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Critical and Postcritical Behavior of Thin–walled Multicell Column of Open Profile

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In the paper thin–walled isotropic and orthotropic multicell columns with open cross–section are consisdered. The local buckling and load carrying capacity analysis of these type columns under uniform compression was performed. Columns built of square shape cells with equal thickness of all walls and constant cross–section area were analyzed. The width of walls depends on the number of cells present in a column. The obtained results – it is the cell number and cross–section shape influence on the buckling load and load carrying capacity, are presented in the form of graphs and tables. The relatively high increase of local buckling stress value is demonstrated as well as the decrease of relationship between the load carrying capacity and buckling load – both events connected with increase of cell number.

Keywords: Multicell, thin–walled column, buckling, load carrying capacity

1. Introduction and basic assumptions

The paper deals with the local buckling and load carrying capacity analysis of multicell columns of open cross–section. The column is axially uniformly compressed whereas the shortening of the column takes place with loaded edges mutually parallel. The single cell is of square cross–section and its dimensions depend on the number of cells. The one cell column (box type column) has a square cross–section of b_1 width and t_1 wall thickness. Therefore its cross–section area equals to:

$$A_1 = 4b_1t \tag{1}$$

The double–cell column has the cross–section sides defined as $b_2 \times 2b_2$. The equal–leg angle shape column can be built of three cells with b_3 cell side width. The four cell column can posses the shape of T–profile, Z–profile or an angle with unequal–legs. Then the one side length is termed as b_4.

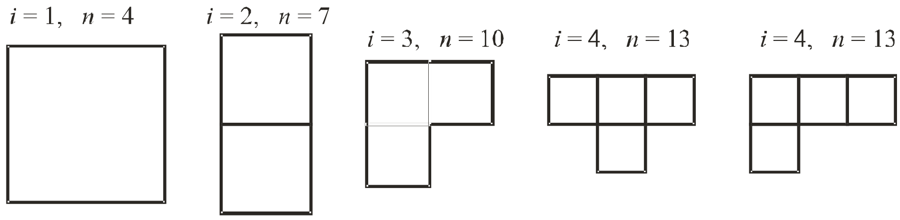

Figure 1 Cross–sections of single–, double–, three– and four–cell columns

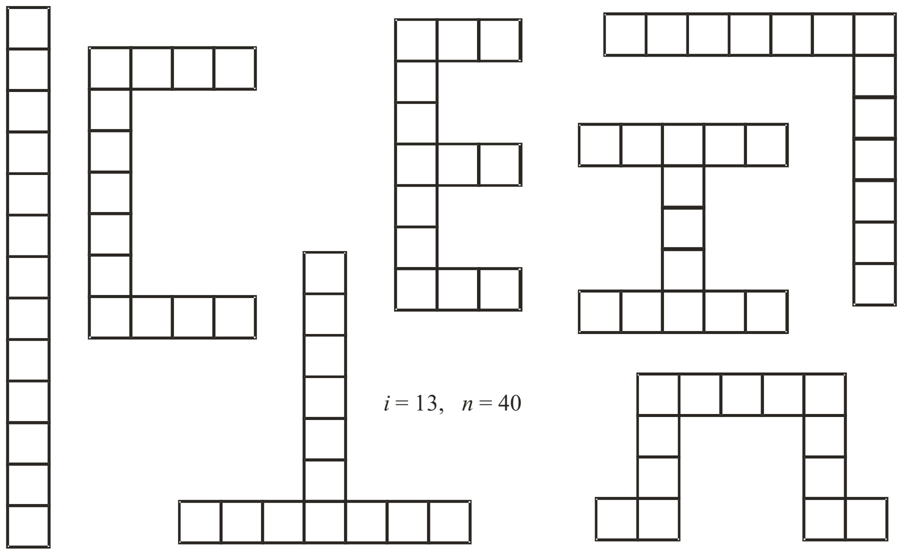

Figure 2 Cross–sections of open profile columns built of 13 cells

The cross–section shapes of mentioned columns with equal thickness of all walls and equal cross–section areas are presented in Fig. 1.

The one cell column has four walls and extending the cross–section with one more cell required adding of three walls more. Then the relationship between n number of walls and i number of cells is following:

$$n = 4 + 3\,(i - 1) \qquad i \ = \ 1, 2, 3, \ldots \tag{2}$$

The column compounded of five cells can be of C–shape, Z–shape or an angle either equal–leg or unequal–leg shape. The column of I–shape cross–section requires application of seven cells whereas unequal I–section profile requires at least nine cells. In Fig. 2 – as an example, there is presented a set of multicell column cross–sections consisting of 13 cells.

The analyzed columns can be made of isotropic material or orthotropic material, where the principal axes of orthotropy are parallel to the column edges. Fiber reinforced plastic is an ordinary orthotropic material that can be applied for multicell columns walls [5].

For pronounced and easy comparison of local buckling stress values and load carrying capacity, in the following considerations it was assumed for the analyzed columns:

- columns are made of the same material,
- the thickness t of all walls is equal,
- for all shapes of column profile the cross–section area is equal,
- the width of single cell wall is dependent on the column cell number.

The last requirements is done for comparison reasons only.

The cross–section area of a column with i number of cells (independent of profile shape) is defined by the following formula:

$$A_i = nb_it = [4 + 3\,(i - 1)]\, b_it \tag{3}$$

Comparing the cross-section area of i–cell column with the section area of a single cell square column, it was determined the breadth of a cell wall of i–cell column as a function of b_1 and i:

$$b_i = \frac{4b_1}{4 + 3\,(i - 1)} \tag{4}$$

In Tab. 1 some exemplary values of b_i/b_1 quotients for i = 1,2,. . . ,13 are calculated.

Table 1 Quotients values of cell wall width

b_1/b_1	b_2/b_1	b_3/b_1	b_4/b_1	b_5/b_1	b_6/b_1	b_7/b_1
1,000	0,571	0,400	0,308	0,250	0,211	0,182
b_8/b_1	b_9/b_1	b_{10}/b_1	b_{11}/b_1	b_{12}/b_1	b_{13}/b_1	
0,160	0,143	0,129	0,118	0,108	0,100	

The local buckling and load carrying capacity for multicell columns of closed section were analyzed in papers [1], [3], [4].

2. Local buckling stress

To establish formulas for local buckling stress the following assumptions were made:

- the thickness t of column walls and material properties allow elastic local buckling of analyzed column,
- there is no initial imperfections in column walls,
- all loaded column edges are simply supported,
- the loaded column ends remain plane and undergo evenly shortening,

- the global (Euler type) buckling will not occur (The global buckling could be ”prevented” by compound column design),

- the interaction between different buckling modes will not occur.

According to mentioned above assumptions, to determine the local buckling stress, all column walls can be considered as thin rectangular plates, simply supported along all edges.

In case of open section profile columns with high number of cells with small dimension b_i and free edges – for example angle section or T–section, local buckling of whole wall or coupling buckling can occur. Such wall can work as a structural corrugated core sandwich plate however this type of structure wasn't considered in this paper. It was assumed that all dimensions of analyzed columns are selected to avoid described above buckling possibility.

For columns which fulfill all listed conditions and which are made of orthotropic materials, Królak et al. [4] gave an approximated formula for local buckling stress value for wall of b_i width:

$$\sigma_{cr\ i}^{loc} = k\frac{\pi^2 E_1}{12\gamma}\left(\frac{t}{b_i}\right)^2 \tag{5}$$

In (5) k is a stability factor, which for long plates, simply supported at all edges, has a value of $k = 4$, γ is a factor which depends on orthotropic material moduli E_1, E_2, G and Poisson's ratios ν_{12} and ν_{21}. The inverse of factor γ is given by the relation (6):

$$\frac{1}{\gamma} = \frac{\frac{1}{2}\sqrt{\frac{E_1}{E_2}} + \frac{1}{2}\frac{E_2\nu_{12}}{E_1} + \frac{G}{E_1}\left(1 - \frac{E_2\nu_{12}^2}{E_1}\right)}{1 - \frac{E_2\nu_{12}^2}{E_1}} \tag{6}$$

where E_1 is elastic modulus in the longitudinal direction (column compression direction), E_2 is elastic modulus in the transverse direction (perpendicular to compression direction), G is Kirchhoff modulus and ν_{12}, ν_{21} are Poisson's ratios. According to Betty's theorem it is valid $\nu_{12}E_1 = \nu_{21}E_2$. Equations (5) and (6) were established after some rearrangements of expression given in Volmir work [2]. For special case – isotropic plates, there are well known equalities $E_1 = E_2 = E$, $\nu_{12} = \nu_{21} = \nu$ and $G = E/2(1+\nu)$.

Substituting relation (4) into (5) finally gives:

$$\sigma_{cr\ i}^{loc} = k\frac{\pi^2 E_1}{12\gamma}\left[\frac{4+3\,(i-1)}{4}\frac{t}{b_1}\right]^2 \tag{7}$$

or

$$\sigma_{cr\ i}^{loc} = \frac{\left[4+3\,(i-1)\right]^2}{16}k\frac{\pi^2 E_1}{12\gamma}\left(\frac{t}{b_1}\right)^2 \tag{8}$$

Describing in (8) new terms as:

$$\alpha_i = \frac{\left[4+3\,(i-1)\right]^2}{16} \tag{9}$$

and

$$\sigma_{cr\ 1}^{loc} = k\frac{\pi^2 E_1}{12\gamma}\left(\frac{t}{b_1}\right)^2 \tag{10}$$

leads to writing this formula in the following shorter form:

$$\sigma_{cr\ i}^{loc} = \alpha_i \sigma_{cr\ 1}^{loc} \tag{11}$$

In expression (11) $\sigma_{cr\ 1}^{loc}$ is critical stress of local buckling for single cell column with b_1 width of cell wall. Whereas for orthotropic rectangular plate with b_1 width, wall thickness t and longitudinal modulus E_1 and coefficient γ given by (6), $\sigma_{cr\ 1}^{loc}$ is determined according to (8). Numerical (discrete) values of coefficient α_i are presented in Fig. 3.

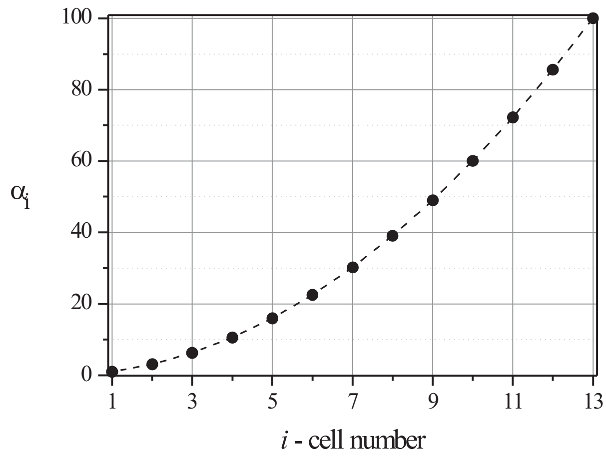

Figure 3 α_i coefficient as a function of cell number

It is worth noting that the length of longitudinal buckling half–waves in compressed orthotropic plates strictly depends on the stiffness ratio i.e. D_1/D_2 quotient. For orthotropic plate D_1 is the bending stiffness in the loading (longitudinal) direction and D_2 is the bending stiffness in the direction (transverse) normal to loading. For $D_1/D_2 < 1$ the length of half-waves is shorter where as for $D_1/D_2 > 1$ longer than in isotropic plates ($D_1 = D_2$) with equal geometry and identical boundary conditions [2].

3. Stability and load carrying capacity calculation

The numerical calculations were performed for isotropic columns with following geometrical and material properties: $i = 5 \div 13$ – cell number, $t = 0.5$ mm – wall thickness,the width of cell walls was determined from relation (4), the cross–section area was assumed equal to $A = 1000$ mm^2, the total column length was $l_i = 5 \times b_i$ with elastic modulus $E = 2 \times 10^5$ MPa, Poisson's ratio – $\nu = 0.3$ and yield limit $\sigma_{pl} = 200$ MPa.

The considered columns are subjected to uniform axial compression (even shortening of a total column length). The local buckling stress values are obtained from formula (7) and determined with finite element method application. The load carrying capacity for all columns was calculated with finite element software ANSYS ver 11.1 usage. For comparison, the results for C–section column with cell number varying from 5 to 13 (with equal area A) are listed in Table 2. From these values, the strong influence of cell number on the local buckling stress and load carrying capacity is clearly visible.

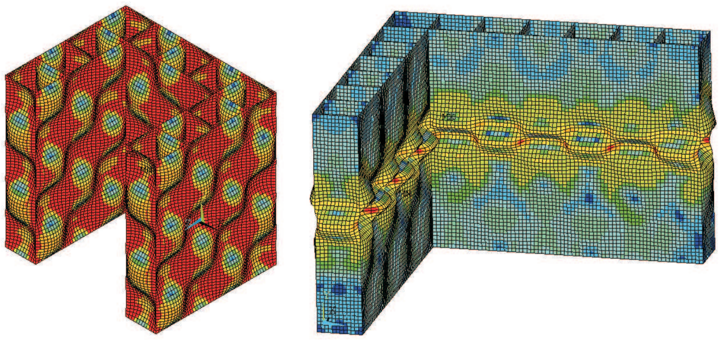

Figure 4 7– and 11–cells columns buckling mode

In Fig. 4 the examples of buckling mode and deformations in failure - post–buckling state are presented, for C–section multicell and angle profiles, respectively. Both results were obtained with finite element solutions.

In Tab. 3 the numerical and analytical results are compared for 13–cells column with different section profiles. The aim of performed calculations was to determine the influence of column open cross–section shape on the buckling stress value and load carrying capacity of axially compressed multicell columns.

In spite of small differences in load carrying capacity values for columns with equal cells number (fourth column of Tab. 3), the load shortening curves obtained for these profiles prove similar behavior of compared columns along the postbuckling path (Fig. 7). Even failure modes for these profiles show similar shapes and location (Fig. 8) in the structure.

The stability problem and load carrying capacity of similar multicell columns subjected to bending and/or eccentric compression will be considered in separate paper. It is well known that for bending type loading the cross-section profile shape is of great significance (keeping $A = \text{const.}$).

Table 2 Buckling load and load carrying capacity as a function of cell number

i – cell number	Section profile Fig. 5	N_{cr} [kN] Eq. (7)	N_{cr} [kN] FEM	$\Delta = \left\lvert \frac{N_{FEM}-N_{(7)}}{N_{(7)}} \right\rvert$	N_{lim} [kN]	$N_{\text{lim}}/N_{cr(7)}$
5	1	11.570	11.871	2.53	55.675	4.81
6	2	16.310	15.949	1.87	57.097	3.54
7	3	21.870	21.124	2.10	71.441	3.27
7	4	21.87	21.423	2.10	77.209	3.53
8	5	28.240	27.483	2.77	88.111	3.12
9	6	35.430	34.130	3.81	95.257	2.69
10	7	43.420	41.954	3.50	101.864	2.35
11	8	52.450	50.092	4.71	107.598	2.05
12	9	61.870	58.636	5.51	113.050	1.83
13	10	72.300	67.489	7.13	118.713	1.64

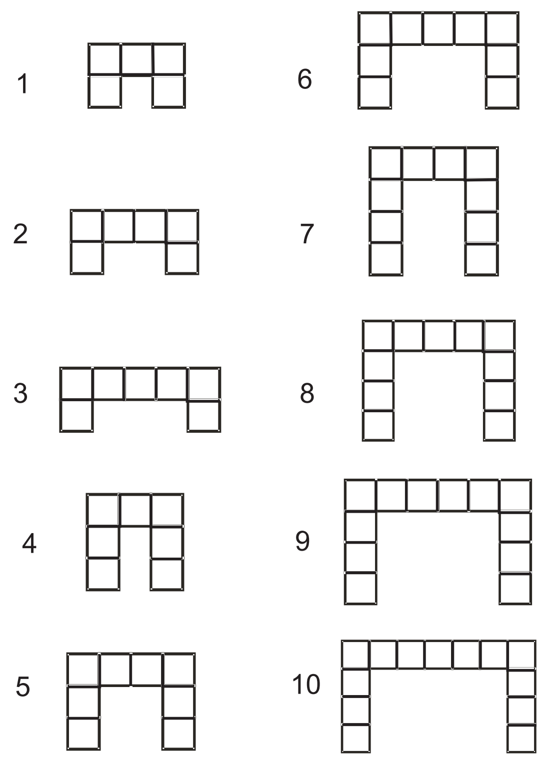

Figure 5 Section profile for Tab. 2

Table 3 Buckling load and load carrying capacity of 13-cells column profile

Cell number	Profile section Fig. 6	N_{cr} [kN]	N_{lim} [kN]
13-cells chanel shape	1	67,489	118,713
13-cells prismatic bar	2	61,398	119,481
13-cells equal-leg angle	3	67,239	119,484
13-cells T-shape	4	66,994	119,383
13-cells I-shape	5	68,792	118,735
13-cells *hat*	6	69,341	114,967

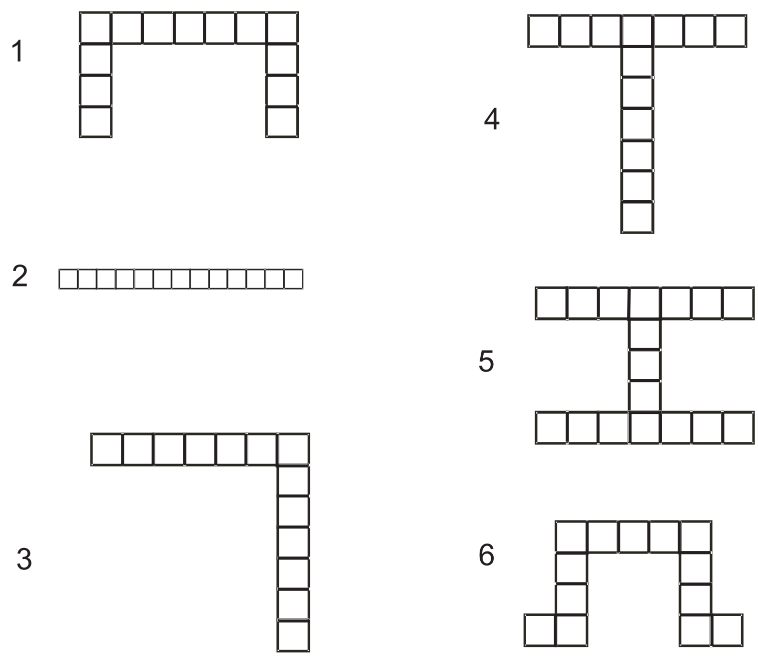

Figure 6 Section profile for Tab. 3

4. Analysis of results

Finite Element Method allows to analyze stability and load carrying capacity problems of multicell columns of different shapes of cell section and different material properties (isotropy and orthotropy). In this paper the cells of square section – as a pronounced example, were considered. The aim of this analysis was to prove that multicell columns can withstand much greater local buckling load than single cell column with equal cross–section area and posses much greater load carrying capacity as single cell column, as well.

The obtained results of numerical calculation show that the number of cells is of essential influence on the local buckling stress value. For 13–cells column their critical stresses are approximately 6 times greater than for 5–cells column. The profile section shape with given number of cells doesn't influence the buckling stress value or load carrying capacity.

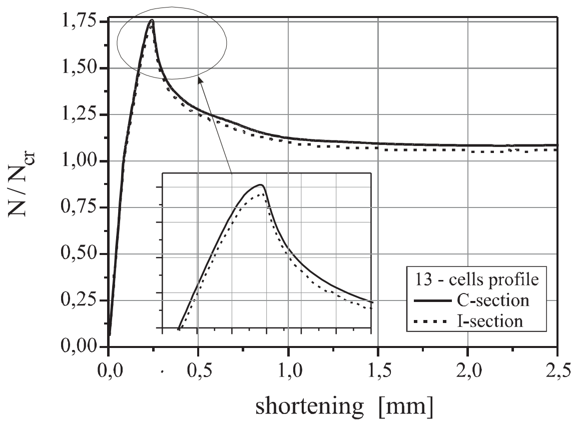

Figure 7 Comparison of 13–cells I–section and C–section columns load–shortening curves

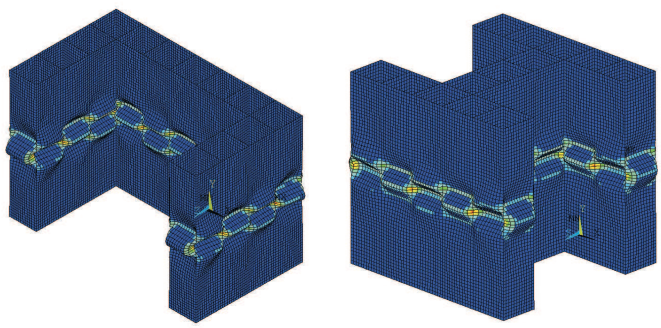

Figure 8 Failure modes of 13–cells C–section and I–section columns

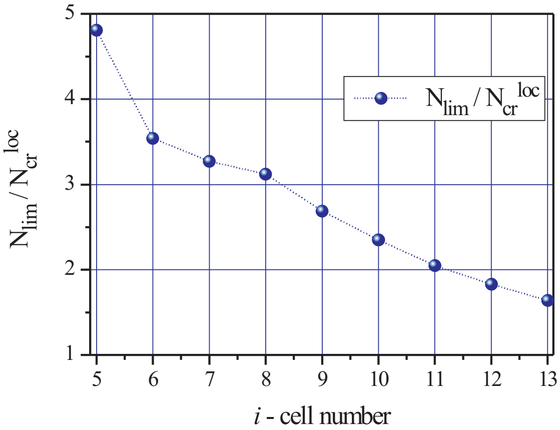

Figure 9 Load carrying capacity and buckling load quotient

The quotient of load carrying capacity and buckling load decreases when the cell number increases what can be directly confirmed by column 7 of Tab. 2 data and Fig. 9.

Simple comparison of critical stress values established with the application of formula (7) with those obtained from FEM solution, proves that the approximated formula (7) can be applied in practice to calculate σ_{cr}^{loc}, despite the difference increase up to 7.1% for 13–cells column.

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