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Membrane–Flexural Coupling Effect in Dynamic Buckling of Laminated Columns

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The purpose of this paper is the analysis of dynamic stability of thin–walled laminated columns of closed rectangular cross–section, subjected to in–plane pulse loading of finite duration. In the analysis with the FE Method the Lagrange strain tensor is assumed and various material characteristics are applied. In the solution the shear influence is considered according to the First Shear Deformation Theory displacement field. In the performed analysis the influence of walls initial imperfections, pulse shape and pulse duration on the dynamic buckling load are examined as well as the stacking sequence of laminated walls, the orientation of principal directions of separate layers and orthotropy ratio. The applications of some dynamic criteria are compared as well.

Keywords: Thin–walled structure, pulse load, dynamic buckling

1. Introduction

A variety of positive factors, especially strength–to–weight and stiffness–to–weight ratios are the reasons of rising usage of fiber reinforced composite laminates in many structural and engineering applications. These materials permit the designer to tailor make the structure or components to achieve high performance structural objectives. The composite columns or girders are subjected to various types of loads which may cause buckling under static or suddenly applied loads. This dynamic loading could be discrete type loading of finite duration caused also by natural forces like wind or water in terms of solid-fluid interaction. The dynamic stability of composite columns encompasses many classes of problems and physical phenomena, for example parametric resonance, parametric excitation and impulse buckling.

The aim of this paper is the analysis of dynamic behavior of composite laminated columns of closed rectangular cross–section, built of thin-walled rectangular plates subjected to in-plane pulse loading of finite duration (Fig. 1). The analysis of dynamic stability of composite plated structures under in–plane pulse loading depends on pulse characteristic – it means pulse duration, pulse shape and magnitude of its amplitude [9]. The dynamic impulse buckling occurs when the loading process is of an intermediate amplitude and the pulse finite duration is close to the period of fundamental natural flexural vibrations. Usually the effects of dumping are neglected in such cases [8]. The dynamic behavior of a column, consisting of rectangular composite laminated plates, under in–plane loads involves rapid deflections growth of walls, which are initially not flat but imperfect. There is no buckling load and there is no bifurcation point over the loading path, as in the static case. Therefore the dynamic critical load is defined on the basis of an assumed dynamic buckling criterion. Very popular and most often used, adopted from shell structures to plate columns, Budiansky–Hutchinson [7] criterion assumes that dynamic stability loss occurs when the maximal plate deflection grows rapidly with the small variation of the load amplitude. The other one, Petry–Fahlbusch [17] failure criterion states that a dynamic response caused by a pulse load is defined to be dynamic stable if the condition that the effective stress σ_{eff} is not greater than the limit stress σ_L is fulfilled at every time everywhere in the structure. Ari Gur and Simoneta [3] analyzed laminated plates behaviour under impulse loading and formulated own criteria of dynamic buckling, two of them of collapse–type conditions. There are publications [2],[8],[19] which deal with dynamic stability of columns but the beam model is there employed in the stability analysis. The papers [1],[5],[6],[16] are some examples of dynamic stability analysis with application of different methods and tools in solution of shell structures but there are very few works which were published of dynamic behaviour of thin-walled plated composite laminated columns [8]. In [9] the dynamic response of thin orthotropic plates subjected to in–plane pulse loading were analyzed. Mania and Kowal–Michalska [14] considered isotropic columns of platted walls under axial pulse compression, especially the influence of cross–section shape (square versus rectangle) and pulse shape (rectangle, triangle and sinus) on dynamic critical load value. Recently in [11] some results of investigations of orthotropic thin walled columns were presented. The influence of strain rate effect on the columns response in the framework of the incremental flow theory was considered: in [12] for isotropic materials, whereas in [15] for orthotropic column walls.

This paper thereby deals with thin–walled composite plate model of column axially loaded by compression impulse of finite duration.

2. Solution of the problem

The subject of the study is the short thin–walled column commented above and presented in Fig. 1. The walls are multi–layered laminate with different stacking sequence, composed of orthotropic laminas, which principal axes of orthotropy - arbitrary orientated with respect to column edges, are defined by the lamination angle $\theta \in < 0^o, 90^o >$. The material of lamina is modeled as linear elastic however in some cases a bilinear characteristic is employed for calculations in elastic–plastic range. It is assumed that the loaded edges of the column are simply supported and remain straight and mutually parallel during loading. The shape of walls' initial imperfections fulfill the boundary conditions along all edges of component plates and correspond directly to the first static buckling mode. The ratio of imperfection amplitude to the thickness of column walls (all walls are assumed of equal thickness) was in the range (0.01, 0.1). The column was assumed a thin–walled structure with walls of equal thickness therefore the maximal in–plane dimension of wall (length or width) to its thickness was taken equal to $\max\{l/h_1,\, l/h_2,\, b_1/h_1,\, b_2/h_2\} = 1/100$.

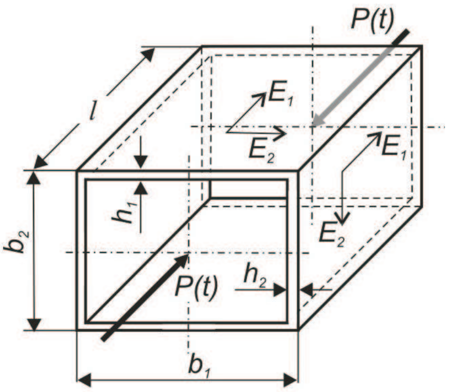

Figure 1 Geometry, dimensions and loads of multilayered column

The transient analysis was performed for pulse of rectangular and sine shapes with time duration T_p equal to or close to the period of fundamental natural flexural vibrations T. For each considered structure this period was obtained from modal analysis (eigenvalue problem). Zero initial conditions were assumed for velocity and initial imperfection with chosen amplitude was applied as initial deflections.

The presented solution was obtained on the basis of the first shear deformation theory ($FSDT$). The displacement field was assumed as follows:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\varphi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\varphi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

where u_0, v_0, w_0 are the displacement components along the coordinate directions of a point on the midplane ($z = 0$) and φ_x, φ_y denote rotations about the y and x axes, respectively [13]. In order to determine both out–of–plane and in–plane geometric plate behaviour under dynamic loading, the strain tensor with all terms present (Green–Lagrange strain tensor) for in–plane deformation was employed:

$$\begin{aligned} \varepsilon_{xx} &= u_0,_x + \tfrac{1}{2}\left(u_0^2,_x + v_0^2,_x + w_0^2,_x\right) + z\varphi_x,_x \\ \varepsilon_{zz} &= 0 \\ \varepsilon_{yy} &= v_0,_y + \tfrac{1}{2}\left(u_0^2,_y + v_0^2,_y + w_0^2,_y\right) + z\varphi_y,_y \\ \gamma_{yz} &= 2\varepsilon_{yz} = w_0,_y + \varphi_y \\ \gamma_{xz} &= 2\varepsilon_{xz} = w_0,_x + \varphi_x \\ \gamma_{xy} &= 2\varepsilon_{xy} = u_0,_y + v_0,_x + u_0,_x\, u_0,_y + v_0,_x\, v_0,_y + w_0,_x\, w_0,_y + z\left(\varphi_x,_y + \varphi_y,_x\right) \end{aligned} \tag{2}$$

The preceding strains in equation (2) can be divided into the membrane strains $\left\{\varepsilon^{(0)}\right\}$ and the flexural strains $\left\{\varepsilon^{(1)}\right\}$, what can be written in a shorter form as the matrix sum:

$$\left\{\begin{array}{c}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{array}\right\}=\left\{\begin{array}{c}\varepsilon_{xx}^{(0)}\\ \varepsilon_{yy}^{(0)}\\ \gamma_{yz}^{(0)}\\ \gamma_{xz}^{(0)}\\ \gamma_{xy}^{(0)}\end{array}\right\}+z\left\{\begin{array}{c}\varepsilon_{xx}^{(1)}\\ \varepsilon_{yy}^{(1)}\\ \gamma_{yz}^{(1)}\\ \gamma_{xz}^{(1)}\\ \gamma_{xy}^{(1)}\end{array}\right\}=\left\{\varepsilon^{(0)}\right\}+z\left\{\varepsilon^{(1)}\right\} \tag{3}$$

The Euler–Lagrange equations obtained from the Hamilton's principle for a single column wall have the form:

$$\begin{aligned}
&-N_{xx},_x-N_{xy},_y-\left(N_{xx}u_0,_x\right),_x-\left(N_{yy}u_0,_y\right),_y-\left(N_{xy}u_0,_x\right),_y-\left(N_{xy}u_0,_y\right),_x\\
&+I_0u_0,_{tt}+I_1\varphi_x,_{tt}=0\\
&-N_{xy},_x-N_{yy},_y-\left(N_{xx}v_0,_x\right),_x-\left(N_{yy}v_0,_y\right),_y-\left(N_{xy}v_0,_x\right),_y-\left(N_{xy}v_0,_y\right),_x\\
&+I_0v_0,_{tt}+I_1\varphi_y,_{tt}=0\\
&-Q_x,_x-Q_y,_y-\left(N_{xx}w_0,_x\right),_x-\left(N_{yy}w_0,_y\right),_y-\left(N_{xy}w_0,_x\right),_y-\left(N_{xy}w_0,_y\right),_x\\
&+q+I_0w_0,_{tt}=0\\
&-M_{xx},_x-M_{xy},_y+Q_x+I_1u_0,_{tt}+I_2\varphi_x,_{tt}=0\\
&-M_{xy},_x-M_{yy},_y+Q_y+I_1v_0,_{tt}+I_2\varphi_y,_{tt}=0
\end{aligned} \tag{4}$$

In (4) N_{xx}, N_{yy}, N_{xy} are resultants of membrane force, M_{xx}, M_{yy}, M_{xy} are moment resultants, Q_x, Q_y denote the transverse force resultants [13], I_0, I_1, I_2 are the mass moments of inertia defined in (5) and $N_{xx},_x$ is used instead of $\frac{\partial N_{xx}}{\partial x}$ notation.

$$\left\{\begin{array}{c}I_0\\ I_1\\ I_2\end{array}\right\}=\int\limits_{\frac{-h}{2}}^{\frac{h}{2}}\left\{\begin{array}{c}1\\ z\\ z^2\end{array}\right\}\rho dz \tag{5}$$

Generally, for complex geometry, arbitrary boundary conditions and/or nonlinearities exact (analytical or variational) solution to equations (4) cannot be developed. Therefore a finite element method was chosen to solve the problem. After some transformations of presented above set of equations (4) (integrating by parts, rearranging etc) the weak form of equations of motion, associated with the $FSDT$ could be obtained. The explicit expression of equation (4) can be found in [13]. Finally after approximation of the primary variables u_0, v_0, w_0, φ_x, φ_y with interpolation Lagrange functions of the form:

$$\{\mathbf{u}\}=\left\{\begin{array}{c}u(x,y)\\ v(x,y)\\ w(x,y)\end{array}\right\}=\sum_{i=1}^{4}N_i\left\{\begin{array}{c}u_i\\ v_i\\ w_i\end{array}\right\}+\sum_{i=1}^{4}N_i\frac{rt_i}{2}\left[\begin{array}{cc}a_{1,i} & b_{1,i}\\ a_{2,i} & b_{2,i}\\ a_{3,i} & b_{3,i}\end{array}\right]\left\{\begin{array}{c}\theta_{x,i}\\ \theta_{y,i}\end{array}\right\} \tag{6}$$

the FEM model of analyzed multilayered column was developed. Equation (6) is defined in s, t, r local coordinates system and $\underline{u}_i$ is motion of node i of a four node quadrilateral Mindlin-type shell element, applied for the meshing. N_i is for shape function, $[a]$ and $[b]$ are unit vectors of the reference coordinates and $\theta_{x,i}$; $\theta_{y,i}$ are rotations with respect to $[a]$ and $[b]$ vectors, respectively [20]. To perform the calculations for some chosen cases the ANSYS software was employed [20].

The column of a shape of a four wall cubic structure was meshed with SHELL181, four nodes isoparametric nonlinear multilayered element (Fig. 2). It possesses six degrees of freedom: three displacements along the axes of local coordinate system and three rotations around these axes, respectively. However the in–plane rotation around the axis normal to the element surface is controlled in Allman's sense [8]. The uniformly distributed mesh was applied with up to 10000 quadrilateral elements [4] what allowed to analyze even the high frequency modes in dynamic response of the column, giving acceptable time of computations. The simply supported boundary conditions of S2 type [10] were chosen for the loaded edges of the column and the uniform compression of impulse type of finite duration, parallel to the walls, dynamically loaded the structure [11].

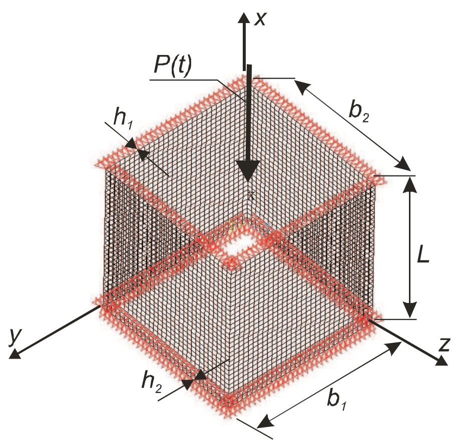

Figure 2 Meshed column model in reference coordinate system (dimensions and loading are repeated as well)

The critical conditions for dynamic buckling were determined on the basis of Budiansky–Hutchinson criterion [7] and the quadratic interaction failure criterion reported by Tsai and Hahn [18]. According to Tsai–Wu theory the 3–D strength ratio also called I_F failure index, is defined as [20]:

$$I_F = \left[-\frac{B}{2A} + \sqrt{\left(\frac{B}{2A}\right)^2 + \frac{1}{A}} \right]^{-1} \tag{7}$$

with (in contracted notation):

$$
\begin{aligned}
A = {} & \frac{\sigma_1^2}{F_{1t}F_{1c}} + \frac{\sigma_2^2}{F_{2t}F_{2c}} + \frac{\sigma_3^2}{F_{3t}F_{3c}} + \frac{\sigma_4^2}{F_4^2} + \frac{\sigma_5^2}{F_5^2} + \frac{\sigma_6^2}{F_6^2} + \\
& + c_4 \frac{\sigma_2\sigma_3}{\sqrt{F_{2t}F_{2c}F_{3t}F_{3c}}} + c_5 \frac{\sigma_1\sigma_3}{\sqrt{F_{1t}F_{1c}F_{3t}F_{3c}}} + c_6 \frac{\sigma_1\sigma_2}{\sqrt{F_{1t}F_{1c}F_{2t}F_{2c}}}
\end{aligned}
\tag{8}
$$

$$
B = \left(F_{1t}^{-1} - F_{1c}^{-1}\right)\sigma_1 + \left(F_{2t}^{-1} - F_{2c}^{-1}\right)\sigma_2 + \left(F_{3t}^{-1} - F_{3c}^{-1}\right)\sigma_3 \tag{9}
$$

In equations (8) $F_{ic/t}$ are strength of lamina in i = 1,2,3 principal direction, in tension and compression respectively, where c_j, j = 4,5,6 are the Tsai–Wu coupling coefficients, taken by default equal to –1 [20].

3. Numerical results

Some chosen results of all performed numerical calculations are presented in diagrams. The material properties of considered unidirectional composites are collected in Table 1.

Table 1

Property	Unit	Carbon/epoxy	Boron/epoxy
E_1	GPa	133.860	137.900
$E_2 = E_3$	GPa	7.706	8.963
$G_{12} = G_{13}$	GPa	4.306	7.102
G_{23}	GPa	2.760	6.205
$\nu_{12} = \nu_{13}$		0.301	0.250
ν_{23}		0.396	0.450
ρ	kg/m^3	1520	1450

It is seen from the Table 1 that the composite strength properties was taken as for transversely isotropic material what is common practice for fiber reinforced composites [18].

In the performed analysis for the first time occurred the highest deflection of column walls was registered. Usually it took place during the acting pulse load or immediately after it has been released. The difference was connected with the time duration of pulse load and pulse amplitude [8]. Unloaded structure after impulse time vibrated freely what is presented in phase diagrams for pre–buckling *DLF* amplitude (Fig. 3) and post–buckling *DLF* value (Fig. 4). The introduced dynamic load factor - *DLF* is a quotient of pulse load amplitude to the static buckling load for perfect structure. The static buckling load was determined in the first step of performed investigation, it is in linear eigenbuckling analysis.

Recalling the laminate constitutive equations [18], which relate the force and moment resultants to the strains:

$$
\left\{ \begin{array}{c} \{N\} \\ \{M\} \end{array} \right\} = \left[\begin{array}{cc} [A] & [B] \\ [B] & [D] \end{array} \right] \left\{ \begin{array}{c} \{\varepsilon^{(0)}\} \\ \{\varepsilon^{(1)}\} \end{array} \right\} \tag{10}
$$

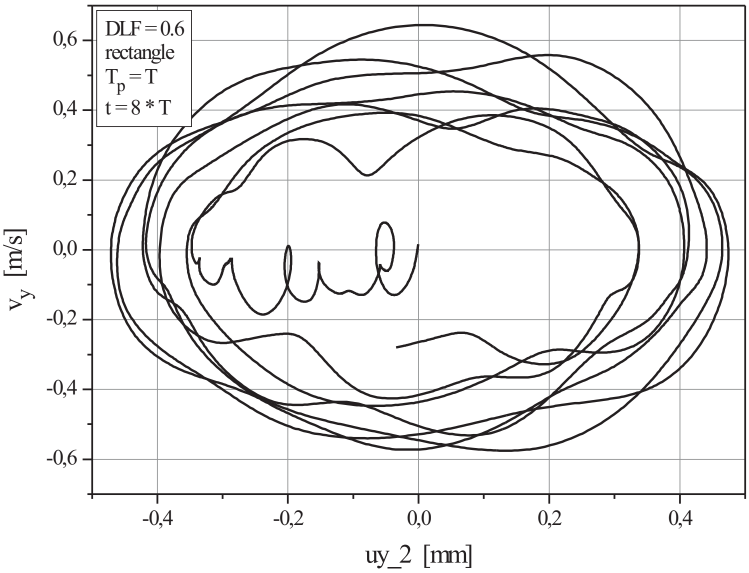

Figure 3 Phase diagram for rectangular pulse and different *DLF* value: $DLF = 0.6$

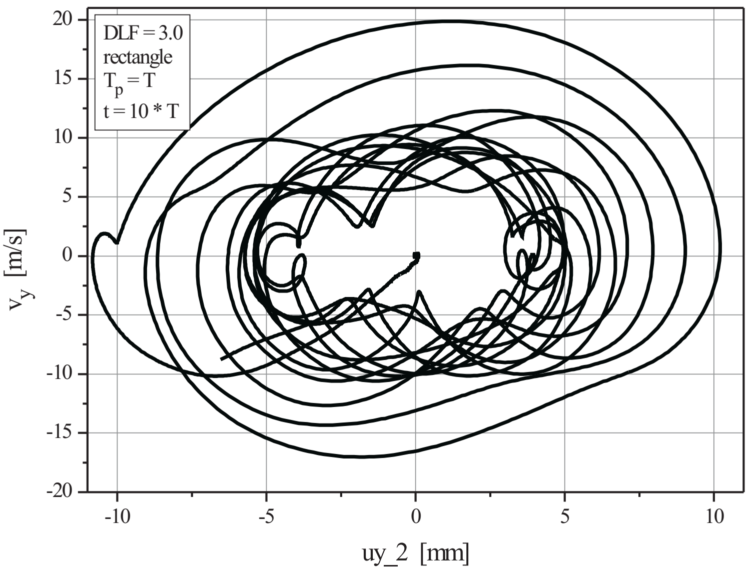

Figure 4 Phase diagram for rectangular pulse and different *DLF* value: $DLF = 3.0$

we can distinguish three matrices: A_{ij} which is called extensional stiffness, D_{ij} the bending (flexural) stiffness and B_{ij} the bending–membrane coupling stiffness. For some chosen cases of multi–layered laminates made of plies of equal thickness, considered in this paper, coefficients of matrix $[B]$ have non zero values. Especially for cross–ply antisymmetric laminate non zero are B_{11} and B_{22} elements, whereas for antisymetric angle–ply laminate, B_{16} and B_{26} coefficients have non–trivial values. These coefficients are responsible for the coupling effects although for balanced and/or symmetric stacking they vanished. In some applications the membrane–coupling effect meets the design requirements but in buckling phenomena – in statics and dynamics – influences the critical load and changes the response.

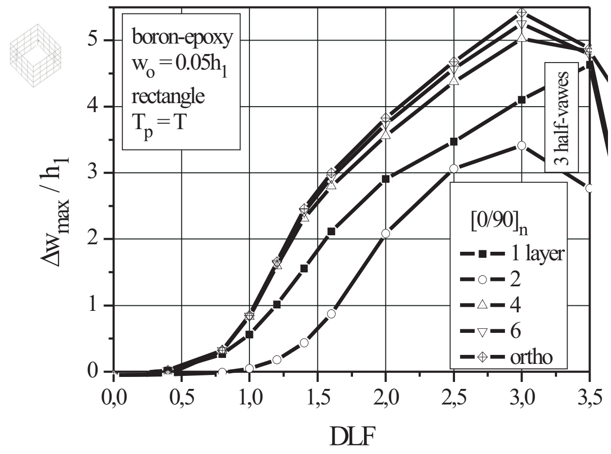

Figure 5 Deflection as a function of impulse amplitude

The study of effect of stacking sequence in cross–ply unsymmetric laminate on dynamic response is presented in Fig. 5. For comparison reasons the thickness of all column walls was assumed unchanged and the number of plies was increased. In this study boron–epoxy composite lamina mechanical properties were used in calculations. The lay–up influences the flexural stiffness of the wall and in limit of infinite number of plies tends to orthotropic, square–symmetric solution. For high number of plies the critical *DLF* value does not depend of plies number. For higher amplitudes of dynamic pulse load (*DLF* greater than 3) the change in dynamic buckling mode is observed, from imperfect one half-wave shape in dynamic response three half–waves deflection of walls occurred.

Comparing the dynamic critical loads for laminates used in above considerations, with critical loads values obtained for them in static analysis, one can observed the same range of values for both cases. Although for low number of layers (up to 8) the critical dynamic loads are visibly higher (even 12% for 4 layers column wall) than those for static but this effect diminishes when the number of layers increases.

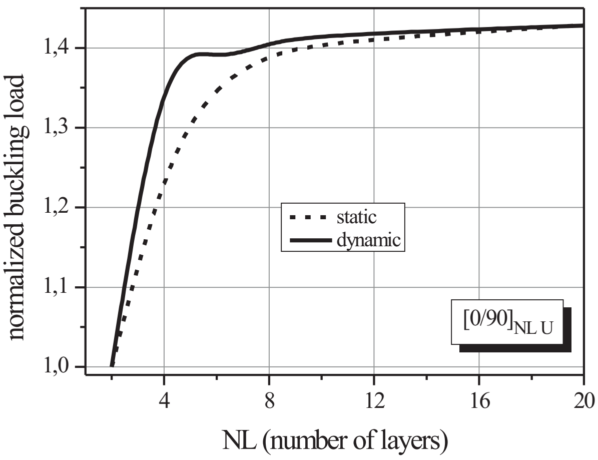

Figure 6 Comparison of buckling loads for static and dynamic cases, for cross-ply antisymetric laminates, rectangular pulse

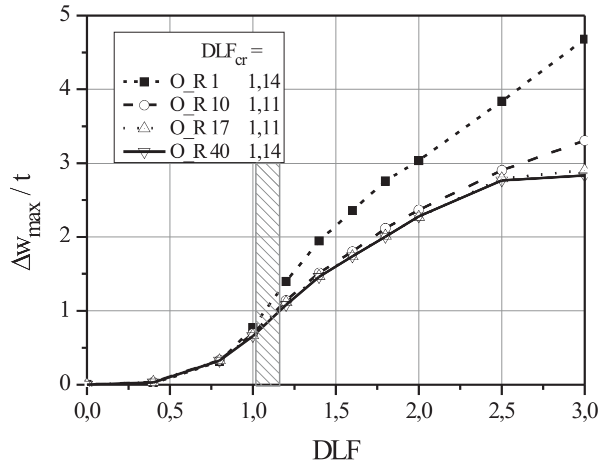

Figure 7 The influence of orthotropy ratio on buckling loads for cross-ply antisymetric laminates, rectangular pulse

For multilayered composites, mean the number of layers NL is greater than 16, in practice, both dynamic and static critical loads are of the same value. In Fig. 6 the buckling loads are related to the minimal critical load (in static and dynamic loading) obtained for columns with two–layered composite walls of the same thickness. Introducing the definition of orthotropy ratio as a quotient of Young's modulus in composite plane with E_1 in fiber direction and E_2 in direction perpendicular to fibers, which coincides with principal orthotropy direction as well:

$$O\_R = \frac{E_1}{E_2} \tag{11}$$

and conducting appropriate numerical calculations, the anisotropy effect in column dynamic response can be analyzed. The results of these calculations for 20 layers cross–ply antisymetric carbon–epoxy laminate with the same lamination angle, present the graphs in Figure 7. Curves for all otrhotropy ratio grater than 10, for dynamic load factor DLF value up to 2.5 show little differences. The dashed rectangle in Fig. 7 interprets the critical range of DLF according to Budiansky–Hutchinson criterion [7], [8]. However, considering each line of dynamic response as a separate discrete function and determining the second derivatives for it (for example with Finite Difference method), the deflection point can be found [13]. Its abscissa gives the numerical value for critical dynamic load factor for material with given orthotropy ratio. It is obvious that this value is located in the marked period in Fig. 7. Among the engineering applications of fiber reinforced composites for most cases the orthotropy ratio is grater then 10 so it could be stated that CFRP are less sensitive to own anisotropy in dynamic response history. The maximal deflections of column walls for increasing orthotropy ratio are lower comparing with deflections for column with equal moduli in wall plane what of course is connected with increased stiffness. In connection with the orthotropy sensitivity and deflections, the analysis of boron–epoxy column gave similar conclusions.

The influence of membrane–flexural coupling on buckling load of angle-ply laminate plate was analyzed in [10]. In Figs 8–9 results of analogous but dynamic investigation for rectangular impulse loading with $T_p = T$, for column of carbon–epoxy walls is presented. T_p denotes time of acting pulse load equal to time T – it is a period of fundamental vibrations. According to Budiansky–Hutchinson criterion the highest critical value of dynamic load factor (DLF_{cr}) was obtained for $[-45/45]_{2A}$ stacking, similarly as for static loading. However the relation to unidirectional fiber orientation is ca 40% for dynamic response, while only 26% for static case. The plot of deflection for the case when the dynamic load amplitude for different lay–ups is related to buckling load of $[0/0]_{2A}$ laminate (Fig. 9), shows the difference for $[-90/90]_{2A}$ walls. This is due two half–waves buckling mode which was applied as initial imperfection for this stacking and comparatively low stiffness due to perpendicular on–axis orientation.

In the response of column with $[-60/60]_{2A}$ stacking the change in buckling mode is connected with the transition from initial two half–waves into three half–wave deflection of the column wall. It occurred for amplitudes of pulse loading higher than $DLF = 1.8$.

For angle–ply antisymetric laminates the range of critical dynamic loads is visibly wider than those for static buckling (Fig. 10). Where the static buckling loads for the orthotropy directions orientated in angles 30÷60 degrees are practically the same, those for impulse loading differ in more than 14%. Also the total range of critical dynamic loads is 10% wider than static buckling loads range. The critical loads for impulse loading were determined with Budiansky–Hutchinson criterion application.

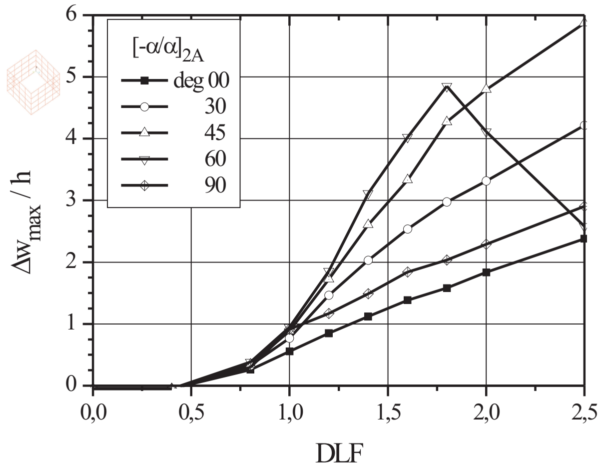

Figure 8 Column with carbon–epoxy walls, coupling effect in dynamic response

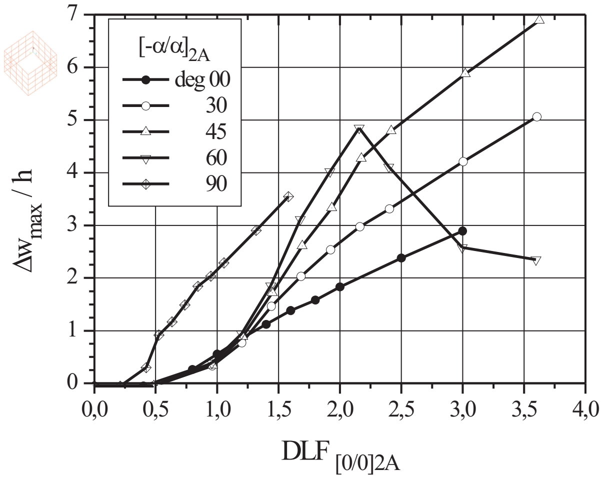

Figure 9 Column with carbon–epoxy walls, coupling effect in dynamic response, normalized to $[0/0]_{2A}$ solution

For both cases the load values were normalized to critical loads (static and dynamic) obtained for column with walls of $[90/90]_{2A}$ lay–up, which were the lowest loads. As it was mentioned earlier in dynamic buckling analysis none buckling–bifurcation load can be determined and bifurcation point as well. The dynamic response history has to be analyzed with incorporation of arbitrary chosen criterion to establish the critical dynamic load. The well–known from literature criteria were referenced in the introduction. It is interesting to compare the results of their application in finding critical dynamic load. As an example in Fig. 11, the results of calculations

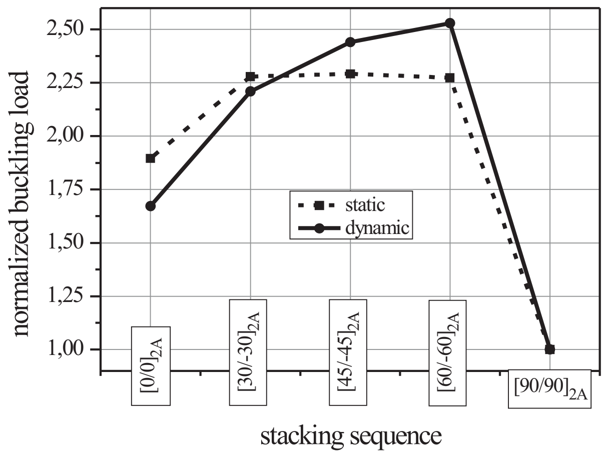

Figure 10 Comparison of buckling loads for static and dynamic cases, for angle–ply antisymetric laminates

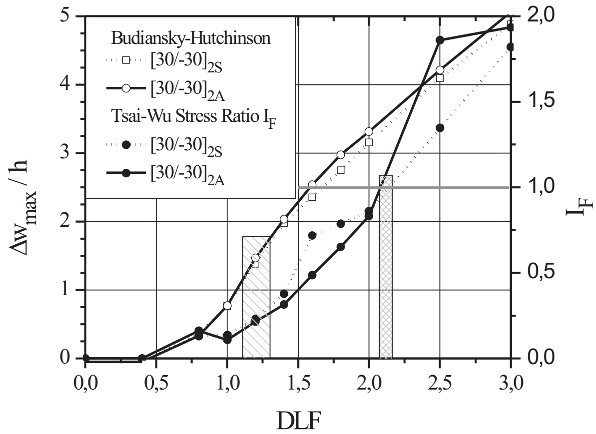

Figure 11 Application of different dynamic buckling criteria

for angle–ply laminates are presented, where both Budiansky-Hutchinson and Tsai-Wu criteria were employed. The last one was applied according to Petry–Falhbusch approach for orthotropic materials. Although the membrane–flexural coupling is not present in symmetric angle–ply multi–layered laminate, their results are included for comparative study. The range of critical values of dynamic load factor obtained with application of Budiansky–Hutchinson criterion is indicated by the left, lower dashed rectangle. Those critical values determined with (7) Tsai–Wu stress ratio (failure index I_F) application are marked by right, higher and slender rectangle. The values for two ranges differ almost twice. One can consider Budiansky–Hutchinson criterion as more conservative as relate to Tsai–Wu/Petry–Falhbusch criterion but Budiansky–Hutchinson criterion is generally in a good agreement with Ari Gur–Simoneta and not mentioned previously Volmir criteria [8]. It can be assessed as safer than Tsai–Wu or Petry–Fahlbusch criterion which is rather of failure type, and in such case it should be assumed that the material is not fully exploited even for plated structures which have stable postbuckling branch.

3.1. *Conclusions*

In the paper the representative results of dynamic buckling analysis of thin–walled laminated columns under impulse compression were presented. The plated walls of columns were made of fiber reinforced composites. Different layers architecture of FRP type composites were analyzed in order to assess their dynamic behavior. The special emphasis has been placed on the consideration of the effect of membrane–flexural coupling. The numerical calculations were obtained with Finite Element Method and ANSYS software package applications. The results of carried out analyses confirm the sensitiveness of the column dynamic response on the initial imperfections, pulse shape [8], [12] (and pulse duration not presented in the paper) on the dynamic buckling load as well as the stacking sequence of laminated walls, the orientation of principal directions of separate layers and their influence on the dynamic critical load value. In the case of cross–ply antisymetric laminates with high number of plies the critical DLF value does not depend on plies number. Also the rising orthotropy ratio does not influence the critical DLF range. For angle–ply antisymetric lay–up the dynamic load range is wider then in static case but similarly the highest buckling load was obtained for lamination angle $\theta = 45^o$, similarly for static and dynamic loading. Due to lack of bifurcation buckling load in the dynamic analysis there is the necessity of dynamic buckling criterion usage. The applied criterion influences the draw conclusions and DLF critical value.

The complex of the problem of dynamic stability of composite thin–walled columns induces the need of application of different numerical technique combination to reach the solution. In the presented study it was obtained – among others – due to FE application.

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