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Buckling Resistance Assessment of a Slender Cylindrical Shell Axially Compressed

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The paper deals with some considerations focused on resistance assessment of slender cylindrical shells subjected to the axial compression. The load carrying capacity of such shells is determined by stability criterion. It is not enough to determine the critical load in order to assess the load carrying capacity. It is necessary to apply the whole procedure recommended by designing codes and other design recommendations. Details of this procedure were presented in the paper. The correctness of the resistance assessment was verified experimentally on segments of cylindrical shells made of stainless steel.

Keywords: Buckling resistance, cylindrical shell, design recommendations, experimental test, numerical solution, analytical solution

1. Introduction

The buckling problem of axially compressed, slender, elastic cylindrical shell was solved at the beginning of twentieth century (R. Lorentz, 1908 and 1911, S. Timoshenko, 1910, R. V. Southwell, 1913) and was probably the first analytical solution of any shell buckling problem. This classical solution can be traced on the basis of monographs by Timoshenko [1] and by Fluegge [2].

The designer who wants to construct safely a cylindrical shell has to asses its resistance taking into account various criteria. In a case of relatively slender cylindrical shells the buckling criterion is the most decisive as far as the resistance is concerned. The knowledge of the classical buckling solution constitutes very important indication but of course is not sufficient. To design safely a structure in a form of cylindrical shell it is required to take into account design recommendations like Eurocode [3] or European design recommendations (EDR) published by ECCS

[4], in which experiences of many designers and scientists working in this field were converted into recommendations warranting the safe designing of shell structures.

The paper deals with an attempt of resistance assessment of steel cylindrical shells subjected to the axial compression. Shells were fabricated from stainless steel sheets, cold–rolled and welded by the single longitudinal seam.

At the beginning of the paper the calculation of the critical load according to the initial buckling theory was presented. It is the load which evokes the buckling of considered shell. The presented procedure corresponds exactly to the proposal of Fluegge [2] together with a graphical method of determination of critical load on the basis of the so called garland curve. The critical load of considered shell was determined also numerically by means of the COSMOS/M system [5] with the initial buckling option.

It was shown that the presented numerical solution was nearly identical as the classical, analytical solution.

The buckling resistance assessment of the considered shell was made also on the basis of European code EN 1993: Part 1.6 [3] and on the basis of European design recommendations (EDR) [4]. This approach was presented in the paper in details. The resistance determined by this way turned out to be much smaller than determined earlier values of critical loads.

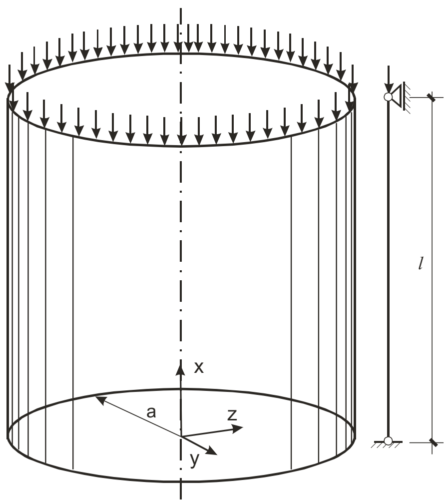

Figure 1 The cylinder under compression

Results of experimental investigations of three cylindrical shells supported consistently with a classical case of Lorentz, Timoshenko and Southwell were presented in the paper as well. The resistance obtained experimentally was much smaller than the value of the critical force for the ideal cylindrical shell. The reason was obvious: inevitable geometrical imperfection were present in examined shells. It is worth mentioning that the resistance prediction which followed from codes [3] and [4] was always smaller than resistances obtained experimentally.

2. Stability of axially compressed cylindrical shell. Analytical solution

The presented below algorithm, based on analytical solution, was taken from the monograph of Fluegge [2].

The problem of initial stability of cylindrical shell compressed in longitudinal direction can be reduced to the following relationship (Eq. (7–13) from [2]):

$$q_2 = \{(1-\nu^2)\lambda^4 + k[(\lambda^2+m^2)^4 - 2(\nu\lambda^6 + 3\lambda^4 m^2 + (4-\nu)\lambda^2 m^4 + m^6) \\ +2(2-\nu)\lambda^2 m^2 + m^4]\} / \left[\lambda^2(\lambda^2+m^2)^2 + \lambda^2 m^2\right] \tag{1}$$

in which the following notations were used:

$$q_2 = \frac{P}{D} \qquad k = \frac{t^2}{12a^2} \qquad D = \frac{Et}{1-\nu^2} \qquad \lambda = \frac{n\pi a}{l} \tag{2}$$

where:

t – the shell thickness,
l – the length of cylindrical shell,
a – the radius of cylindrical shell,
n – the number of half–waves in longitudinal direction,
m – the number of full waves in circumferential direction,
P – the distributed load acting in longitudinal direction,
E – the Young's modulus,
ν – the Poissons's ratio.

For the defined shell geometry (l, a and t are known) and material parameters (E and ν are known) the value of q_2 depends on pair of two integer numbers m i n. Characteristics shown in Fig. 2 refer to the number m taken from the interval 0–15 and were obtained for the following data:

E = 193 GPa,
ν = 0.3,
l = 400 mm,
a = 200 mm (comp. Fig. 1).

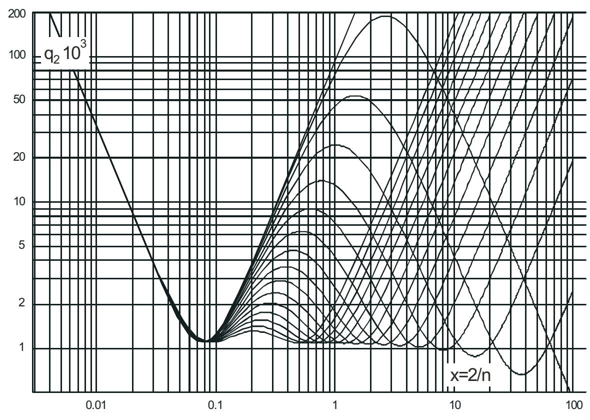

Figure 2 The collection of solutions for various m as functions of x

To determine the critical value of the load one should find such a pair of integer numbers m i n, for which the value of q_2 attains the minimum. The minimum value of q_2 obtained in this manner is the critical load which was looked for and the pair of integer numbers m and n determines the buckling mode corresponding to the primary bifurcation point. The procedure leading to determination of the lowest value of q_2 can be performed graphically and it is the easiest approach.

Let us introduce the auxiliary variable

$$x = \frac{l}{na} \quad \text{hence} \quad \lambda = \frac{\pi}{x} \tag{3}$$

For the presented above data the variable

$$x = \frac{l}{na} = \frac{400}{n200} = \frac{2}{n}$$

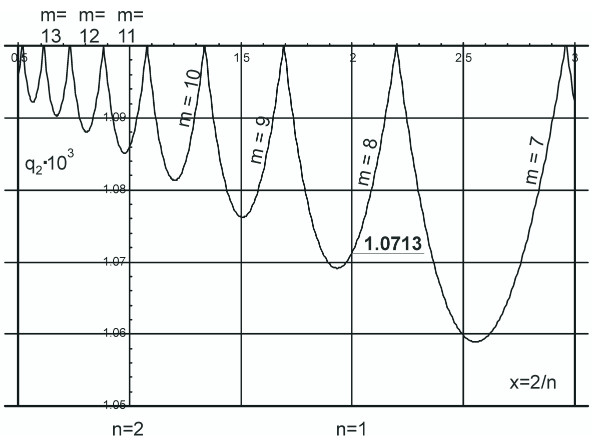

Figure 3 The detail from the Fig. 2

In this particular case $q_2^{cr} = 1,0713 \cdot 10^{-3}$ (comp. Fig. 3) and the buckling form is defined by the one half–wave in longitudinal direction and eight waves in circumferential direction. This buckling form was presented in Fig. 4 in which only one half of the cylinder was presented (the symmetry plane is located on the lower edge).

The critical value of the distributed load acting on the edge of the cylinder will be calculated from the relationship:

$$P_{cr} = q_2^{cr} D = 1,0713 \cdot 10^{-3} \frac{193 \cdot 10^9 \cdot 0,0004}{1 - 0,3^2} = 90,84 \cdot 10^3 \mathrm{N/m} \tag{4}$$

and the meridian critical stress can be calculated from the formula:

$$\sigma_{cr} = \frac{P_{cr}}{t} = \frac{90,84 \cdot 10^3}{0,0004} = 227,21 \cdot 10^6 \mathrm{N/m}^2 = 227,21 \mathrm{MPa} \tag{5}$$

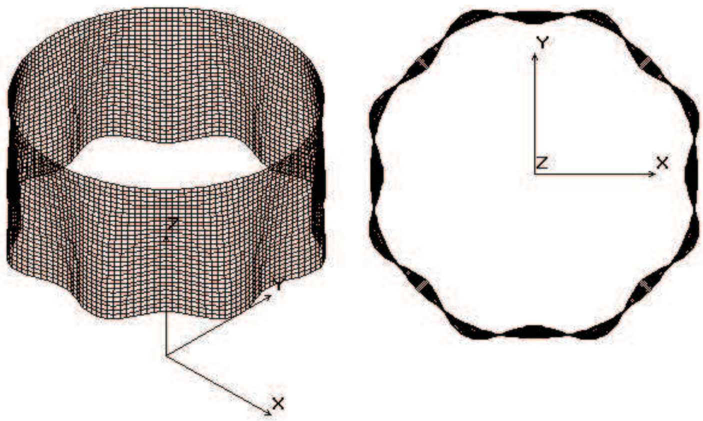

Figure 4 The first buckling mode: $m = 8$, $n = 1$

The well known from the literature (cf. [1], [6]), approximate formula on critical value of longitudinal stress leads to the result:

$$\sigma_{cr} = 0,605E\frac{t}{a} = 0,605 \cdot 193 \cdot 10^9 \cdot \frac{0,4}{200} = 233,53 \cdot 10^6\text{N/m}^2 = 233,53\text{MPa} \quad (6)$$

It is the value only 2,6% higher than the accurate value defined in Eq. (5).

The initial buckling problem of the considered shell was solved also numerically by means of the COSMOS/M [5] system which is based on finite element method. Due to symmetry only one half of the cylinder was modeled. Appropriate boundary conditions were adopted on the symmetry plane and on the upper edge on which the external load was applied.

Calculated values of critical stresses and corresponding buckling modes are presented in the Tab. 1.

Table 1 Critical stresses and buckling modes

No. of the mode	1	2	3	4	5
σ_{cr}[MPa]	228,6	232,3	233,7	235,0	236,5
m, n	8, 1	13, 3	16, 5	18, 7	19, 7

The consistency with the analytical solution in respect to critical pressure value and the buckling mode is pretty good.

3. Checking of the buckling limit state of cylindrical shells according to design recommendations

The presented below procedure is consistent with the clause 8.5 of the code EN 1993–1–6:2007 [3] and the chapter 10 of European design recommendations [4].

At the first step of this procedure the dimensionless length parameter ω is calculated:

$$\omega = \frac{l}{\sqrt{rt}} = \frac{400}{\sqrt{200 \cdot 0,4}} = 44,72 > 1,7 \tag{7}$$

$$0,5\frac{r}{t} = 0,5\frac{200}{0,4} = 250 \tag{8}$$

Because ω lays in the interval $1.7 < \omega < 250$, it means that the considered shell is the cylinder of intermediate length. Hence $C_x = 1.0$.

The elastic critical meridional buckling stress will be calculated from the formula (cf. Eq. (6)):

$$\sigma_{x,Rcr} = 0,605EC_x\frac{t}{r} = 0,605 \cdot 193000 \cdot 1,0 \cdot \frac{0,4}{200} = 233,53\text{MPa} \tag{9}$$

The relative shell slenderness parameter in longitudinal direction is expressed by the formula:

$$\lambda_x = \sqrt{\frac{f_{y,k}}{\sigma_{x,Rcr}}} = \sqrt{\frac{241}{233,53}} = 1,016 \tag{10}$$

where $f_{y,k}$ is the yield stress of the applied steel.

Let us adopt the fabrication tolerance quality class C (normal quality). Hence, the fabrication tolerance quality parameter $Q = 16$.

We can calculate now the elastic imperfection reduction factor:

$$\alpha_x = \frac{0,62}{1 + 1,91\left(\frac{1}{Q}\sqrt{\frac{r}{t}}\right)^{1,44}} = \frac{0.62}{1 + 1,91\left(\frac{1}{16}\sqrt{\frac{200}{0,4}}\right)^{1,44}} = 0,151 \tag{11}$$

Let us adopt recommended values of the squash limit relative slenderness λ_{xo}, the plastic range factor β and the interaction exponent η: $\lambda_{x0} = 0,2$, $\beta = 0,6$, $\eta = 1,0$.

Hence, the plastic limit relative slenderness $\lambda_{x,p} = \sqrt{\frac{\alpha_x}{1-\beta}} = \sqrt{\frac{0,151}{1-0,6}} = 0,615$. The case $\lambda_x > \lambda_{x,p}$ takes place, and hence, the buckling reduction factor

$$\chi_x = \frac{\alpha_x}{\lambda_x^2} = \frac{0,151}{1,016^2} = 0,147 \tag{12}$$

The characteristic value of critical stresses we will obtain from the formula:

$$\sigma_{x,Rk} = \chi_x f_{y,k} = 0.147 \cdot 241 = 35,38\text{MPa} \tag{13}$$

and the design value from the relationship

$$\sigma_{x,Rd} = \frac{\sigma_{x,Rk}}{\gamma_{M1}} = \frac{35,38}{1,1} = 32,16\text{MPa} \tag{14}$$

It is the value sevenfold smaller than the critical value which follows from the solution of initial buckling problem. This significant reduction is result of high imperfection sensitivity of longitudinally compressed cylinders. The presented here design procedure of course takes this fact into account.

4. Experimental investigations of resistance of compressed cylindrical shells

The test rig, on which the compression of steel cylindrical shells were investigated, was shown in the Fig. 5. The compressive load was transmitted by the rigid plate and the steel hinge attached to the upper traverse of hydraulic strength machine. Such a loading method guarantees the uniform distribution of the load on the whole edge at least at the initial state of the loading process.

The shell was very flexible. To preserve the ideal shape of cross-section, on both edges of cylinders addition internal wheels made of thin laminated plastic were placed inside cylinders and attached to its walls. External diameters of these wheels were equal to internal diameters of examined steel cylinders.

Three cylinders of the same dimensions but different inevitable and unknown geometrical imperfections were tested. Maximum compressing loads and corresponding critical stresses were presented in the Table 2. The exemplary plot the load versus displacements of the upper edge of the cylinder relationship was shown in Fig. 6. The sudden drop of load was observed at the instant of buckling of the upper part of the shell. The deformation mode at the stage of the total resistance exhaustion is well visible in the Fig. 5.

Table 2 Critical forces and corresponding critical stresses

	Cylinder 1	Cylinder 2	Cylinder 3
F_{cr} [kN]	15,5	32,1	27,0
σ_{cr} [MPa]	61,7	127,4	107,5

Figure 5 The test rig. The collapse of the tested cylinder

5. Final remarks

The buckling criterion is the most decisive as far as the resistance of a compressed, flexible, slender cylindrical shell is concerned. The critical load obtained as a result of solution of initial buckling problem can not be the basis of buckling resistance assessment of the shell. The critical load obtained in such a way should be significantly reduced due to presence of unavoidable geometrical imperfections. The manner in which this reduction should be accomplished is presented in code [3] and in design recommendations [4] published in 2008 by ECCS (European Convention for Constructional Steelwork).

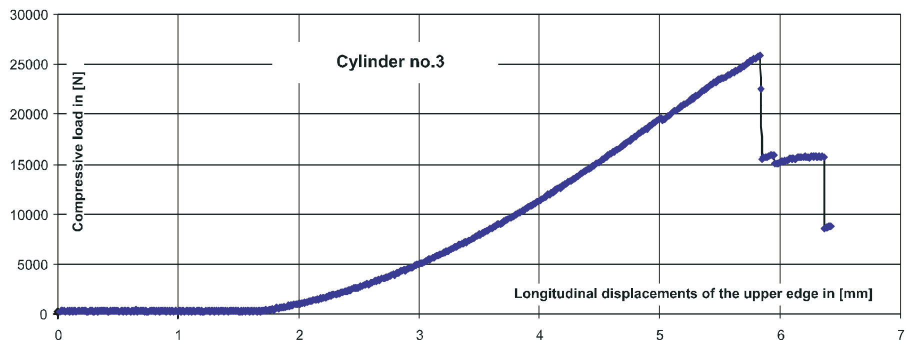

Figure 6 The compressive load versus the upper edge displacement for the cylinder no. 3

The accurateness and the engineering safety of this approach was confirmed by tests which were performed on slender cylinders made of stainless steel. In all performed tests the registered resistance was higher than the predictions resulting from codes and design recommendations. It means that the procedure of resistance assessment recommended in [4] works correctly and should be used by designers of tanks and silos.

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