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Application of Different Dynamic Stability Criteria in Case of Columns with Intermediate Stiffeners

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In this paper, the dynamic responses of the open cross section columns with intermediate stiffeners subjected to in–plane pulse loading of a rectangular shape were concerned. Columns made of isotropic material were assumed to be simply supported at both loaded ends. The problem of the interactive buckling was solved by finite element method (FEM) – ANSYS 9. The critical dynamic load factors DLF has been determined using the most popular Budiansky–Hutchinson's criterion and they have been verified with author's versions of the phase portraits criterion.

Keywords: Stiffener, stability criterion, thin–walled structures, buckling, pulse loading, FEM

1. Introduction

The restoration of the economics and its rapid development after ending II World War took place. New industry showed growing interest in light weight structural elements. The interest has been driven for enhanced structural efficiency and reduced material use. In the event of such structures the most important problem is buckling. Thin–walled structures may have many buckling modes differ from one another both in quantitative and in qualitative (e.g. by global and local buckling) respects. The different buckling modes may be interrelated. Such special case of buckling is so–called interactive or coupled buckling. The problem of the interaction of the global mode with the local ones is of great significance. The numerical calculations carried out by Kolakowski [1–2] have proven that the interaction of local modes having considerably different wavelengths is either very weak or does not occur at all. Moreover, one can see that the interaction of two global modes of buckling is very weak or even does not occur at all. In the case when the post–buckling behaviour of each mode taken separately is stable, their interaction may result in an unstable behaviour. A more comprehensive review of the literature concerning the interactive buckling analysis of an isotropic structure can be found in [3–18].

The intermediate and edge stiffeners can improve the local buckling resistance of a structure. Turning the free edge of an unstiffened web inwards (Fig. 1b) and outwards (Fig. 1a) forms a 'lip'. The lip is the most common type of edge stiffeners used in open thin–walled sections. The intermediate stiffener is a U-bent or V–bent which subdivides the plate element into smaller sub–elements. The shape, size and position of intermediate stiffeners and edge stiffeners in thin-walled structures exert a strong influence on the stability and the postbuckling behaviour of thin–walled structures.

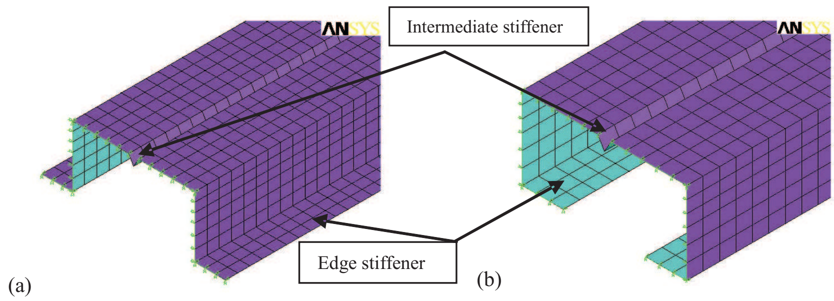

Figure 1 The MES models of channel thin–walled columns with stiffeners: a) outwards (denoted OL), b) inwards (denoted IL)

The advances in computing technology have enabled analysis of nonlinear solutions of thin–walled members under static or dynamic compression. The applications of finite element method (FEM) to solve interaction buckling problems are on–going. FEM proved to be the most successful numerical method to analyse static or dynamic buckling of complex thin–walled structures. Such analyses involve a discretisation of the structure and nonlinear solutions of large system of equations. This method is very important in practice, because it is becoming standard practice to use FEM in conjunction with experiments in improving and preparing new engineering standards for thin–walled structures. Early 1970's papers [19–20] described elastic global buckling using beam elements. The bifurcation analysis is described in [21–22]. The FEM models using plate or shell elements permit analyses coupling between local and global bucking in the elastic range [23]. There is no distinction between the analyses of plate and shell structures. In both cases one can use shell elements in geometric and material nonlinear analyses [24–28].

The dynamic coupled buckling of complex thin–walled structures has not been sufficiently investigated. There are known solutions of dynamic buckling problem for columns [29–38]. The investigations should be continued. The dynamic buckling takes place when the duration of the compressive pulse is less than two times of period of fundamental natural flexural vibrations of the structure. The assumption was made that dumping have been neglected in the dynamic study. When the amplitude of pulse load is low, the thin-walled structure vibrates around the static equilibrium position. The behavior of structure is stable. Otherwise the structure can vibrate very strongly or can move divergently.

The dynamic stability loss of plated structures may occur only for structures with initial geometric imperfections. Therefore the dynamic bifurcation load does not exist and the dynamic buckling load should be defined on the basis of the assumed buckling criterion. Simitses [39] voiced a view that there are not precise buckling criteria for structures with stabile post–buckling path. For the plate structures without imperfection the critical buckling amplitude tends to infinity.

In literature one can found a lot of stability criteria. The most popular one was formulated by Budiansky and Hutchinson [29–30]. The next one is the phase portrait criterion [29, 38, 41]. The failure criteria [40] or quasi-bifurcation criterion [34–36, 40] will not be discussed in this paper.

The purpose of this paper is to determine the critical value of dynamic loads using FEM and to compare Budiansky-Hutchinson's criterion and author's versions of the phase portraits criterion.

2. Formulation of the problem

In this study, there is presented an analysis of the dynamic coupled buckling of long, prismatic, isotropic, thin–walled columns of open cross–section with V-intermediate stiffener under axial pulse compression (Figs. 1). In the numerical calculations, a rectangular shape of in–plane pulse $\sigma(t)$ loading with the duration t_o is considered.

$$\sigma(t) = \begin{cases} \sigma_0 & \text{for} \quad 0 \le t \le t_0 \\ 0 & \text{for} \quad t > t_0 \end{cases} \tag{1}$$

The dimensionless dynamic critical load factor DLF using different criteria was determined. The DLF's factor has been defined as the quotient of the dynamic loading σ_o (1) and minimal static critical stress σ_{min}:

$$\mathrm{DLF} = \sigma_o / \sigma_{\min} \tag{2}$$

In the world literature, one can find many criteria allowing to determine the critical value of DLF's factor. The most popular one is Budiansky–Hutchinson's criterion that states: "*The dynamic stability loss occurs when the velocity with which displacements grow is the highest for certain force amplitude*". The results have been verified using the phase portraits criterion with author's modifications. This criterion was formulated as follows: *the dynamic buckling load for the time interval* $(0,\ 1.5\,t_0)$ *has been defined as the minimum value of the pulse load such that phase portrait is an open curve.*

Numerical calculations are using commercial software based on finite element method – ANSYS 9. The structures have been treated as simply supported at the ends of columns. The four–node shell element (SHELL 43) of six degrees of freedom has been chosen. In the FEM model only the boundary conditions corresponding to the flexural global buckling mode and the local buckling were modeled. So, the global torsional–flexural buckling mode was neglected.

In the first step the critical stresses for the global flexural buckling mode and the local modes and the natural frequencies (without a compressive force) have been calculated. Lanczos eigenvalue extraction method has been used [42]. The imperfections have been neglected in this step.

In the next step the critical values of the dynamic load factor DLF for the coupled buckling analysis of the structures with imperfections were determined. The amplitude of the initial deflection corresponding to the i–th buckling mode are equaled to ζ_{oi} where the indices take the following values: global buckling mode (i =1), the minimum local one (i =2). The amplitudes of the pulse force have been applied as multiples of the lowest static buckling load. The structural dynamic analysis using FEM, which has allowed us to find the response of a structure to the pulse loading, has been conducted using the "Full Transient Dynamic Analysis". Newmark's formula for the time integration has been used. The Newton–Raphson scheme was used to solve the nonlinear equations of motion [42–43].

3. Analysis of the results

The detailed numerical calculations have been carried out for the thin–walled channel column OL and IL (Fig. 1) subject to uniform pulse compression. The geometrical dimensions of the cross–section of the column are given in Fig. 2: b_s =4 mm, b_1 = 50 mm, b_2 =25 mm, b_3 =12.5 mm, $h_1 = h_2 = h_3$ =1 mm. The length L of structures equals 650 millimeters. The material is subject to Hooke's law. Material properties: Young's modulus $E = 2 \cdot 10^5$ MPa, Poisson's ratio $\nu = 0.3$ and density $\rho = 7850$ kg/m^3. The following values of the amplitudes imperfections: $\zeta_{o1} = h_1$, $\zeta_{o2} = 0.2\,h_1$.

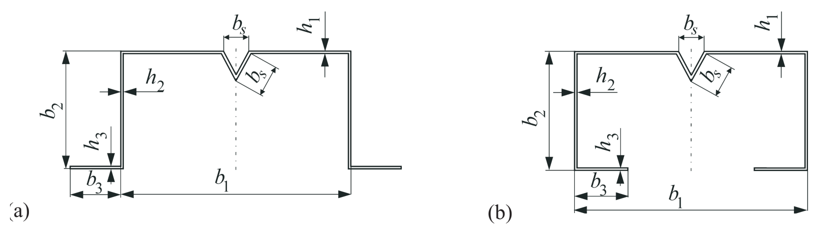

Figure 2 Dimension of the cross–section of the thin–walled columns with stiffeners: a) channel column OL; b) channel column IL

The values of the critical stress σ_2^* and the natural frequencies σ_1^* of the free vibrations for global buckling mode and minimum local ones are presented in Tab. 1 (in brackets the corresponding numbers of half–waves, m, are given). The open columns reinforced with V–stiffeners show two local minima for two different local buckling modes. The first minimum refers to the smaller number of half–waves m=9 and the second one to the greater number of half–waves m=21. Fig. 3 show buckling modes for channel column OL. In the case of channel column IL the buckling modes are identical.

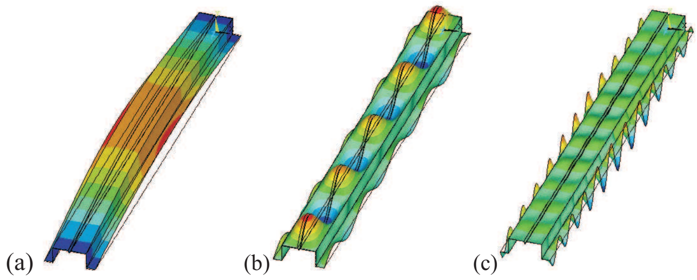

Figure 3 Buckling modes for the channel column OL: a) global one (m=1); b) fundamental local one for m=9; c) fundamental local one for m=21

Table 1 Values of the critical stress σ_2^* and the natural frequencies ω_0 of free vibrations

Results for channel column OL presented in Fig. 2a		Results for channel column IL presented in Fig. 2b	
σ_2^* [MPa]	ω_o[rad/s]	σ_2^* [MPa]	ω_o [rad/s]
497.8(1)	1 217.2(1)	455.6(1)	1 168.6(1)
878.1(9)	14 413.6(9)	872.3(9)	14 438.8(9)
980.2(21)	34 475.8(21)	978.7(21)	34 488.4(21)

The critical values of the dynamic load factor DLF for the coupled buckling analysis of the structures with imperfections were determined. The pulse duration equals $t_o = \alpha T_1$, proportional to the period of the global flexural mode $m = 1$, where: $T_1 = 5.2$ ms for the column presented in Fig. 2a and $T_1 = 5.4$ ms – Fig. 2b. So, it is the interaction between dynamic global flexural buckling mode and quasi-static local buckling ones. Number α is equal to 1.0 or 0.5. The minimal static critical stress σ_{min} corresponding to the global flexural buckling mode (m =1, Table 1) equals 497.8 MPa for the column presented in Fig. 2a and 455.6 MPa – Fig. 2b. The dimensionless amplitude of imperfections equals 1.0 for global flexural mode (m =1) and 0.2 for minimal local ones ($m \neq 1$).

The dynamic critical load factor DLF$_{cr}$ was determined using the Budiansky and Hutchinson criterion. The dynamic stability loss occurs when the maximum plate deflection grows rapidly at a small variation in the load amplitude. To determine the dynamic critical load factor DLF$_{cr}$, one has to plot the maximum dynamic deflections ζ_{max} versus the DLF (Fig. 6). In order to plot the maximum dynamic deflections ζ_{max} versus the DLF, one has to prepare the time function of dynamic dimensionless deflections ζ of selected points vs. the dynamic load factor DLF (Figs 4–5).

According to the Budiansky–Hutchinson dynamic stability criterion (denoted B–H) for coupled dynamic buckling, the value of the critical dynamic load factor (DLF$_{cr}$) equals 0.9 (Fig. 6a) for channel column OL and 1.1 (Fig. 6b) for channel column IL ($t_o = T_1$). There was made an assumption that dimensionless computation time equals $\tau = t/T_1 = 1.5\alpha$. Analogously, the critical values of DLF have been defined for $t_o = 0.5T_1$. The results have been put together in Tab. 2.

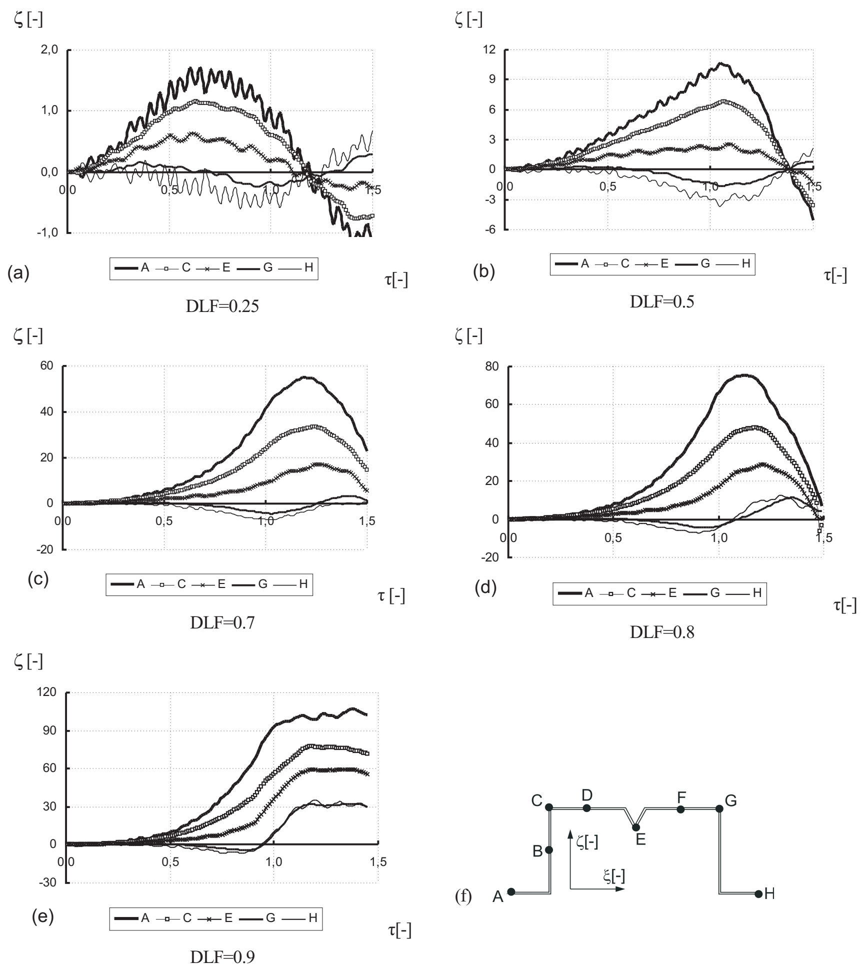

Figure 4 Time function of dynamic dimensionless deflections of selected points vs. the dynamic load factor DLF ($t_o = T_1$ t.e. α=1) for channel column OL

The critical values of the dynamic load factor DLF for the coupled buckling analysis of the structures with imperfections were determined using the phase portraits criterion (denoted P–P), when computation time is always equal to $t = 1.5T_1$ for all values of number α. The phase portraits of the dynamic response for the open columns are shown in Fig. 7. The duration of pulse load is equal to the period of the global flexural free vibrations multiplied by factor α ($t_o = \alpha T_1$). The following notations are applied in the phase portrait: ζ denotes the dynamic dimensionless deflections of selected point and $(\dot{\zeta})_{\max}$ its velocity, $V_o = h/T_1$ – comparative velocity, h=1 mm thickness of the wall of the column.

Table 2 Values of critical dynamic load factor DLF_{cr}

Mode interaction m[-]	Criterion B-H	Criterion P-P	Number α [-]
Results for columns presented in Fig. 2a			
1; 9 or 1; 21	0.9	1.0	1
	1.7	1.8	0.5
Results for columns presented in Fig. 2b			
1; 9 or 1; 21	1.1	1.0	1
	1.9	2.0	0.5

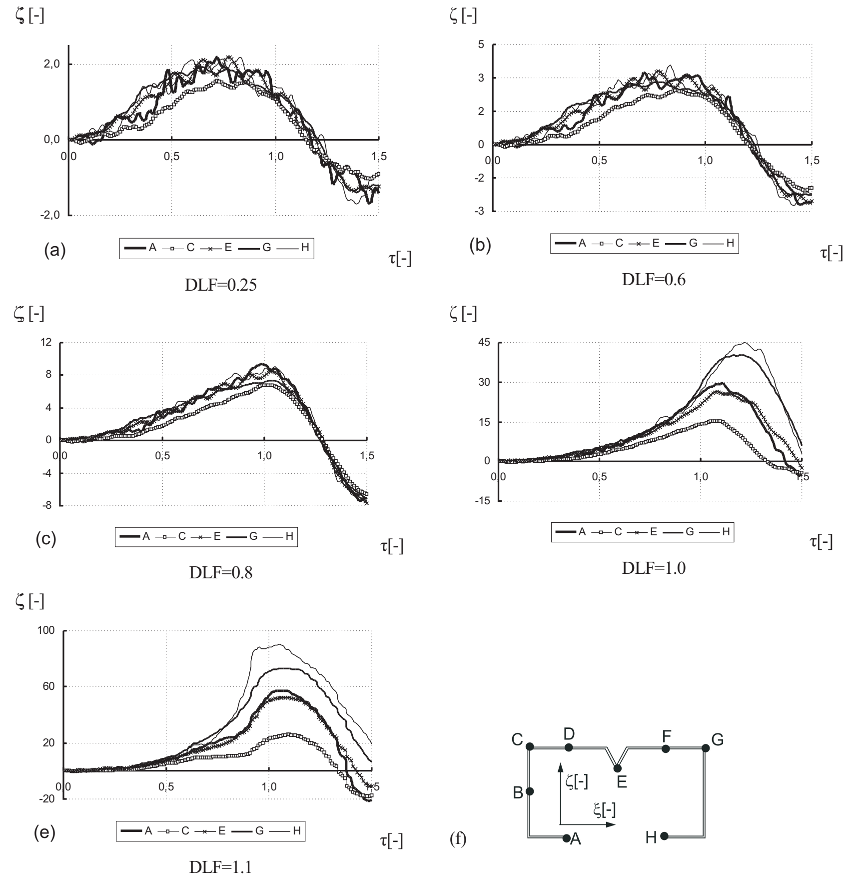

Figure 5 Time function of dynamic dimensionless deflections of selected points vs. the dynamic load factor DLF ($t_o = T_1$ t.e. α=1) for channel column IL

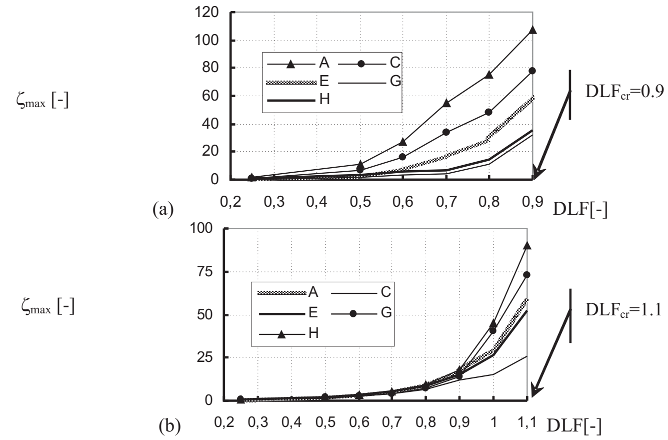

Figure 6 Peak–dynamic dimensionless deflections of selected points vs. the dynamic load factor DLF (t_o=T_1) for: a) channel column OL; b) channel column IL

If a value of DLF factor is less than critical one, the trajectory is a closed curve indicating periodic oscillations. Otherwise, at the DLF factor equal to critical one, the curve is no longer closed and the response is unbounded. According to the new phase portrait stability criterion, the value of the critical dynamic load factor (DLF_{cr}) can be found. Tab. 2 include the results calculated using the phase portrait criterion. All values of critical dynamic load factor DLF_{cr} are similar.

Fig. 8 shows total, dynamic deflections for the open columns with stiffeners versus time. When the pulse amplitude is nearly equal to its critical value, the structure vibrates very strongly. In the initial phase, deformations of the column correspond to the initial imperfections. Then the global mode grows violently. Finally columns are destroyed. The form of initial imperfections is unimportant.

4. Conclusions

The analysis of dynamic stability with the finite element method (FEM) renders the modal analysis of coupled dynamic buckling of thin–walled structures possible. It can be helpful in the analysis of the influence of various parameters of structures. It has been possible to verify the results obtained in the FEM by the analytical–numerical method e.g. [34-36, 38].

The results obtained by phase portraits criterion have been verified using the Budiansky–Hutchinson criterion. The results obtained from both criteria are similar. The greatest advantage of using the phase portraits criterion in relation to Budiansky–Hutchinson criterion is that one can analyse displacements as well as velocity variation.

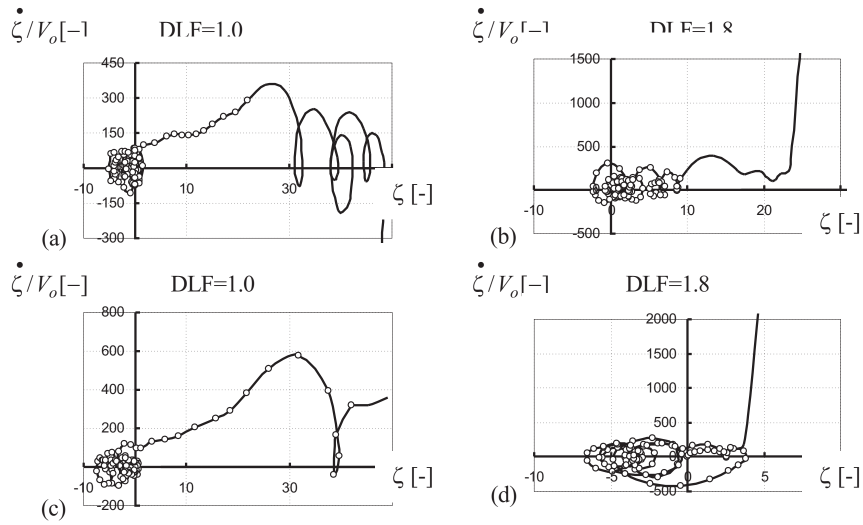

Figure 7 Typical phase portraits of dynamic response for open columns presented in: a) Fig. 2a for α=1; b) Fig. 2a for α=0.5; c) Fig. 2b for α=1; d) Fig. 2a for α=0.5

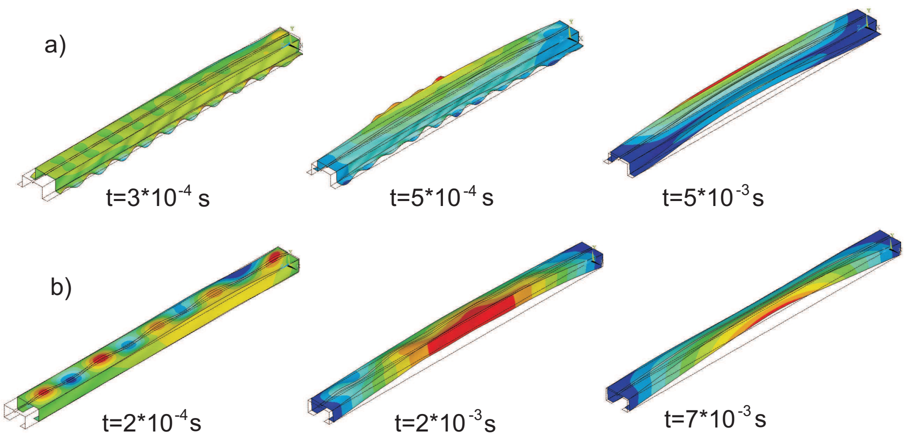

Figure 8 Comparison of dynamic deflections for the open columns versus time. The pulse amplitude is near equal to its critical value: a) channel column OL; b) channel column IL

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