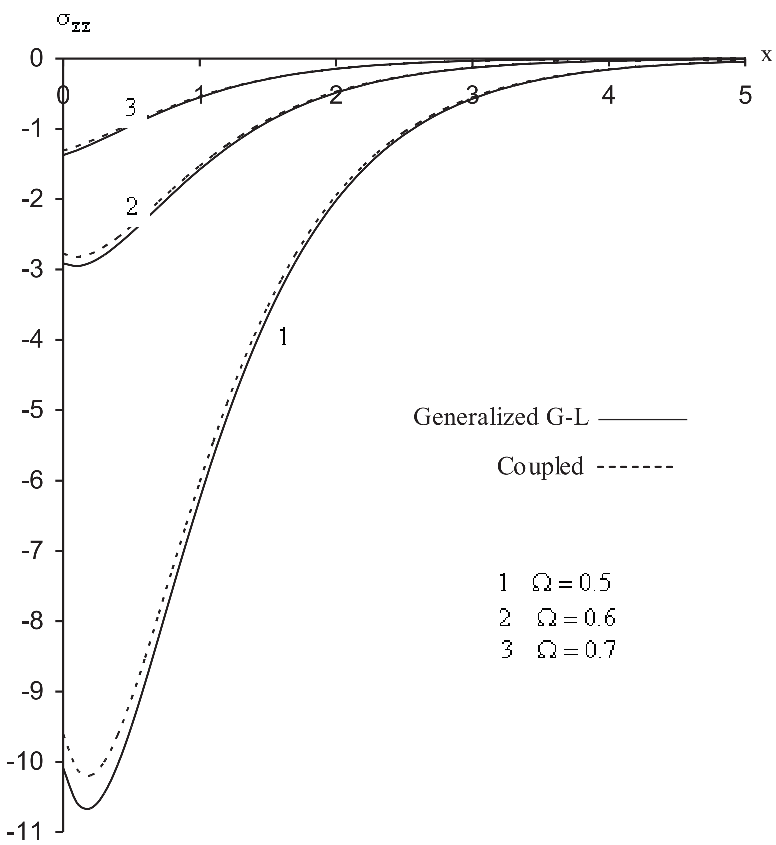

Figure 11 Distribution of stress component σ_{zz} for $z = 0$ and $\alpha = 1.02$

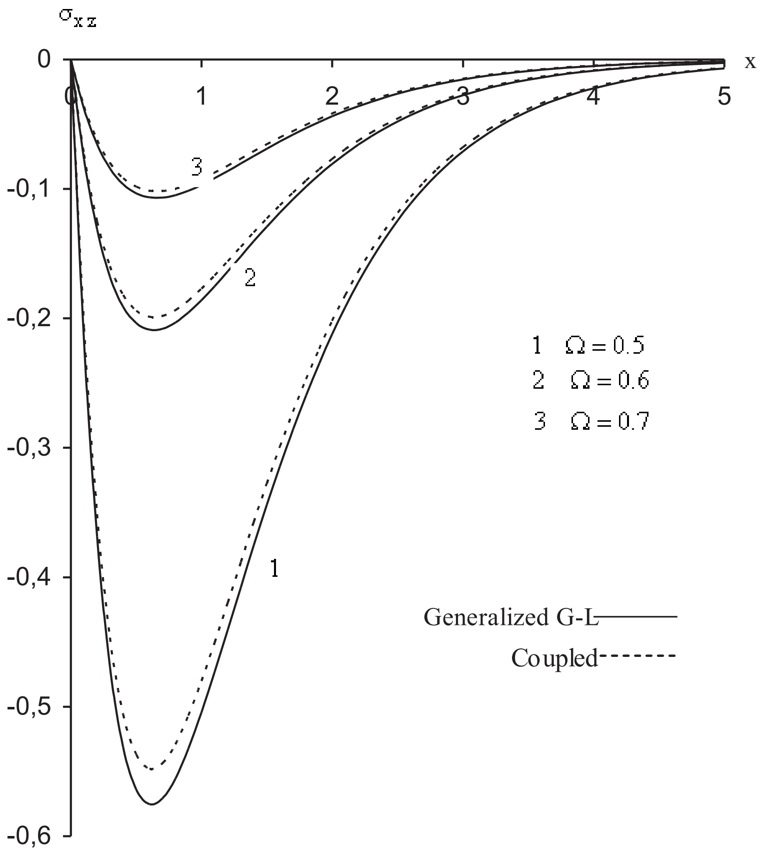

Figure 12 Distribution of stress component σ_{xz} for $z = 0$ and $\alpha = 1.02$

4. Numerical Results

The copper material is chosen for numerical evaluations. In the calculation process, the material constants necessary to be known can be found in [19].

The thermal shock $f\,(y,t)$ applied on the surface, is taken of the form

$$f\,(y,t) = \theta_o H\,(\,L - |\,y\,|\,)\exp(\,-\,b\,t\,) \tag{84}$$

where H is the Heaviside unit step function and θ_o is a constant. This means that heat is applied on the surface of the half–space on a narrow band of width 2L surrounding the y–axis to keep it at temperature θ_o,while the rest of the surface is kept at zero temperature. The other constant of the problem are taken as

$$L = 4, \quad \theta_o = 1, \quad b = 1, \quad \nu_o = 0.05, \quad \tau_o = 0.03, \quad \omega_o = 1, \quad a = 1.2$$

Considering the distributions of temperature, displacement and stresses for $z = 0$. Calculated results of the real part of the non-dimensional temperature θ, displacement components u, w and stress components σ_{xx} ,σ_{zz}, σ_{xz} are shown in Figs 1–12 respectively. The solid lines represent the solution obtained by Green–Lindsay's theory ($\nu_o = 0.05$, $\tau_o = 0.03$) and the dashed lines represent the solution obtained by the coupled theory ($\nu_o = 0, \tau_o = 0$).

Figs 1–6 give the field quantities distribution at time $t = 10$, $\Omega = 0$, $\alpha = 1.02$. and $R_H = 8120$, respectively. Figs 7–12 give the field quantities distribution at three different values of the rotation namely, $\Omega = 0.5$, 0.6, 0.7. Also Figs 7–12 indicate the effect of the rotation on the field quantities distribution. The phenomenon of finite speeds of propagation is manifested in all these figures. The medium deforms because of thermal shock, and due to the application of the magnetic field, there result an induced magnetic field in the medium.

5. Concluding remarks

In all figures, it is clear that all the distributions considered have a non–zero value only on a bounded region of space. Outside this region the values vanish identically and this means that the region has not felt thermal disturbance yet. From the distributions of temperature, it can be found the wave type heat propagation in the medium. The heat wave front moves forward with a finite speed in the medium with the passage of time. This is not the case for the classical theories of thermo–elasticity where an infinite speed of propagation is inherent and hence all the considered functions have a non–zero (though may be very small) value for any point in the medium. This indicates that the generalized heat conduction mechanism is completely different from the classic Fourier's in essence. In generalized thermo–elasticity theory heat propagates as a wave with finite velocity instead on infinite velocity in medium.

Owing to the complicated nature of the governing equations for the generalized electromagneto–thermoelasticity with two relaxation times few attempts have been made to solve problems in this field, these attempts utilized approximate methods valid for only a specific range of some parameters.

In this work, the method of normal mode analysis is introduced in the field of electromagneto–thermoelasticity and applied to a specific case in which the temperature, displacement, stress and magnetic field are coupled for a rotating medium.

It is important to observe that rotation has high influence on all the field quantities in the generalized electromagneto-thermoelasticity.

References

[1] **Lord, H. and Shulman, Y.**: A Generalized dynamical theory of thermoelasticity, *J. Mech. Phys. Solid*, 15, 299–309, **1967**.

[2] **Puri, P.**: Plane waves in thermoelasticity and magneto–elasticity, *Int. J. Engng Sci.*, 10, 467–476, **1972**.

[3] **Nayfeh, A.H. and Nasser, S.N.**: Thermoelastic waves in solids with thermal relaxation, *Acta Mech.*, 12, 53–69, **1971**.

[4] **Agarwal, V.K.**: On surface waves in generalized thermoelasticity, *J. Elasticity*, 8, 171–177, **1978**.

[5] **Ezzat, M.A. and Othman, M.I.A.**: State–space approach to generalized magneto–thermo–elasticity with thermal relaxation in a medium of perfect conductivity, *J. Thermal Stresses*, 25, 409–429, **2002**.

[6] **Othman, M.I.A.**: Lord–Shulman theory under the dependence of the modulus of elasticity on the reference temperature in two–dimensional generalized thermo–elasticity, *J. Thermal Stresses*, 25, 1027–1045, **2002**.

[7] **Othman, M.I.A.**: Generalized thermo–elasticity plane waves in rotating media with thermal relaxation under the temperature dependence properties, *Mechanics and Mechanical Engineering*, 9, 89–110, **2005**.

[8] **Othman, M.I.A.**: Generalized electro-magneto-thermoviscoelastic in case of 2-D thermal shock problem in a finite conducting medium with one relaxation time, *Acta Mechanica*, 169, 37–51, **2004**.

[9] **Green, A.E. and Lindsay, K.A.**: Thermoelasticity, *J. Elast.*, 2, 1–7, **1972**.

[10] **Agarwal, V.K.**: On plane waves in generalized thermoelasticity, *Acta Mech.*, 31, 185–198, **1979**.

[11] **Agarwal, V.K.**: On electromagneto–thermoelastic plane waves, *Acta Mech.*, 10, 181–191, **1967**.

[12] **Schoenberg, M. and Censor, D.**: Elastic waves in rotating media, Quart, *Quart of Applied Math.*, 31, 115–125, **1973**.

[13] **Roy Choudhuri, S.K. and Chatterjee Roy, G.**: Temperature-rate dependent magneto–thermoelastic waves in a finitely conducting elastic half–space, *Computers Math. Applic.*, 19, 85–93, **1990**.

[14] **Ezzat, M.A. and Othman, M.I.A.**: Electromagneto–thermoelastic plane waves with two relaxation times in a medium of perfect conductivity, *Int. J. Engng. Sci.*, 38, 107–120, **2000**.

[15] **Ezzat, M.A., Othman, M.I.A. and Samaan, A.A.**: State space approach to two–dimensional electromagneto–thermoelastic problem with two relaxation times, *J. Thermal Stresses*,39, 1383–1404, **2001**.

[16] **Dhaliwal, R. and Rokne, J.**: Thermal shock problem in generalized thermoelastic, *J. Thermal Stresses*, 12, 259–279, **1989**.

[17] **Othman, M.I.A.**: Effect of rotation on plane waves in generalized thermo–elasticity with two relaxation times, *Int. J. Solids and Structures*, 41, 2939-2956, **2004**.

[18] **Nowacki, W.**: Dynamic problem of thermoelasticity, *Noordhoff International the Netherlands*, p. 399, **1975**.

[19] **Sherief, H.H.**: Problem in electromagneto thermoelasticity for an infinitely long solid conducting circular cylinder with thermal relaxation, *Int. J. Engng. Sci.*, 32, 1137–1149, **1994**.

Nomenclature

λ, μ	Lame"s constants
ρ	density
C_E	specific heat at constant strain
t	time
T	absolute temperature
T_o	reference temperature chosen so that $\left\lvert \frac{T-T_o}{T_o} \right\rvert << 1$
σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
u_i	components of displacement vector
k	thermal conductivity
J	current density vector
μ_o	magnetic permeability
ε_o	electric permeability
$C_T^2 = \frac{(\lambda+2\mu)}{\rho}$	
$C_L = \sqrt{\mu/\rho}$,	velocity of transverse waves
$c^2 = 1/\mu_o \varepsilon_o$	sound speed
ν_o, τ_o	two relaxation times
e	cubical dilatation
α_t	coefficient of linear thermal expansion
$\gamma = (,3\lambda + 2\mu)\alpha_t$	
$\varepsilon = \frac{\gamma^2 T_o}{\rho^2 C_E C_T^2}$,	is the usual thermo–elastic coupling parameter
$C_A^2 = \frac{\mu_o H_o^2}{\rho}$	is the so–called Alfven speed
$\alpha = 1 + \frac{C_A^2}{c^2}$	
$\beta^2 = \frac{C_T^2}{C_L^2}$	
$\omega^* = \frac{k}{\rho C_E C_T^2}$	
$R_H = \frac{C_A^2}{C_T^2}$	