Mechanics and Mechanical Engineering
Vol. 14, No. 2 (2010) 291–308

Tension Field in Thin–walled Aeronautical Girders

Jarosław Mańkowski
Jerzy Osiński
Piotr Żach

Instytut Podstaw Budowy Maszyn
Technical University of Warsaw
Narbutta 84, 02–524 Warsaw, Poland

Received (15 June 2010)
Revised (18 July 2010)
Accepted (26 July 2010)

The paper describes the influence of the field of draws on the state of effort of thin–walled girders, particularly on the state of burdening elements of connections of metal sheets of web of an I–beam with elements of the framework, e.g.: of riveted joints appearing in structures of this type. In the paper were presented results of analyses of girders without and with lightening holes. A fact brought up in this paper, is that implementing lightening holes is raising the critical strength of covering but an also causes increase of normal stresses appearing in elements of the skeleton and of shear stresses in connections.

Keywords: Tension field, thin–walled girders, FEM

1. Introduction

In this paper, were presented results of analyses of the thin–walled double spar flange girder, subjected to bending by crosswise force, acting in the wall plane of web of an I–beam. The considered girder can be a structural team of the wing or a fragment of the body e.g. of transport aircraft. The shear burden is supporting coming into existence of the phenomenon called the tension field. The girder was designed, as the classical semi–monocoque structure, in which among others: is allowed exceeding critical force, causing buckling of coverage.

The tension field is a phenomenon appearing in thin–walled structures. The first publications on this subject came in at the beginning of the last century [7, 9]. Although this phenomenon is often appearing in semi–monocoque structures, there is a little of works and publications concerning examinations and analyses of it. It's demonstrated, even if, by the document published in 2002 by the NASA, containing guidelines for the air structures design [8]. So far, in world literature, there are not many publications about methods of analysis of the influence of the tension field for the effort state of semi–monocoque structures. The tension field can be a reason for coming into existence of many local phenomena, causing concentrations of stresses in connections – especially it is regarding rivet and screw connections which are still universally used in aviation. With reference to the above, a sequence of examinations of both analyses, being aimed at determining the influence of the tension field on the condition of stresses and deformations coming into existence in the zone of the connection between framework and cover. In the paper results of analyses of the influence of holes carried out were also presented in riveted covers, thin–walled girders and ribs, to the condition of burdens for riveted joints appearing in these structures. In order to apply recalled holes is raising the critical strength on buckling. This issue is known for many years and willingly applied in air structures for at least two reasons: for first raising the critical strength of cover; secondly reducing mass.

For the sake of strongly non-linear character examined phenomena, analyses were made using the Finite Element Method. An Abaqus system was used for calculations.

2. Tension field

Tension field (Diagonal tension) – a special case of buckling of coatings of thin–walled structures, coming into existence as a result of the effect of tangent forces, was first mentioned in English nomenclature [7, 11].

In 1928 H. Wagner presented the trial, in which the thin cover with crosswise stiffenings was subjected to the burden which caused oblique pleats. He demonstrated, that the buckling of coverage isn't destroying the structure, until crosswise stiffenings are working only for compression (to the moment of loss of the stability). It had an intense influence on the attempt at the thin–walled structures design.

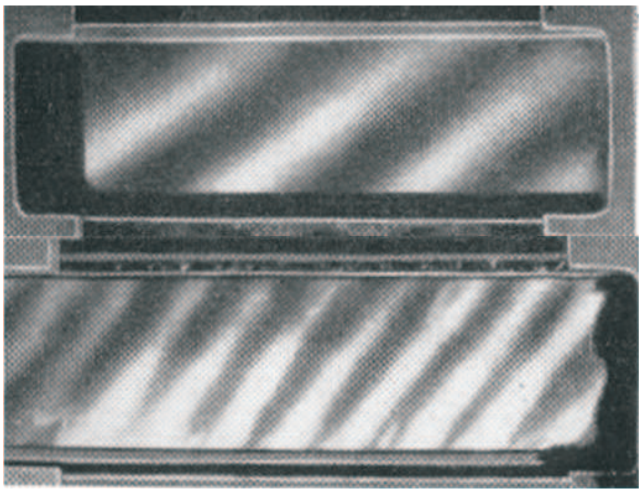

Figure 1 Example of tension field – Figure from [[4]]

Essence of the phenomenon called the tension field is, that in spite of burdening the coverage with only tangent forces, in cover, after exceeding critical force, is appearing complex state of stress (Fig. 2).

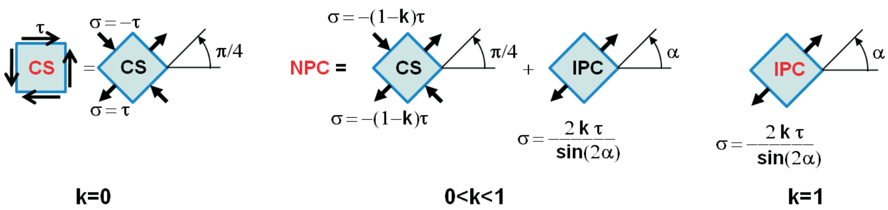

Figure 2 State of stress in the tension field

In case of the tension field one should distinguish three phases dependent on the value of the burden:

- the first phase, in which the burden doesn't exceed the value of critical force, therefore, in cover is only appearing pure shear CS (1); in principle it isn't necessary still to talk about the tension field;

$$\sigma_{PC} = \sigma_{CS} = \tau \tag{1}$$

- the second phase, in which the burden exceeds the value of critical force; in this case the state of the tension in cover is a superposition of pure shear CS and of stretching stress resulting from folding of coverage, so that is from appearing of the tension field IPC (2); the second phase is being called the incomplete tension field and indicated NPC[1];

$$\sigma_{PC} = \sigma_{CS} + \sigma_{IPC} \tag{2}$$

where

$$\sigma_{CS} = (1-k)\tau \qquad \sigma_{IPC} = \frac{2k\tau}{\sin(2\alpha)} \qquad 0 < k < 1$$

- the third phase, in which the load is so great that isn't any shear stress but only pure stretching IPC is already appearing (3); of course it is purely theoretical case, however can be taken into consideration for very thin covers.

$$\sigma_{PC} = \sigma_{IPC(k=1)} = \frac{2\tau}{\sin(2\alpha)} \tag{3}$$

Destroying the structure as a result of tension field effect can appear when:

- we'll exceed tensile strength of material of coverage – tearing material will take place;

[1] NPC – incomplete tension field, definition by Zbigniew Brzoska [2].

- exceeding critical force for elements of the framework will take place, on account of growing normal force and the appearance of perpendicular strain to the axis of bars of the framework.

Kuhn Paul described detailed concerning suming up this phenomenon in National Advisory Committee for Aeronautics in 1952[2], and then, fully systematized description of this phenomenon, placed in the extensive monograph concerning analyses of thin–walled structures [7], published in 1956. They are next important positions in which the row of the information is about the tension field, among others: [4] published in 1960[3], [2] published in 1961[4] and [10] published in 1965[5]. A changed and completed edition of the book [3] was published in 1965. However about the tension field, perhaps on account of the complexity of the issue, relatively not a lot of works came into existence, and incurred in consecutive years works, usually were based on works of J. P. Timoshenko, P. Kuhn, Z. Brzoska, and mainly concerned simply cases.

It isn't possible of course to state that there is no publication about the tension field. Especially twenty last years resulted in the certain number of publications concerning this issue directly. Certainly, a published in 1994 work (by William L. Ko and Raymond H. Jackson [6]) is meriting attention, which analysis of the ribbed, thin–walled plate loaded by tangent forces was described in. Also works which in 2003–2004 Alexander Tessler, David W. Sleight and John T. Wang published are deserving the special attention. At these works they described research findings and FEM analyses of the very thin shell, which as a result of the shearing force, underwent folding [12].

Moreover in the work [13] was described analysis of the solar sail subject both for burden acting in the plane of the shell (plane stress) and for burdening with gravity forces, perpendicular to the shell. In real structures, the problem of the tension field most often appears in thin–walled girders and in shells of outside planes, especially in covers of fuselage, which among others are exposed to turning.

3. Analysis of the Thin–Walled Beam

In order to evaluate the impact of the tension field on the state of the effort of thin–walled beam, a lot of analysis were made, which also served to evaluate the possibility of using different analytical models. Occurring in the construction thin–walled beams are usually equipped with struts, whose aim is to increase the overall stiffness of the structure and increase the critical force. However, struts make increased amount of disturbance zones caused by the co–operation between cover and the framework [3, 7, 10]. Accordingly, the analysis was performed with the beam with struts in which the ratio of length to the height of the panels is approximately equal to one. Beams were designed to have elongation factor profile equal 5, and its basic dimensions are 1000mm x 200mm – Fig. 3a. Load was realized by concentrated force, placed at the bottom left corner of the structure, facing down – Fig. 3b. Restraint was carried out in a way that enabled shifting of vertically

[2] *A Summary of Diagonal Tension*, Kuhn Paul (information base on [[2]]).
[3] Author: Hertel Heinrich, Dr. Ing. O. Professor an der Technischen Universitët Berlin.
[4] Author: Zbigniew Brzoska Dr In. Profesor IPPT PAN, PW.
[5] Author: Jerzy Teisseyre Dr In. Profesor IPPT PAN, Politechnika Wrocawska.

upper, right corner of the structure. The lower-right corner was designed as immovable. Elements of the skeleton were made from hot-drawn angles, made from material D16TN. The main elements of the skeleton, both longitudinal and transverse were made from 25x25x3 former [14] – Fig. 3c, while the spacers were made from 18x18x2 former [14] – Fig. 3d. Cover sheet was a 1mm thick, made from the material D16TN. Spacers were placed on only one side of coverage.

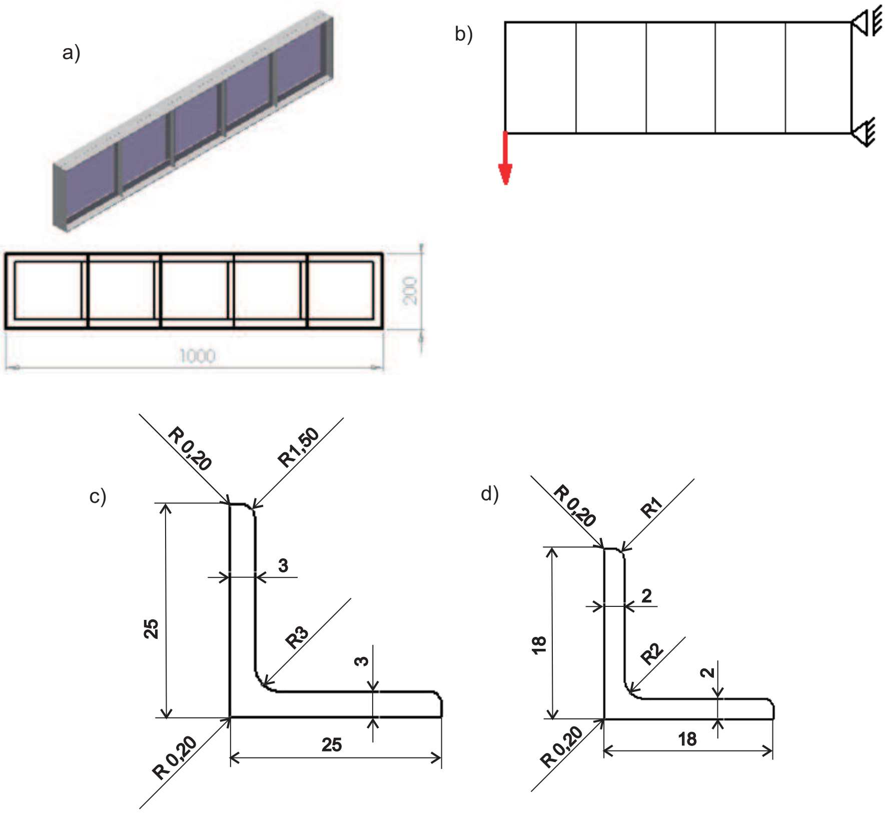

Figure 3 Thin–walled beam with struts: a) general diagram; b) loading and fixing schema; c) 25x25x3 PN-84/H-93669; d) 18x18x2 PN-84/H-93669

The main goal of analyses of this structure was checking the possibility of applications of different analytical models applied in analysis of thin–walled structures, appointing critical forces and checking conditions of coming into existence of the tension field. Aspiring for carrying the put task out, analyses were performed for the following analytical models:

- Model 1 – semi–monocoque model,
- Model 2 – model beam–shell – linear analysis,

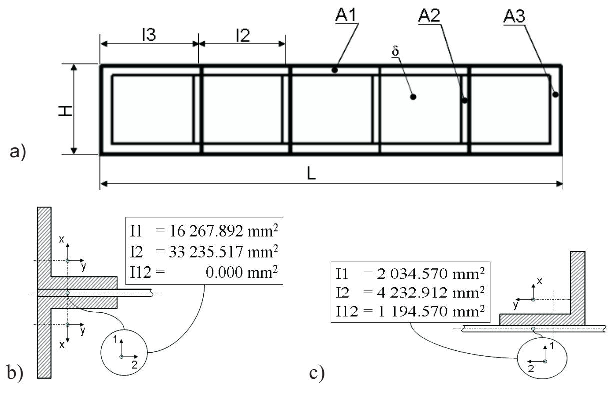

Figure 4 Thin–walled beam with struts – diagram of markings and moments of inertia: a) marking accepted in FEM models (semi–monocoque, monocoque, beam–shell), b) moments of inertia of elements of the framework with regard to the local system of coordinates of the element for the semi–monocoque and beam–shell model, c) moments of inertia of struts with regard to the local system of coordinates of the element for the semi–monocoque and beam-shell model

- Model 3 – model beam–shell – non-linear analysis,
- Model 4 – cover model with the connection on stiffly of skeleton with cover,
- Model 5 - cover model with partial connecting the framework with cover and with the contact.

FEM models were built according to assumptions of right analytical models. For the model with struts – Fig. 3, the following parameters were assumed:

L = 985.48 mm, H = 185.48 mm, l2 = 198.51 mm, l3 = 193.60 mm.

Face areas of elements of the framework were enlarged according to assumptions of the semi-monocoque model and took out appropriately:

A1 = 317.213 mm^2, A2 = 132.913 mm^2, A3 = 317.213 mm^2.

Thickness of the metal sheet was 1 mm.

Face areas of elements of the outside framework for the semi–monocoque and beam–shell models were increased by the fragment of the metal sheet of cover being outside the centres of gravity – Fig. 4b and amounted to A1 = 291.14 mm^2. Face area of struts for these models amounted to A2 = 68.38 mm^2. Moments of inertia and putting the pivot, towards which they were calculated, are presented in pictures: Fig. 4b and Fig. 4c.

For the semi–monocoque model analysis of the state of stress with the solution at using linear procedures was made.

For remaining models analyses were performed in the linear scope, for the purpose of determine critical forces, and then non–linear analyses, of which determining the condition of stresses and deformations after exceeding critical forces was a purpose. Linear analysis of the beam–shell model was also performed (Model 2).

Non–linear analyses weren't performed for the semi–monocoque model, on account of the fact of the practical unfitness of these models for that kind of analyses.

Until recently, on account of the small computing power of computers, above models were also being used for the simplified evaluation of the condition of stresses, appearing after exceeding critical force. It is possible to carry it out through introducing the substitute stiffness for cover and changing her in the iterative way, after every change of bending the structure [7]. Such a solution allows the analysis of the state of normal stresses in the plane of cover and shearing stresses. However it doesn't allow the analysis of stresses and deformations on direction perpendicular to the plane of cover. In relation to above was regarded pointless.

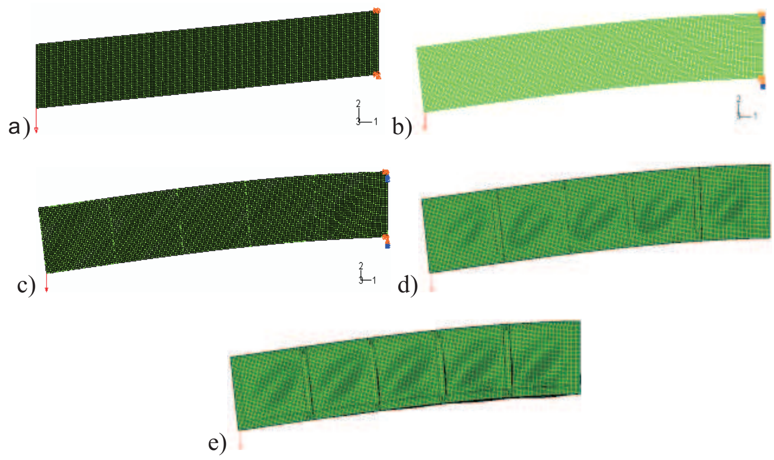

Figure 5 Thin–walled beam with struts – the FEM model, the load and boundary condition: a) semi–monocoque model; b) beam–shell model – linear analysis; c) beam–shell model – non-linear analysis; d) shell model with stiffly connection between framework and cover (TIE); e) shell model with partial stiffly connection between framework and cover (TIE) and partly modeled contact between the framework and cover

Load was realized by concentrated force with value 1000N. Load was selected in order to during linear analyses were made, stresses in cover didn't exceed the 60% yield point R_{02}.

Presented results are confirming, that at the proper selection of the ratio of the framework stiffness to the cover stiffness (of dimensions of the face area to the metal sheet thickness) a very low sensitivity of the structure to exceeding critical forces is received – Fig. 5. For each of analysed models, in spite of exceeding critical force of cover, decrease of the general stiffness is small – Fig. 6. During analyses a great sensitivity of examined FEM models to too big disproportions of the cover stiffness towards the framework stiffness was stated. Reducing the thickness of metal sheets of covering in discussed models by the 20% resulted in the lack of abilities of getting the convergence of the solution in the over–critical range.

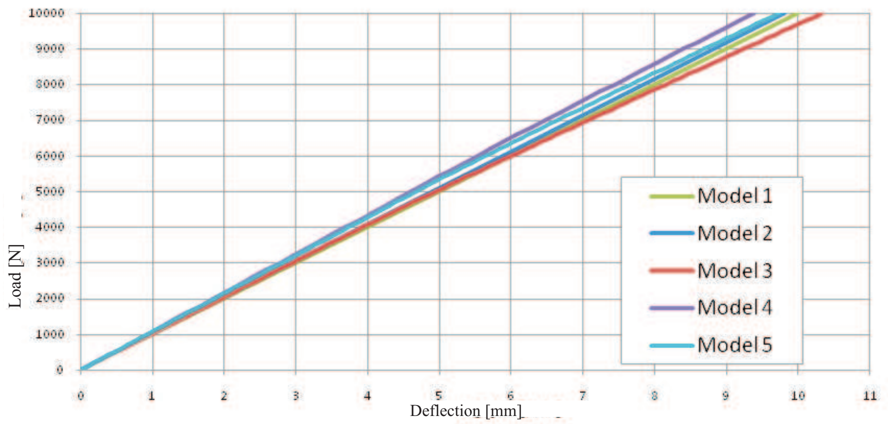

Figure 6 Graph showing the course of load depending on the bending function: Model 1 – semi-monocoque model; Model 2 – beam–shell model – linear analysis; Model 3 – beam–shell model - non-linear analysis; Model 4 – shell model with stiffly connection between framework and cover (TIE); Model 5 – shell model with partial stiffly connection between framework and cover (TIE) and partly modeled contact between the frame–work and cover

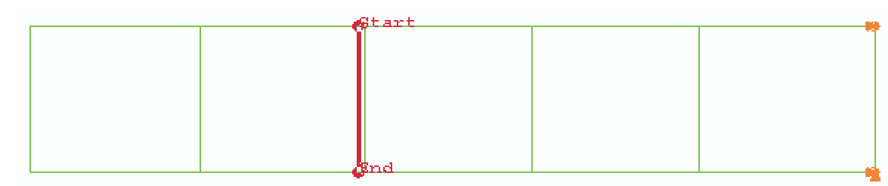

Figure 7 Thin–walled beam with struts - chosen section for comparing results of current analyses

3.1. *Achieved results for next models*

Summary of analyses results was described for the chosen face area – Fig. 7. A course of reduced stresses and shear stresses was compared. Reduced stresses (according to H–M–H) – Fig. 8, will be deciding about the resistance of cover to tearing; however shear stresses – Fig. 9, are decisive load of riveted joints.

A fact of appearing very big deflections on direction perpendicular to the plane of cover is deserving the special attention – Fig. 10. Achieved results are confirming the significant influence of the tension field on the state of loads and deformations near connections of the metal sheet of web of an I–beam with elements of the framework. Comparing results of analyses one should notice that the rise in elements of the framework reaches 10% of normal stresses – however the rise of shearing stresses in the cover reaches almost a 100% towards linear analysis. An also substantial increase in reduced stresses causes it.

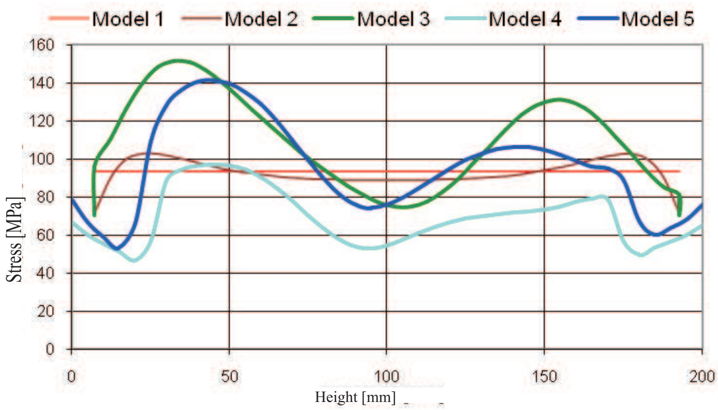

Figure 8 Thin–walled beam with struts – graph showing the course of reduced stresses (according to H–M–H) for the chosen section – comparing results: Model 1 – semi–monocoque model; Model 2 – beam-shell model – linear analysis; Model 3 – beam–shell model non–linear analysis; Model 4 – shell model with stiffly connection between framework and cover (TIE); Model 5 – shell model with partial stiffly connection between framework and cover (TIE) and partly modeled contact between the framework and cover

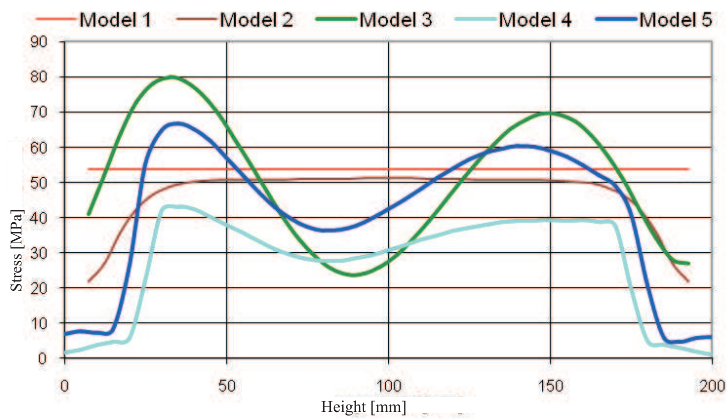

Figure 9 Thin–walled beam with struts – graph showing the course of shear stresses for the chosen section – comparing results: Model 1 – semi–monocoque model; Model 2 – beam–shell model – linear analysis; Model 3 – beam–shell model non–linear analysis; Model 4 – shell model with stiffly connection between framework and cover (TIE); Model 5 – shell model with partial stiffly connection between framework and cover (TIE) and partly modeled contact between the framework and cover

4. Analysis of the Influence of Lightening Holes to the Condition of the Loaded Joints, on the Example of Analyses of the Girder

This chapter is devoted to analysis of the influence of lightening holes made in girders, ribs, and the similar elements of the structure, on effort of material of cover and connections of metal sheets of cover with elements of the framework. Applying lightening holes is known for many years and willingly applied in air structures, for at least two reasons: for first raising the strength critical of cover; secondly reducing mass.

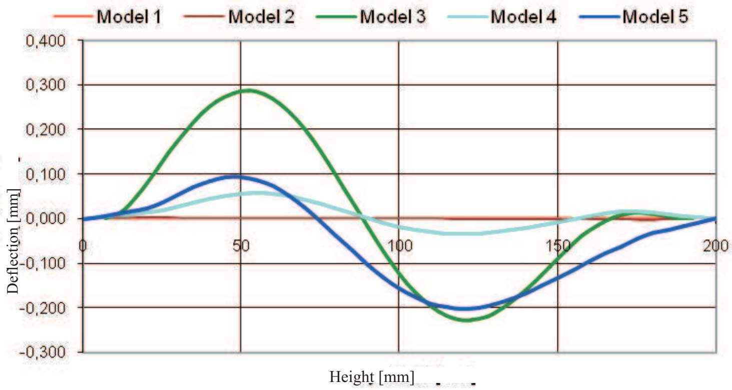

Figure 10 Thin–walled beam with struts – graph showing the course of bending for the chosen section on direction perpendicular to the selected plane area – comparing results: Model 1 – semi–monocoque model; Model 2 - beam–shell model – linear analysis; Model 3 – beam–shell model non–linear analysis; Model 4 – shell model with stiffly connection between framework and cover (TIE); Model 5 – shell model with partial stiffly connection between framework and cover (TIE) and partly modeled contact between the framework and cover

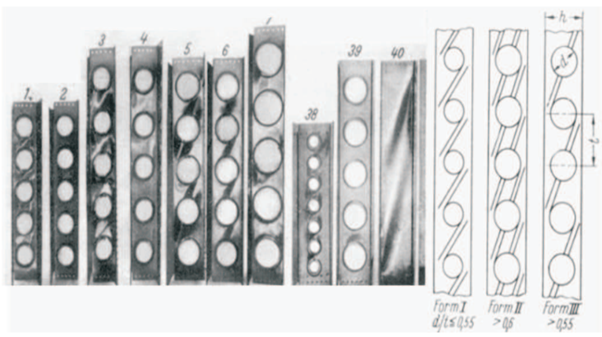

Figure 11 Buckling forms of bending thin–walled beam with lightening holes

There are undoubted advantages. In spite of the universality of solutions of this type, there aren't solid analytical studies of the recalled problem described in literature, especially related to riveted joints. All of the solutions are based mainly on former research works, or internal studies – unpublished. For the example it is possible to mention works performed for the SAAB company which Heinrich Hertel carried out in 1960 [4].

One of achievements of mentioned works, was the statement that form of buckling of girder with lightening holes is dependent on the ratio of the diameter of holes to the pitch of a riveted joint – Fig. 11.

On account of the too small amount of published data didn't manage to confirm the above statement. From the context of these publications, we can suppose that apart from the influence of the ratio of the diameter of holes to the pitch, to the buckling form a way of joining elements of covering with framework has considerable influence. Not a lot it is also known about characteristics of materials used for examinations. However, it is possible to maintain that implementing lightening holes is raising the critical strength of cover, but also causes the substantial increase of normal stress appearing in elements of the framework and increase of shear stress in joints.

At this work, Authors presented results of analysis of the girder with dimensions 750x200mm. The framework was made from the 25x25x3 former, and cover from metal sheet of thickness 1 mm. Framework and covers are made from an aluminium alloy D16TN, applied also in previous analyses.

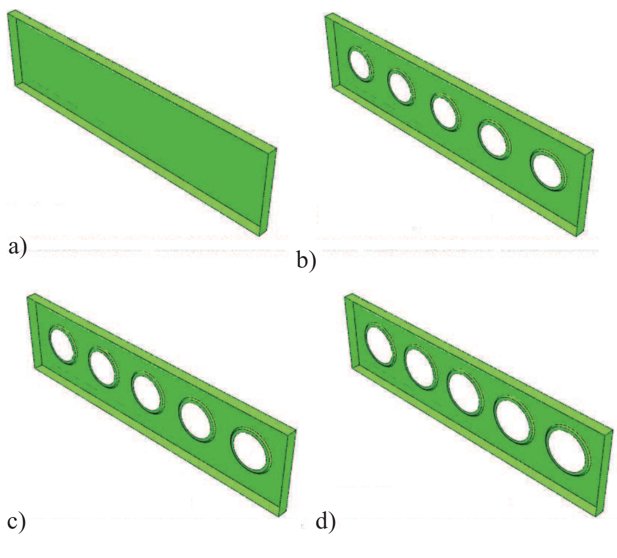

Figure 12 Thin–walled beam – models for analysis of the influence of holes on the state of stress of rivet connections: a) without holes, b) hole with the diameter of 70 mm, b) hole with the diameter 80.5 mm, b) hole with the diameter of 91 mm

Four models were made – Fig. 12: girder without holes and three models of the girder with holes with diameters: 70.0 mm, 80.5 mm, 91.0 mm. Holes were made according to SAAB–Anevi assumptions – Fig. 13. Models were made, as shell models with applying S4 elements. Connections between framework and covers were carried out with TIE elements.

Load was carried out as a continuous, however boundary conditions were defined in the way making impossible the migration of the right side of the girder in any three directions. Fig. 14 is showing the scheme of the load and boundary conditions. The value of the load was selected this way so that for complete analysis we received the reaction on vertical direction equal 10 kN.

For every model two types of analyses were made. Firstly analysis of critical force and buckling forms (for every model was counted twenty first forms) – linear analysis. Secondly, analyses of bending the girder were performed – non-linear analysis.

Hereinafter of work results, showing four first buckling forms are placed for every girder, with critical forces suiting the given form – Fig. 15. Get values of critical forces were placed in Tab. 1. Negative values of the part of critical forces, mean, that given buckling form will appear at force (in this case – at continuous load) directed in the opposite direction.

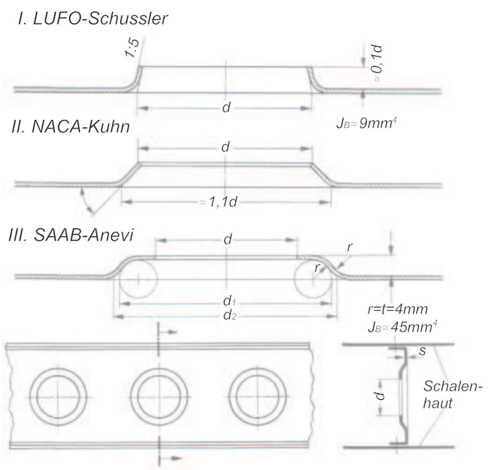

Figure 13 Different ways of carrying lightening holes out

How it was mentioned, both results of analyses of the buckling form, as well as displacements on direction perpendicular to the plane of cover – Fig. 16a, didn't confirm assumptions given by Hertel. However achieved results allow for drawing essential concerning conclusions both of effort state of cover as a result of bending the inspected girder, as well as loads appearing in connections between girder and elements of the framework. Cover effort state is very well illustrated by reduced stresses – Fig. 16b. One should pay attention to the high increase of cover effort.

For loading the girder without holes corresponding to the first critical force, the maximum reduced stresses according to the Mises hypothesis are growing about over the 250% – Tab. 2.

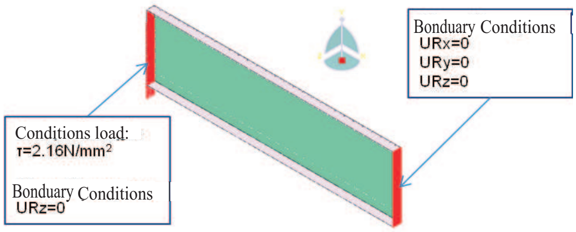

Figure 14 Girder – models for analysis of the influence of holes on the state of stress of rivet connections – boundary conditions and loads

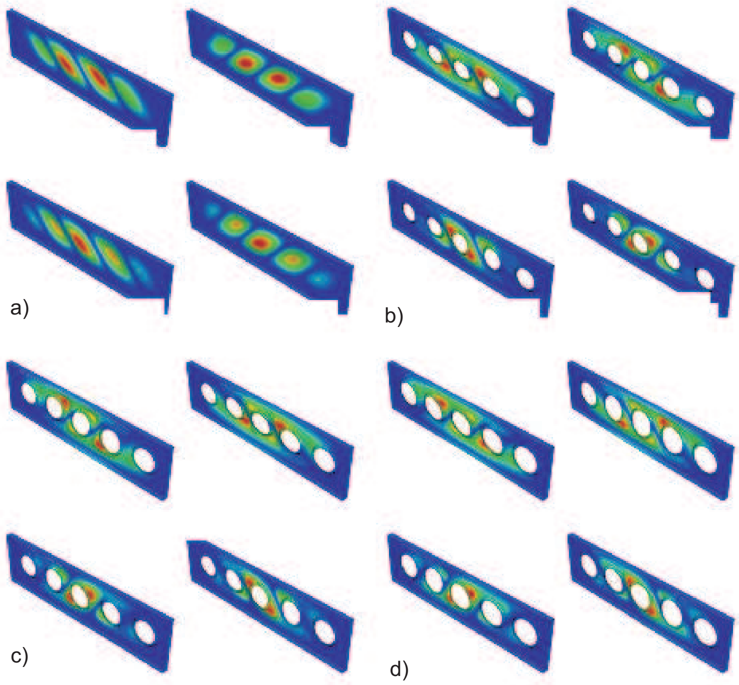

Figure 15 Girder – analysis of the influence of holes on the state of stress of rivet connections – four first buckling forms:
a) model without holes;
b) model with holes with the diameter of 70 mm;
c) model with holes with the diameter 80.5 mm;
d) model with holes with the diameter of 91 mm

Table 1 Statement of the value of critical forces

diameter of the hole	I critical force [kN]	II critical force [kN]	III critical force [kN]	IV critical force [kN]
lack	-0,39546	0,39547	-0,39596	0,39596
70,0 mm	-0,46921	0,46940	-0,48714	0,48742
80,5 mm	0,48038	-0,48045	0,51403	-0,51413
91,0 mm	0,51275	-0,51278	0,56774	-0,56794

Table 2 Girder with lightening holes – statement of reduced stresses (according to H–M–H) for loading the girder without holes corresponding to the first critical force (3.9 kN)

Without hole	hole – 70 mm	hole – 80,5 mm	hole – 91 mm
122 MPa	176 MPa	233 MPa	315 MPa

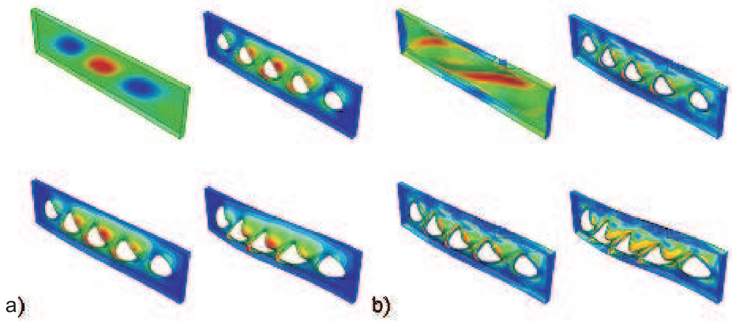

Figure 16 Girder – analysis of the influence of holes on the state of stress of rivet connections: a) bendings after exceeding critical force on direction perpendicular to the plane of cover; b) reduced stress (according to H–M–H) after exceeding lowest critical force [MPa]

Such a great increase of cover tension is having its impact for loading of elements linking the cover with the framework. A course of shear stresses in cover depicts it. For the example results were presented for the upper, internal line of the joint of the metal sheet of cover with the angle – Fig. 17. From the described graph one can see, that the maximum values of shear stresses, for all models with holes, are over twice as bigger than stresses appearing in the model without the hole.

Additionally, one should pay attention to the fact of appearing of the bending moment in cover, causing bending metal sheets in the area of the joint with the angle. As the example, we present results for torsion angle of the metal sheet for the upper, internal line of the joint of the metal sheet of cover with the angle – Fig. 18. As can be seen, for the girder without holes, the torsion angle is very small and is fluctuating near zero degrees. However for models with holes, the torsion angle reaches almost one degree.

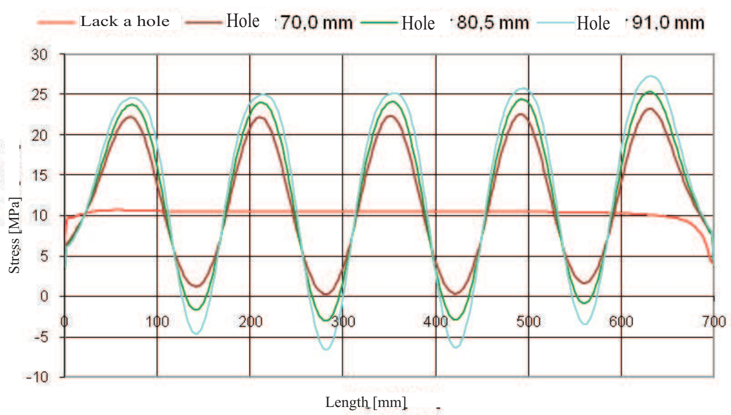

Figure 17 Shear stresses in cover - for the upper, internal edge of contact between metal sheet and the angle

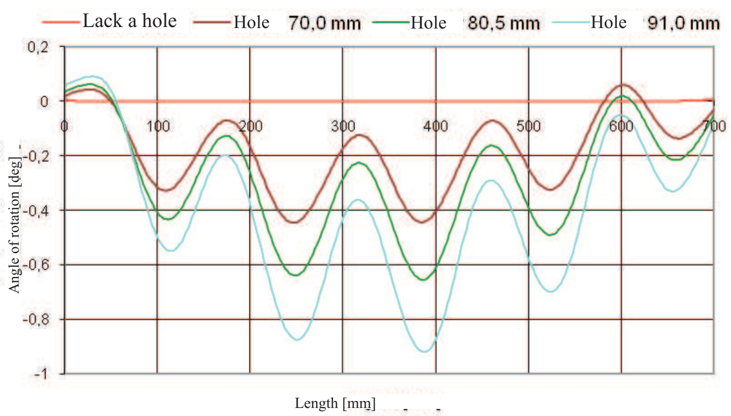

Figure 18 Torsion angle of the metal sheet for the upper, internal line of the joint of the metal sheet of cover with the angle, with regard to the longitudinal axis

Recapitulating conducted analyses, analysis of the fall in mass was also performed for individual models. A fall in mass was get for following models about: 55.8 g, 73.8 g, 92.2 g. What at mass of the girder without holes equal of 1176.8 g, constitutes the profit from the 4.7% up to the 7.8%. If it will be taken into account, that mass of cover is only 465.0 g – it is a fall marking it in mass. Presented above results of analyses allow for stating, that lightening holes, applied in thin–walled girders, ribs, and the similar structures, cause an increase in critical force and meaning lowering mass, but with themselves are also bringing danger of the substantial increase of load in connections of web of I–beams with elements of the skeleton – e.g.: of riveted joints.

Moreover, in spite of the load not–exceeding critical, state of the deformation and effort is providing covers about coming into existence of the tension field. It constitutes the cause of the meaning growth of shear forces. Additionally, character of the work of the rivet joint is changing, in which apart from shearing the rivet, perhaps also to appear bending and stretching. This thesis is confirmed by fact that the cover metal sheets are bending at the close neighbourhood of riveted joints. It has a great meaning in introduced models because the rivet joints were replaced with the connection "on stiffly". At a detailed analysis, including the phenomenon of the contact and the discontinuous joint of cover with the framework, perhaps it to turn out that the torsion angle is much bigger and constitutes essential part of the load.

5. Summary

In classic methods of analyses of thin–walled semi–monocoque structures (which is allowed buckling of cover during the normal use), is assumed that the cover deformations forming in surroundings of connections between metal sheets of an web of I–beams with elements of the framework, as a result of the appearance of the tension field, are very small and therefore, their influence on joint effort is being omitted. An also uniform distribution of the load is assumed (caused by folding of cover metal sheets) along the edge of elements of the framework. In the destination of checking the influence of above simplifications on the effort state, the structure was performed sequence of analyses and examinations. Majority of calculations with the application of FEM were made (Finite Element Method).

Analyses were performed on different examples of thin–walled girders. A usefulness of different models was examined in FEM analysis of the structure of this type. On the basis of achieved results we stated, that semi–monocoque model allows for determining general effort of the structure. These results are in accordance with results achieved with classic methods of analysis of thin–walled structures, but practically don't permit the deformation of covers analysis. A very short time of calculations is their virtue. More information was obtained on the basis of monocoque and shell models. The registered time of calculations of models of this type was longer even ten times and depended on the type of analysis. Provided results of linear analyses coincided in the majority with results achieved from the semi–monocoque model, but results of non–linear analyses already delivered other information. Relatively a rather small increase in normal stresses was registered in elements of the framework and very much, up to 100% large, rise of shearing stresses in the cover. Therefore, to the purpose of the evaluation of the work of the structure in the over–critical state, in case of analyses performed in the global scale, applying beam-shell models and non–linear procedures is necessary. It will allow for establishing areas of the structure, the most exposed on buckling of covers.

Analysis of the girder was also performed with lightening holes, of which raising the critical force is a purpose. On the basis of achieved results we stated that appropriately formed holes in cover caused the increase in critical force (in case of analyzed models about 18÷43%) and significant fall in mass, but are bringing danger of the substantial increase of riveted joints load. In examined models an increase in shear stresses was get, towards the structure without holes, about 200÷250%. Appearing of the deformation of the metal sheet of cover, causing coming into existence the torsion angles reaching 0.9° (measured in nodes suiting the edge of the framework in length).

Achieved results are confirming the significant influence of the tension field on the effort state both of metal sheets of web of an I–beam, of elements of the framework – particularly of elements of joints. A need to perform detailed analyses more is inserting it, than it results from the classic approach.

References

[1] **ABAQUS Analysis User's Manual**, ABAQUS, Inc. United States of America, **2004**.

[2] **Brzoska, Z.** Statics and stability of the rod and thin-walled structures, Warsaw, *National Scientific Publishers*, **1961**.

[3] **Brzoska, Z.** Statics and stability of the rod and thin-walled structures, Second Edition – amended, Warsaw, *National Scientific Publishers*, **1965**.

[4] **Hertel, H.** Leichtbau Flugzeuge Und Andere Leichtbauwerke, *Springer–Verlag*, Berlin / Göttingen / Heidelberg, **1960**.

[5] **Jachimowicz, J.** Static analysis of rigidity–strength machine housings, Dissertation, Warsaw University of Technology, **1995**.

[6] **Ko, W.L. and Jackson, R.H.** Shear buckling analysis of a hat–stiffened panel, *Edwards*, Kalifornia: NASA Dryden Flight Research Center, Technical Memorandum, NASA-TM-4644, **1994**.

[7] **Kuhn, P.** Stresses In Aircraft and Shell Structures, McGraw–Hill Book Company Inc., New York / Toronto / Londyn, **1956**.

[8] Payload Flight Equipment Requirements and Guidelines for Safety–Critical Structures, National Aeronautics and Space Administration, International Space Station Program, Johnson Space Center, Houston, Teras, Contract No. NAS9–02099 (PA16), **2002**.

[9] **Simon, H. and Wagner, H.** Uber die Krafteinleitung in Dunnwandige Zylinderschalen, Luftfahrtforschung vol. 13, No. 9, **1936**.

[10] **Teisseyre, J.** Construction of bodies. Part I – calculation, Warsaw, Transport and Communication Publishers, **1965**.

[11] **Tessler, A., Sleight, D.W. and Wang, J.T.** Nonlinear shell modeling of thin membranes with emphasis on structural wrinkling, Hampton: NASA Langley Research Center, Analytical and Computational Methods Branch, Structures and Materials Competency, VA 23681–2199, **2003**.

[12] **Tessler, A., Sleight, D.W. and Wang, J.T.** Nonlinear shell modeling of thin membranes with emphasis on structural wrinkling, Hampton: NASA Langley Research

Center, Analytical and Computational Methods Branch, Structures and Materials Competency, VA 23681–2199, **2003**.

[13] **Wang, J.T., Chen T., Sleight, D.W. and Tessler, A.** Simulating non-linear deformations of solar sail membranes using explicit time integration, Palm Springs, California: NASA Langley Research Center, Hampton, 45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, VA 23681-2199, **2004**.

[14] PN-84/H-93669, Aluminum and aluminum alloys. Profiles.