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Dynamic Response of the Truncated Conical Shell Subjected to Pressure Pulse Loading

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The paper deals with an analysis of the dynamic response of the isotropic conical shell subjected to rectangular or triangular pulse loading. This problem was solved with the Bubnov–Galerkin analytical–numerical method, using a two–parameter function of deflection. In the equations of dynamic equilibrium, an orthotropic material was taken into consideration. Moreover, pressure loading and compression force were included. The solution results were compared with the finite element method, using ANSYS 11.

Keywords: Dynamic buckling, dynamic stability, conical shells

1. Introduction

Conical shells as typical thin–walled structures have many applications in building industry as pressure vessels, roof steel structures, etc. An analysis was conducted for conical shells with low capacity.

In the world literature, the dynamic buckling phenomenon is widely described. Many articles were written in the 60's and the 70's of the 20th century. In [3], [4], a vibration analysis was conducted. On the basis of [6], dynamic equilibrium equations were extended to pressure loading.

In this paper, an analysis of the dynamic response to pulse loading was conducted, taking into consideration an influence of pulse amplitude on the highest value of deflection.

As known ([6], [7]), deflection appears before buckling of the conical shell. It takes place for static and dynamic loading. Despite it, to allow dynamic buckling, initial imperfection was assumed.

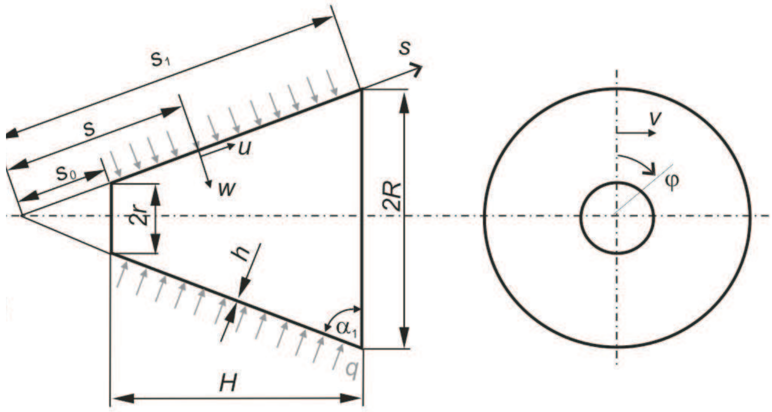

Figure 1 Conical shell – geometric dimensions, a type of loading and displacements

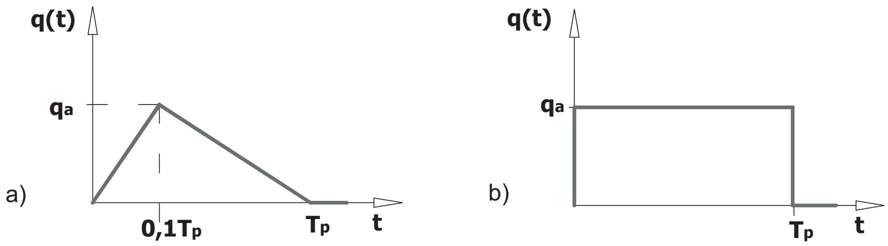

Figure 2 Types of pulse loading: a) triangular, b) rectangular

2. Describtion of the dynamic bucling phenomenon

As mentioned above, a truncated conical shell (Fig. 1) of the following dimensions: $H = 40$ mm, $R = 28$ mm, $r = 5$ mm, $h = 0.5$ mm was considered. It was assumed that the shell under analysis was made of steel with the following elastic properties: Young modulus $E = 2 \cdot 10^5$ MPa, Poisson coefficient $\nu = 0.3$. In the analysis, two shapes of pulse loading were used (Fig. 2); T_p – pulse duration corresponding to the free vibration period, q_a – amplitude.

3. Analytical–numerical method

In the conical coordinate system s, φ, t (Fig. 1), the equilibrium equation obtained from Hamilton's principle can be written as follows [7]:

$$\begin{gathered}\frac{N_2}{R_2} - N_1\kappa_1 - N_2\kappa_2 - 2T_{12}\kappa_{12} + \frac{2}{s}M_{1,s} + M_{1,ss} + \frac{2}{s\cos(\alpha_1)}M_{12,s\varphi} \\ -\frac{1}{s}M_{2,s} + \frac{2}{s^2\cos(\alpha_1)}M_{12,\varphi} + \frac{1}{s^2\cos^2(\alpha_1)}M_{2,\varphi\varphi} + q(t) = \rho h\ddot{w}(s,\varphi,t)\end{gathered} \tag{1}$$

where:

N_1, N_2 – membrane forces along the loop and hoop directions;
T_{12} – membrane shear force;
M_1, M_2 – bending moments;
M_{12} – torsion moment;
q – pressure;
ρ– density,
$R_2 = s/\tan(\alpha_1)$ – curvature radius of the conical shell.
Equation (2) includes curvatures and torsion of the middle surface defined as:

$$\begin{aligned} \kappa_1 &= -w_{,ss} \\ \kappa_2 &= -\frac{1}{s}w_{,s} - \frac{1}{s^2\cos^2(\alpha_1)}w_{,\varphi\varphi} \\ \kappa_{12} &= -\frac{1}{\cos(\alpha_1)}(\frac{w_{,\varphi}}{s})_{,s} \end{aligned} \tag{2}$$

Membrane forces were expressed using the Airy stress function.

Membrane strains of the middle surface according to [6] were written as:

$$\begin{aligned} \varepsilon_1 &= u_{,s} + \frac{1}{2}(w_{,s})^2 \\ \varepsilon_2 &= \frac{1}{s}(u - w\tan(\alpha_1) + \frac{1}{\cos(\alpha_1)}\upsilon_{,\varphi}) + \frac{1}{2}(w_{,s})^2 \\ \gamma_{12} &= s(\frac{\upsilon}{s})_{,s} + \frac{1}{s\cos(\alpha_1)}u_{,\varphi} + \frac{1}{s\cos(\alpha_1)}w_{,s}w_{,\varphi} \end{aligned} \tag{3}$$

In order to obtain the Airy function, the equation of continuum of the middle surface was used:

$$\begin{aligned} &\frac{1}{s^2\cos^2(\alpha_1)}\varepsilon_{1,\varphi\varphi} - \frac{1}{s}\varepsilon_{1,s} + \varepsilon_{2,ss} + \frac{2}{s}\varepsilon_{2,s} - \frac{1}{s\cos(\alpha_1)}\gamma_{12,s\varphi} - \frac{1}{s^2\cos(\alpha_1)}\gamma_{12,\varphi} \\ &= -\frac{\tan(\alpha_1)}{s}w_{,ss} + \left(\frac{1}{s\cos(\alpha_1)}w_{,s\varphi}\right)^2 - \frac{1}{s^2\cos^2(\alpha_1)}w_{,ss}w_{,\varphi\varphi} \\ &-\frac{1}{s}w_{,ss}w_{,s} - \frac{2}{s^3\cos^2(\alpha_1)}w_{,s\varphi}w_{,\varphi} + \frac{1}{s^2}\left(\frac{1}{s\cos(\alpha_1)}w_{,\varphi}\right)^2 \end{aligned} \tag{4}$$

Deflection was assumed as a two–parameter function, taking into account deflection in the range of the pre–buckling state [6], [7]:

$$w(s,\varphi,t) = f_1(t)\sin(ms)\sin(n\varphi) + f_2(t)\sin^2(ms) + w_0(s,t) \tag{5}$$

where:
m – number of waves along the loop direction,
n – number of waves along the hoop direction.
Using formula (3), deflection in the pre–buckling range was obtained:

$$\begin{aligned} w_0(s,t) = -\frac{1}{4\tan^2(\alpha_1)}q(t)\left[(s^2 - s_0^2)\left(\frac{1}{Eh} - \frac{2\nu}{Eh}\right.\right. \\ \left.\left. +\frac{2}{Eh}(\nu - 2)\right) + \frac{2s_0^2}{Eh}Ln(s_0/s)\right] \end{aligned} \tag{6}$$

Then, the Airy function was obtained, but in this paper the expression is omitted.

Boundary conditions (B.C.) were formulated for a simply supported conical shell at the top and the bottom:

$$w(s_0, \varphi, t) = 0 \qquad M_s(s_1, \varphi, t) = 0 \tag{7}$$

Assuming the initial shape imperfection and taking into account (5), initial conditions for deflection were assumed as follows:

$$\begin{aligned} &w(s, \varphi, 0) = f_{10} \sin(ms) \sin(n\varphi) + f_{20} \sin^2(ms) + w_0(s, 0) \\ &\dot{w}(s, \varphi, 0) = 0 \end{aligned} \tag{8}$$

and knowing that $w_0(s, 0) = 0$, $f_{20} = 0$, the first of two equations (8) can be written as:

$$w(s, \varphi, 0) = f_{10} \sin(ms) \sin(n\varphi) \tag{9}$$

Equilibrium equation (1) was solved with an analytical–numerical method – the so–called Bubnov–Galerkin method [5].

Description of algorithms and methods to solve equilibrium equations

In the analysis of the conical shell response, the ANSYS software (version 11) was used. This program performs computations with the finite element method. Discretization of the shell was performed with a 4-node shell element (Fig. 3). This type of the finite element has six degrees of freedom (3 displacements and 3 rotations). A numerical model includes boundary conditions close to (7), expressed in the conical coordinate system.

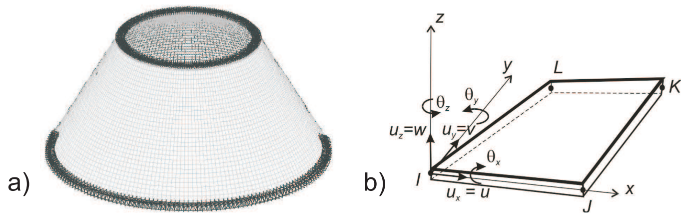

Figure 3 Numerical model of the analysed shell: a) after the discretization, b) type of the finite element

Computations were conducted in two stages. First, frequency of free vibrations and corresponding modes of free vibrations were obtained. Then, a nonlinear analysis was performed using appropriate modes to perturbate the finite element grid. In this analysis equilibrium equation (10)

$$\{\mathbf{P}\} = [\mathbf{M}] \cdot \{\ddot{\mathbf{u}}\} + [\mathbf{K}] \cdot \{\mathbf{u}\} \tag{10}$$

is solved, neglecting the dumping effect. In the equation, $[\mathbf{M}]$ means a mass matrix of the structure.

After substituting derivations of displacements $\ddot{\mathbf{u}}$ for difference quotients of displacements $\mathbf{u}$, a system of nonlinear algebraic equations is obtained for every time step, taking into consideration the Newton's force of inertia $[\mathbf{M}] \cdot \{\ddot{\mathbf{u}}\}$. This approach allows us to use algorithms in the static analysis. Operation of the time integration in the ANSYS code is performed with the Newmark method, but to solve the equations in next steps, the Newton–Raphson algorithm is employed.

4. Results of computations

The results from two methods described in sections 2 were compared to each other and with the results from the literature survey. Upper and lower static critical loads were computed and compared. Then, frequencies of free vibrations were obtained (Tab. 1).

Table 1 Critical forces and frequencies of free vibrations

Type of method	q_{kr} [MPa]	n_{kr} [-]	T_0 [ms]	n [-]
A-N	13	6	0.45	3
MES	14	6	0.11 0.41*	3

*) boundary conditions in the Cartesian coordinate system,

n - number of waves along the circumferential direction

The analysis was conducted for the same duration, independently of the type of pulse loading. In the paper, curves of the response from the analytical–numerical (A–N) and finite element method (FEM) are shown below. The parameter p means an amplitude of the pulse q_a. The duration T_p equals 0.45 ms. In Fig. 4 curves of deflections vs. time are shown. The amplitude of pressure was assumed to be equal to 10 MPa. Initial imperfections were neglected. The highest values of deflection are almost identical and appear at the same time. Oscillations shown in the curves are characteristic of vibrations of the middle surface.

Initial imperfections were taken into consideration in the next part of the analysis. In Fig. 5, curves of deflection vs. time, corresponding to this type of analysis, are shown. Initial imperfections were assumed to be equal to 10 % of the shell thickness.

While comparing curves from Figs 4 and 5, it can be noticed that increments of deflection (a triangular type of pulse) have maximal values of 0.1 mm. In Fig. 6 relative deflection (with respect to thickness) curves vs. the relative amplitude of pressure (to critical static load) are shown. Small differences between relative deflections suggest a neglected influence of initial imperfections.

In Fig. 7 w_{max}/h curves versus p/p_{kr} are shown for a rectangular shape of pulse. In this case of loading, initial imperfections have a significant influence on the behaviour of the shell. When initial imperfections are absent, differences between the results are small. Differences are significant when initial imperfections are assumed. The curve from the FEM goes up rapidly for p/p_{crit}=0.8.

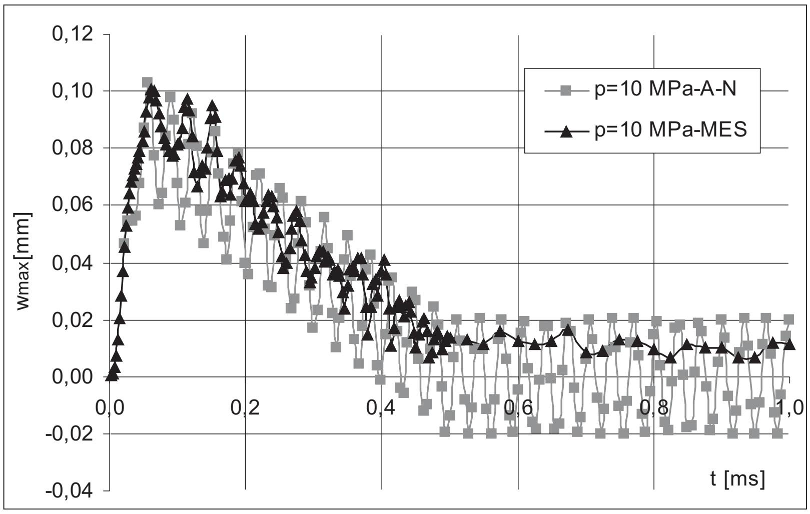

Figure 4 Curves of deflection vs. time for the amplitude of pressure 10 MPa, without initial deflection; duration equals 0.45 ms

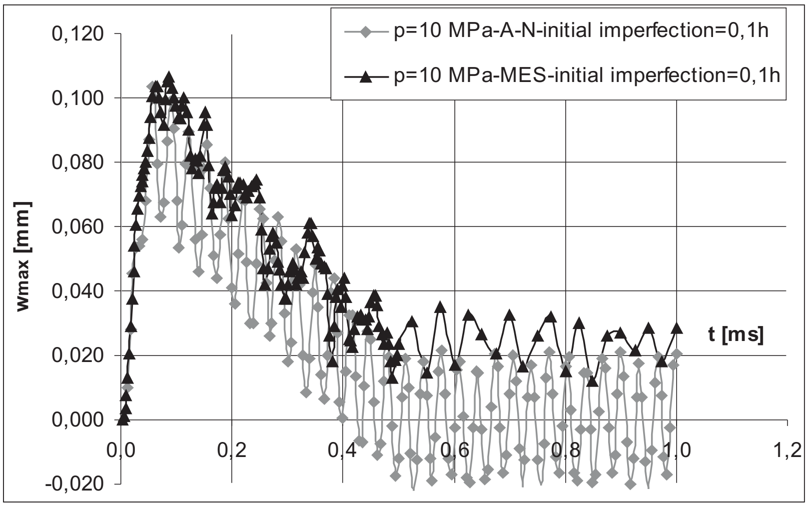

Figure 5 Curves of deflection vs. time for the amplitude of pressure 10 MPa, with initial deflection; duration equals 0.45 ms

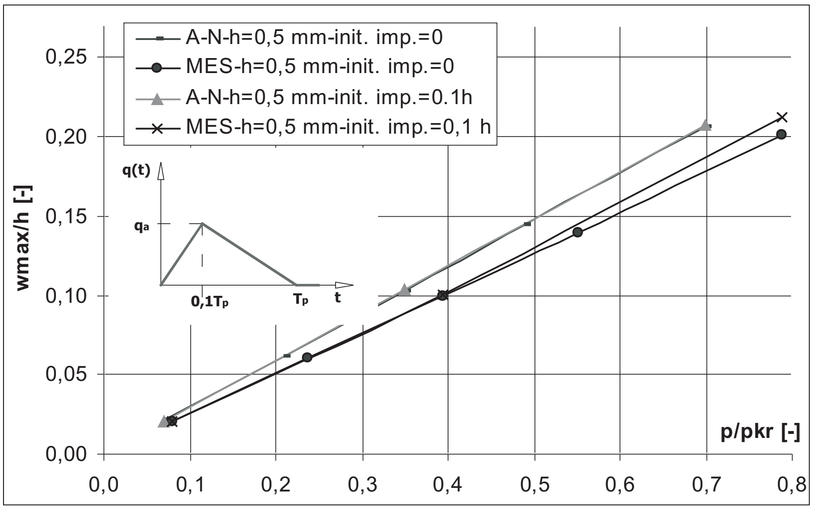

Figure 6 Curves $w_{max}/h = f(p/p_{kr})$ without and with initial deflection – a triangular shape of pulse; duration equals 0.45 ms

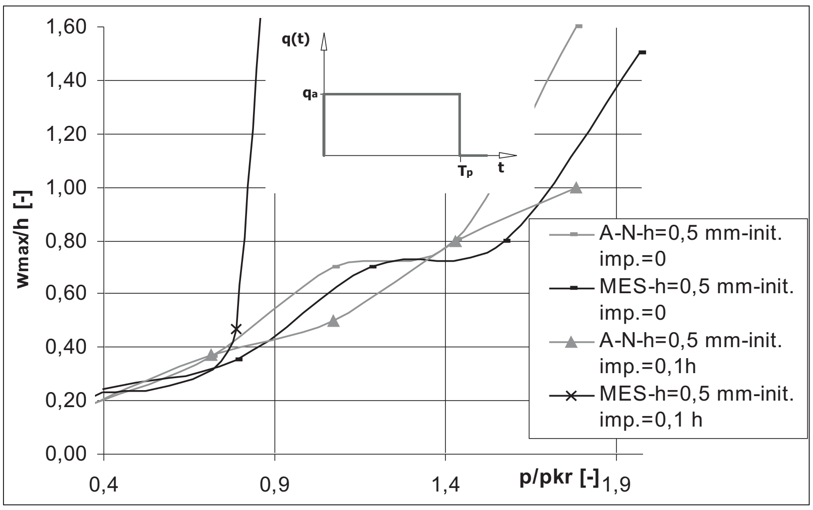

Figure 7 Curves $w_{max}/h = f(p/p_{kr})$ without and with initial deflection – a rectangular shape of pulse; duration equals 0.45 ms

5. Conclusions

The results shown in the paper point to an influence of the initial shape imperfections on the behaviour of conical shells, depending on the pulse loading shape, because energy (connected with its transfer to the structure) of the rectangular pulse is different (higher) than of the triangular one. Moreover, the results suggest that the analytical–numerical method should be modified with respect to a function of deflection (5). It is important to specify boundary conditions in all numerical models.

References

[1] **Budiansky, B. and Hutchinson, J.W.**: Dynamic buckling of imperfection–sensitive structures, *Proceed. XIth Internat. Contr. Applied Mech.*, Berlin, 636–651, **1964**.

[2] **Budiansky, B. and Roth, R.S.**: Axisymmetric dynamic buckling of clamped shallow spherical shell, *Collected papers on instability of shell structures*, NASA TN D–1510, 591–600, **1962**.

[3] **Gupta, A.P.**: Axisymmetic vibrations of conical sandwich shells, University of Roorke, *Roorke*, **1978**.

[4] **Hassan, A.I., Maalawi, K.Y. and Negm, H.M.**: Frequency optimization of conical shells under mass equality constraint, *J. of App. Sciences Research*, 2(11),821–829, **2006**.

[5] **Kowal–Michalska, K. (et al.)**: Stateczność dynamiczna kompozytowych konstrukcji płytowych, *WNT*, Warsaw, **2007**.

[6] **Królak, M.**: Stateczność ortotropowej powłoki stożkowej ściskanej osiowo – w ujęciu nieliniowym, Archiwum Budowy Maszyn, Zeszyt 3, Lodz, **1969**.

[7] **Volmir, A.S.**: Ustoiczivost Deformirujemych Sistem, Moscow **1967**.