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Kinematics Modeling of the Amigobot Robot

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In this article authors presenting problems connected with the kinematics modeling based on Denavit–Hartenberg notation for a wheeled mobile robot. The possibility of sending data between Maple^{TM} and Matlab^{TM} has been discussed. Simulations of the kinematics parameters have been made and the results are shown.

Keywords: Kinematics, Denavit–Hartenberg notation, mobile robots

1. Introduction

In order to describe the kinematics of the mobile robot it is necessary to present kinematics equations. If we take in to consideration radial trajectory for the robot, we are able to split main trajectory from sub–trajectories with can be describe by one system of equations. The problem connected with mathematical description of the kinematics equations derived on the basis of Denavit–Hartenberg notation for mobile robot AmigoBot^{TM} has been presented. The paper suggest the mode of symbolic computations in the environment of Maple^{TM} programme and afterwards the moving of the data to the Matlab^{TM} package. In the Matlab^{TM}–Simulink environment simulation of the robot's kinematics behaviour has been carried out. Such a mode of computations and communicative Matlab^{TM}–Maple^{TM} software have been further discussed in paper [1,2,3]. On the basis of kinematics parameters simulation and comparison of the results have been carried out.

2. Modeling of the kinematics of the wheeled mobile robot AmigoBot^{TM}

The firs step in the process of the kinematics descriptions is connected with robot unit description and system coordinates assumptions. The basic elements of this model are the vehicle frame 4, driving units 1 and 2 and the self–adjusting supported wheel 3.

The individual components of the model are connected with coordinate systems, and so with part 4 of the system we have system coordinates $\mathbf{x}_4$ $\mathbf{y}_4$ $\mathbf{z}_4$. It's beginning in the center of mass of this part. System $\mathbf{x}_0$ $\mathbf{y}_0$ $\mathbf{z}_0$ is a fixed base reference system.

The driving units 1 and 2 are connected with systems of coordinates x_1 y_1 z_1, x_2 y_2 z_2 in characteristic points B and C. Those points are located on axis of the rotation of the particular driving wheels. In order to describe the such a mechatronic system, kinematics equations of the characteristic points have been used. The basic assumption is that robot move with constant velocity of the point A [1,2,3,6,7,8]. The basic assumptions related with modeling have been presented in fig.2.1.

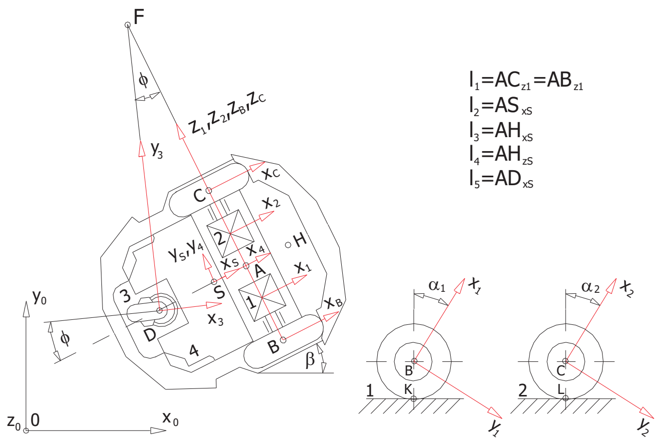

Figure 1 Computation model of AmigoBot^{TM} robot

There are two kinematics problems, simple and inverse, simple describe position and orientation of the system in space with respect reference system while inverse kinematics problem gives as information about kinematics parameters as angles, angular velocities and accelerations on the base of assumed trajectory. In this article we focus on inverse kinematics problem. The all symbolic computations have been leaded in Maple^{TM} environment. For the point H kinematics equation will have the following form:

$$\bar{\mathbf{r}}_H = \mathbf{T}_{4,0,H}\,\bar{\rho}_H \tag{1}$$

In the presented case transformation matrix $\mathbf{T}_{4,0}$ of $\mathbf{x}_4$ $\mathbf{y}_4$ $\mathbf{z}_4$ to $\mathbf{x}_0$ $\mathbf{y}_0$ $\mathbf{z}_0$ system for the point H was entered as [6,7]:

$$\mathbf{T}_{4,0,H} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 & x_A + l_3\cos(\beta) \\ \sin(\beta) & \cos(\beta) & 0 & y_A + l_3\sin(\beta) \\ 0 & 0 & 1 & r_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The vector describing the position of the point H relative to the $\mathbf{x}_4\ \mathbf{y}_4\ \mathbf{z}_4$:

$$\rho_H = \begin{bmatrix} 0 \\ 0 \\ -l_4 \\ 1 \end{bmatrix} \tag{3}$$

After calculating the expression (1) obtained equation of motion of the point H and saved it in two equivalent forms:

$$\mathbf{r}_H = \begin{bmatrix} x_A + l_3\cos(\beta) \\ y_A + l_3\sin(\beta) \\ r_1 - l_4 \\ 1 \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} x_H \\ y_H \\ z_H \\ 1 \end{bmatrix} = \begin{bmatrix} x_A + l_3\cos(\beta) \\ y_A + l_3\sin(\beta) \\ r_1 - l_4 \\ 1 \end{bmatrix} \tag{5}$$

In the next step equations of the trajectory for wheeled mobile robot have been shown. In the current case is a circular path. The equations of motion in a circular path is described as follows:

$$\begin{aligned} x_H &= R\sin(\phi) \\ y_H &= R(1-\cos(\phi)) \\ z_H &= r_1 - l_4 \end{aligned} \tag{6}$$

After compare the coordinates of point H represented by expression (5) with the trajectory equations of motion (6) we obtained the following equation:

$$R\sin(\phi) = x_A + l_3\cos(\beta) \tag{7}$$

$$R(1-\cos(\phi) = y_A + l_3\sin(\beta) \tag{8}$$

Differentiating expression (7) and (8) with respect to time obtained velocity equation x and y coordinates of the H point, which were presented in the form of allowing further calculations using symbolic computation program Maple^{TM}:

$$\dot{x}_A - l_3\dot{\beta}\sin(\beta) - R\dot{\phi}\cos(\phi) = 0 \tag{9}$$

$$\dot{y}_A + l_3\dot{\beta}\cos(\beta) - R\dot{\phi}\sin(\phi) = 0 \tag{10}$$

Another characteristic point is a point A of the system for which it assumes a constant speed and recorded the projections of the speed for particular axis x and y:

$$\dot{x}_A - v_A\cos(\beta) = 0 \tag{11}$$

$$\dot{y}_A - v_A\sin(\beta) = 0 \tag{12}$$

Knowing all the parameters of motion system is also analyzed to determine the angular velocities and angles of the rotation for particular wheels 1 and 2. These angular velocity determined from the equations describing the velocities of the points of contact with the road wheels, assuming cooperation without slipping. To determine the angular parameters of the wheel 1 saved the equation of kinematics point of contact with the road as follows:

$$\bar{r}_K = T_{4,0,K} T_{1,4} \bar{\rho}_K \tag{13}$$

Transformation matrix $\mathbf{T}_{4,0}$ of $\mathbf{x}_4$ $\mathbf{y}_4$ $\mathbf{z}_4$ to $\mathbf{x}_0$ $\mathbf{y}_0$ $\mathbf{z}_0$ system for point K are defined as:

$$\mathbf{T}_{4,0,K} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 & x_A + l_1\sin(\beta) \\ \sin(\beta) & \cos(\beta) & 0 & y_A - l_1\cos(\beta) \\ 0 & 0 & 1 & r_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

Transformation matrix $\mathbf{T}_{1,4}$ of $\mathbf{x}_1$ $\mathbf{y}_1$ $\mathbf{z}_1$ to $\mathbf{x}_4$ $\mathbf{y}_4$ $\mathbf{z}_4$ system for point K stated:

$$\mathbf{T}_{1,4} = \begin{bmatrix} \sin(\alpha_1) & \cos(\alpha_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \cos(\alpha_1) & -\sin(\alpha_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

The vector describing the position of the point K relative to the $\mathbf{x}_1$ $\mathbf{y}_1$ $\mathbf{z}_1$:

$$\rho_K = \begin{bmatrix} -r_1\cos(\alpha_1) \\ r_1\sin(\alpha_1) \\ 0 \\ 1 \end{bmatrix} \tag{16}$$

Differentiating the equation for the kinematics of the point K (13) we received:

$$\bar{v}_K = \dot{T}_{4,0,K} \dot{T}_{1,4} \bar{\rho}_K \tag{17}$$

Substituting into the velocity equation of the point F (17) expressions (14), (15) and (16) we obtained the velocity equation of point K:

$$\begin{bmatrix} \dot{x}_K \\ \dot{y}_K \\ \dot{z}_K \\ 0 \end{bmatrix} = \begin{bmatrix} x_A - r_1\dot{\alpha}_1\cos(\beta) + l_1\dot{\beta}\cos(\beta) \\ y_A - r_1\dot{\alpha}_1\sin(\beta) + l_1\dot{\beta}\sin(\beta) \\ 0 \\ 0 \end{bmatrix} \tag{18}$$

Given that $V_K = 0$, since only wheel 1 cooperates with the road without skidding, the record received a scalar equation:

$$\dot{x}_A - r_1\dot{\alpha}_1\cos(\beta) + l_1\dot{\beta}\cos(\beta) \tag{19}$$

$$\dot{y}_A - r_1\dot{\alpha}_1\sin(\beta) + l_1\dot{\beta}\sin(\beta) \tag{20}$$

With the assigning of second wheel parameters was done similarly as for the wheels 1 it means assumed the kinematics equation of point L, that is the point of contact with the road wheel 2:

$$\bar{r}_L = T_{4,0,L} T_{2,4} \bar{\rho}_L \tag{21}$$

Transformation matrix $\mathbf{T}_{4,0}$ of $\mathbf{x}_4$ $\mathbf{y}_4$ $\mathbf{z}_4$ to $\mathbf{x}_0$ $\mathbf{y}_0$ $\mathbf{z}_0$ system for point L are defined as:

$$T_{4,0,L} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 & x_A - l_1\sin(\beta) \\ \sin(\beta) & \cos(\beta) & 0 & y_A + l_1\cos(\beta) \\ 0 & 0 & 1 & r_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

Transformation matrix $\mathbf{T}_{2,4}$ of $\mathbf{x}_2$ $\mathbf{y}_2$ $\mathbf{z}_2$ to $\mathbf{x}_4$ $\mathbf{y}_4$ $\mathbf{z}_4$ system for point L are defined as:

$$T_{2,4} = \begin{bmatrix} \sin(\alpha_2) & \cos(\alpha_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \cos(\alpha_2) & -\sin(\alpha_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

The vector describing the position of the point L relative to the $\mathbf{x}_2$ $\mathbf{y}_2$ $\mathbf{z}_2$:

$$\rho_L = \begin{bmatrix} -r_2\cos(\alpha_2) \\ r_2\sin(\alpha_2) \\ 0 \\ 1 \end{bmatrix} \tag{24}$$

Differentiating the equation for the kinematics of the point L (21) we received:

$$\bar{v}_L = \dot{T}_{4,0,L}\dot{T}_{2,4}\bar{\rho}_L \tag{25}$$

Substituting into the equation (25) expressions (22), (23) and (24) we obtained the velocity equation of point L:

$$\begin{bmatrix} \dot{x}_L \\ \dot{y}_L \\ \dot{z}_L \\ 0 \end{bmatrix} = \begin{bmatrix} x_A - r_2\dot{\alpha}_2\cos(\beta) - l_1\dot{\beta}\cos(\beta) \\ y_A - r_2\dot{\alpha}_2\sin(\beta) - l_1\dot{\beta}\sin(\beta) \\ 0 \\ 0 \end{bmatrix} \tag{26}$$

Given that $V_L = 0$, since only wheel 2 cooperates with the road without skidding, the record received a scalar equation:

$$\dot{x}_A - r_2\dot{\alpha}_2\cos(\beta) - l_1\dot{\beta}\cos(\beta) \tag{27}$$

$$y_A - r_2\dot{\alpha}_2\sin(\beta) - l_1\dot{\beta}\sin(\beta) \tag{28}$$

The following system of six equations (1,2,3,6,7) allows the designation of all basic motion parameters if the assumed velocity of the point A is known, and the radius R with wheels diameter are also known.

$$\begin{aligned} &\dot{x}_A - l_3\dot{\beta}\sin(\beta) - R\dot{\phi}\cos(\phi) = 0 \\ &\dot{y}_A + l_3\dot{\beta}\cos(\beta) - R\dot{\phi}\sin(\phi) = 0 \\ &\dot{x}_A - v_A\cos(\beta) = 0 \\ &\dot{y}_A - v_A\sin(\beta) = 0 \\ &\dot{x}_A - r_1\dot{\alpha}_1\cos(\beta) + l_1\dot{\beta}\cos(\beta) = 0 \\ &\dot{x}_A - r_2\dot{\alpha}_2\cos(\beta) - l_1\dot{\beta}\cos(\beta) = 0 \\ &\sqrt{l_5^2\dot{\beta}^2 + v_A^2} - r_3\dot{\alpha}_3 = 0 \end{aligned} \tag{29}$$

On the base of the kinematics equations (29) simulations in the Matlab^{TM}–Simulink have been carried out. The following movement model has been assumed: starting, driving straight, driving on circular curve with turning axis of frame β and radius of wheels r, braking. The robot moves with the velocity v_a=0,2 [m/s] (velocity of point A in Fig. 1). The construction data included in Tab. 1 present kinematics parameters necessary to made simulation.

Table 1 The construction data of the AmigoBot^{TM} robot based on kinematics

l_1 [m]	l_3 [m]	l_4 [m]	l_5 [m]	r_1 [m]	r_2 [m]	r_3 [m]
0.12	0.08	0.03	0.12	0.05	0.05	0.03

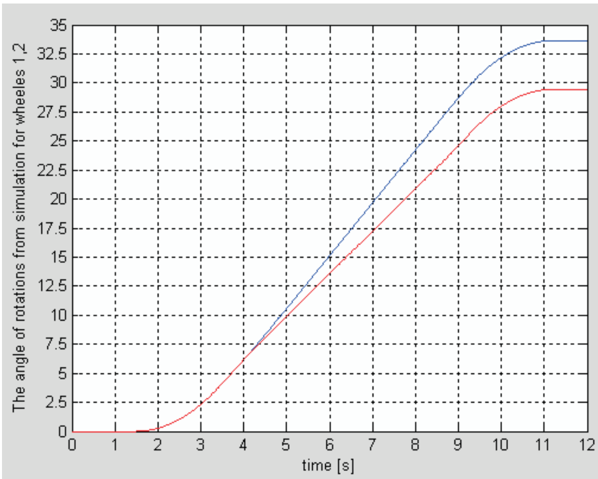

Figure 2 The angles of rotations time courses from simulation: α_1 [rad] – blue, α_2 [rad] – red

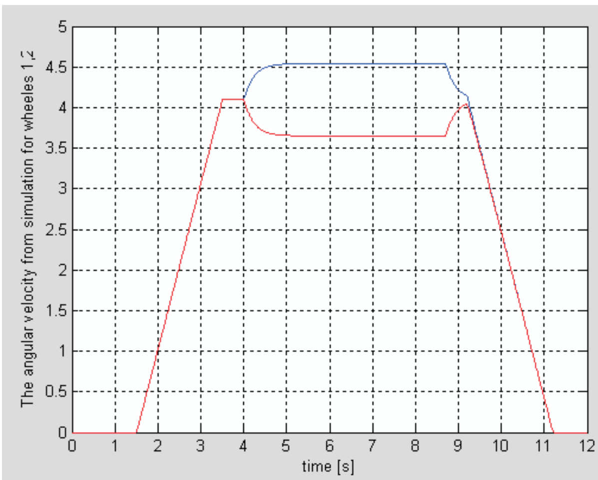

Figure 3 The angular velocities time courses from simulation: $\dot{\alpha_1}$ [rad/s]– blue, $\dot{\alpha_2}$ [rad/s]– red

As an result of the simulation we received kinematics parameters in the form presented in Fig. 2 and Fig. 3.

In order to compare the result of the simulation with real kinematics parameters test rig on real object has been carred out. The result of test rig is presented in Fig. 4 and Fig. 5.

The presented mode of kinematics equations can be applied to structures of various types.

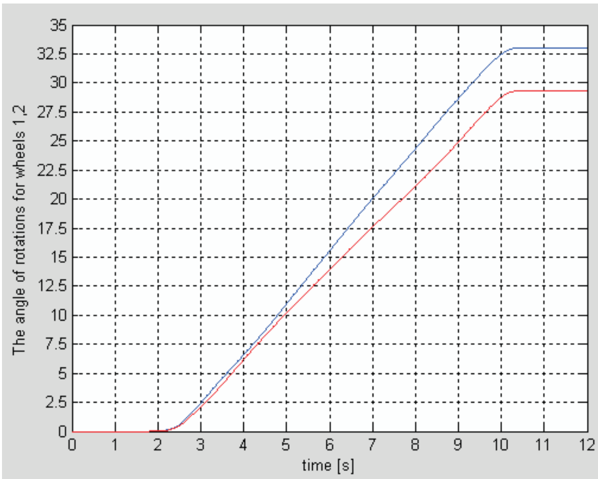

Figure 4 The angles of rotations time courses from test rig: α_1 [rad] – blue, α_2 [rad]– red

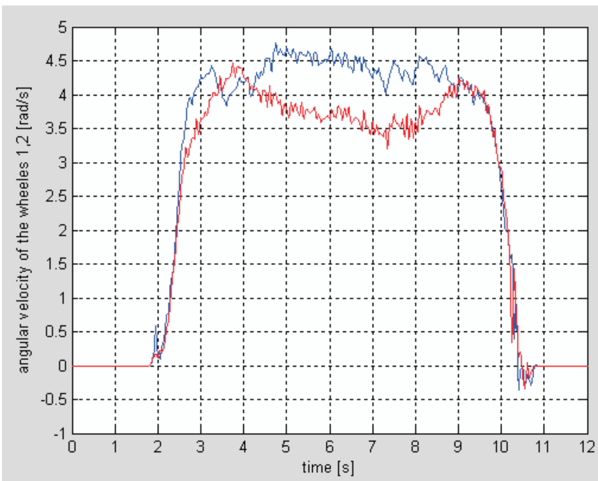

Figure 5 The angular velocities time courses from test rig: $\dot{\alpha_1}$ [rad/s] – blue, $\dot{\alpha_2}$ [rad/s]– red

3. Summary and conclusions

One of the common problems of the analysis of the complex constructions is model parameters creation with means the choice of the best model description in the sense of the accepted quality criterion. The presented kinematics modeling with the use of Denavit–Hartenberg notation allow to received kinematics parameters very similar to the real values. Those parameters we can use in dynamics and control modeling [4]. The advantages of the using Denavit–Hartenberg notation in kinematics modeling are the simplicity of implementation and the formulation of the problem. The presented mode of the modeling of kinematics parameters can be used in order to describe any types of mobile and stationary robots.

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