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Reflection of Magneto–thermo–elastic Waves from a Rotating Elastic Half–Space in Generalized Thermoelasticity under Three Theories

Mohamed I. A. Othman
Faculty of Education, Department of Mathematics,
Salalah-211, P.O. Box 2801, Sultanate of Oman
e-mail:m_i_othman@yahoo.com

Yaqin Song
Department of Engineering Mechanics,
Xi' an Jiaotong University, Xi'an 710049,
People's Republic of China.
e-mail:yqsong@mail.xjtu.edu.cn

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The model of the equations of generalized magneto–thermoelasticity based on Lord–Shulman theory (LS) with one relaxation time, Green–Lindsay theory (GL) with two relaxation times, as well as the classical dynamical coupled theory (CD), is used to study the electro–magneto–thermoelastic interactions in a semi–infinite perfectly conducting solid. The entire elastic medium is rotating with a uniform angular velocity. There an initial magnetic field acts parallel to the plane boundary of the half–space. Reflection of magneto–thermoelastic waves under generalized thermoelasticity theory is employed to study the reflection of plane harmonic waves from a semi–infinite rotating elastic solid in a vacuum. The expressions for the reflection coefficients, which are the relations of the amplitudes of the reflected waves to the amplitude of the incident waves, are obtained. Similarly, the reflection coefficients ratios variation with the angle of incident under different conditions are shown graphically. Comparisons are made with the results predicted by the three theories in the presence and absence of rotation.

Keywords: Reflection, thermal relaxation times, generalized thermo–elasticity theories, electro–magneto–thermoelastic couple

1. Introduction

There are two generalization of the classical theory of thermoelasticity. The first generalization is proposed by Lord-Shulman [1] and is known as LS theory which involves one relaxation time for a thermoelastic process. The second generalization is due to Green and Lindsay [2] and is known as GL theory that takes into account two parameters in relaxation times. The governing equations for displacement and temperature fields in the linear dynamical theory of classical thermoelasticity consist of the coupled partial differential equation of motion and Fourier's law of heat conduction equation. The equation for displacement field is governed by a hyperbolic wave equation, whereas, that of the temperature field is a parabolic diffusion type equation. However, the classical thermoelasticity predicts a finite speed for predominantly elastic disturbances, but an infinite speed for predominantly thermal disturbances that are coupled together. In view of LS theory [1], a part of every solution of the equations extends to infinity. In view of the mathematical difficulty involved in the coupled equations of thermo–elasticity, several authors including Roy Choudhuri and Debnath [3], Roy Choudhuri [4], Chandrasekharaiah and Debnath [5] and Mukhopadhyay and Bera [6] have considered only one–dimensional problems.

To two–dimensional multi–field coupled generalized heat conduction problems, owing to the mathematical difficulties encountered in these coupled problem, the problems becomes too complicated to approach analytically. Numerical techniques, instead of analytical methods, have to be resorted to solving this kind of problems. Based on the theories of generalized thermoelasticity, In [7] a two–dimensional electro–magneto–thermoelastic problem was dealt with for a finitely conducting half–space by Laplace and Fourier transforms. In [8] and [9] the normal mode analysis was applied to a two–dimensional electro–magneto–thermoelastic plane waves problem of a medium of perfect conductivity. Othman [10-12] used the normal mode analysis to study the effect of rotation on plane waves in generalized thermo–elasticity with one and two relaxation times.

The theory of magneto-thermoelasticity is concerned with the interacting effects of applied magnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in many industrial appliances, particularly in nuclear devices, where there exists a primary magnetic field, various investigation are to be carried out by considering the interaction between magnetic, thermal and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geomagnetic studies.

In this paper, the generalized thermoelastic theory is applied to study the reflection of plane wave under a constant magnetic field for a thermally and electrically conducting half-space elastic media nearby a vacuum. The reflection coefficient ratios of various reflected waves with the angle of incidence have been obtained for dynamical coupling theory, LS theory and GL theory. Also the effects of applied magnetic field and thermal coupling are discussed numerically and illustrated graphically.

2. Formulation of the Problem and Basic Equations

We consider the problem of a thermo–elastic half–space ($z \geq 0$). A magnetic field with constant intensity $\mathbf{H} = (\mathbf{0}, \mathbf{H_o}, \mathbf{0})$ acts parallel to the bounding plane (take as the direction of the y–axis). Thus, all quantities considered will be functions of the time variable t and of the coordinates x and z. The elastic medium is rotating uniformly with an angular velocity $\mathbf{\Omega} = \Omega\, \mathbf{n}$, where $\mathbf{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame of reference has two additional terms [4]: Centripetal acceleration,

$\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})$ due to time-varying motion only and the Corioli's acceleration $2\mathbf{\Omega} \times \dot{\mathbf{u}}$ where $\mathbf{u}$ is the dynamic displacement vector. These terms don't appear in non-rotating media.

Due to the application of initial magnetic field $\mathbf{H}$, there results an induced magnetic field $\mathbf{h}$ and an induced electric field $\mathbf{E}$. The simplified linear equations of electrodynamics of slowly moving medium for a homogeneous, thermally and electrically conducting elastic solid are,

$$\operatorname{curl} \mathbf{h} = \mathbf{J} + \varepsilon_o \dot{\mathbf{E}} \tag{1}$$

$$\operatorname{curl} \mathbf{E} = -\mu_o \dot{\mathbf{h}} \tag{2}$$

$$\operatorname{div} \mathbf{h} = 0 \tag{3}$$

$$\mathbf{E} = -\mu_o (\dot{\mathbf{u}} \times \mathbf{H}) \tag{4}$$

where $\dot{\mathbf{u}}$ is the particle velocity of the medium, and the small effect of temperature gradient on $\mathbf{J}$ is also ignored. The dynamic displacement vector is actually measured from a steady–state deformed position and the deformation is supposed to be small.

The displacement equation of motion in a rotating frame of reference is

$$\begin{aligned}\rho\left[\ddot{\mathbf{u}} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}) + 2\mathbf{\Omega} \times \dot{\mathbf{u}}\right] &= (\lambda + \mu)\nabla(\nabla\cdot\mathbf{u}) + \mu\nabla^2\mathbf{u} + \mu_o(\mathbf{J} \times \mathbf{H}) \\ &\quad -\gamma\left(1 + \nu_o \frac{\partial}{\partial t}\right)\nabla T\end{aligned} \tag{5}$$

In the absence of the body force and inner heat source, the generalized electro–magneto–thermoelastic governing differential equations in the context of three different theories are

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e \delta_{ij} - \gamma(T - T_o + \nu_o \dot{T})\delta_{ij} \tag{6}$$

the heat conduction equation

$$kT_{,ii} = \rho C_E \left(1 + \tau_o \frac{\partial}{\partial t}\right)\dot{T} + \gamma T_o \left(1 + \delta \tau_o \frac{\partial}{\partial t}\right)\dot{u}_{i,i}. \tag{7}$$

and strain–displacement relations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{8}$$

In the above equations, a comma followed by a suffix denotes material derivative and a superposed dot denotes the derivative with respect to time, $i, j = x, z$. Moreover, the use of the relaxation times ν_o, τ_o and the parameters δ make the aforementioned fundamental equations possible for the three different theories:

1. The equations of the coupled thermoelasticity, when:

$$\nu_o = \tau_o = 0, \quad \delta = 0 \tag{9}$$

Eqs (5) and (7) has the form

$$\begin{aligned}\rho\left[\ddot{\mathbf{u}} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u}) + 2\mathbf{\Omega} \times \dot{\mathbf{u}}\right] &= (\lambda + \mu)\nabla(\nabla\cdot\mathbf{u}) + \mu\nabla^2\mathbf{u} \\ &\quad + \mu_o(\mathbf{J} \times \mathbf{H}) - \gamma\nabla T\end{aligned} \tag{10}$$

$$kT_{,ii} = \rho C_E \dot{T} + \gamma T_o \dot{u}_{i,i} \tag{11}$$

2. Lord and Shulman's theory [1], when:

$$\nu_o = 0\,, \quad \delta = 1\,, \quad \tau_o > 0\,. \tag{12}$$

where τ_o is the relaxation time and Eq. (5) is the same as Eq. (10) and (7) has the form:

$$kT_{,ii} = \rho C_E \left(1 + \tau_o \frac{\partial}{\partial t}\right) \dot{T} + \gamma T_o \left(1 + \tau_o \frac{\partial}{\partial t}\right) \dot{u}_{i,i} \tag{13}$$

3. Green and Lindsay's theory [2], when:

$$\delta = 0, \quad \nu_o \geq \tau_o > 0. \tag{14}$$

where ν_o, τ_o are the two relaxation times, Eq. (5) remains without change and Eq. (7) has the form:

$$kT_{,ii} = \rho C_E \left(1 + \tau_o \frac{\partial}{\partial t}\right) \dot{T} + \gamma T_o \dot{u}_{i,i} \tag{15}$$

4. The correspondent equations for the generalized thermoelasticity without rotation results from the above mentioned cases by taking $H_o = \Omega = 0$. The displacement components have the following form

$$u_x = u(x, z, t)\,, \quad u_y = 0\,, \quad u_z = w(x, z, t) \tag{16}$$

From Eqs (8) and (16), we obtain the strain components

$$e_{xx} = \frac{\partial u}{\partial x}\,, \quad e_{yy} = 0\,, \quad e_{zz} = \frac{\partial w}{\partial z}, \quad e_{xy} = e_{yz} = e_{yy} = 0.$$

$$e_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \quad e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = u_{i,i} \tag{17}$$

From Eqs (6) and (17), the stress components are given by

$$\sigma_{xx} = (\lambda + 2\mu) u_{,x} + \lambda w_{,z} - \gamma (T - T_o + \nu_o \dot{T}) \tag{18}$$

$$\sigma_{zz} = (\lambda + 2\mu) w_{,z} + \lambda u_{,x} - \gamma (T - T_o + \nu_o \dot{T}) \tag{19}$$

$$\sigma_{xy} = \mu (u_{,z} + w_{,x}) \tag{20}$$

The components of the magnetic intensity vector in the medium are

$$H_x = 0\,, \quad H_y = H_o + h(x, z, t)\,, \quad H_z = 0 \tag{21}$$

The electric intensity vector is normal to both the magnetic intensity and the displacement vectors. Thus, it has the components

$$E_x = E_1\,, \quad E_y = 0\,, \quad E_z = E_3 \tag{22}$$

The current density vector **J** be parallel to **E**, thus

$$J_x = J_1\,, \quad J_y = 0\,, \quad J_z = J_3 \tag{23}$$

From Eqs (1)–(4) and (5), we get (in this paper y–axis is the direction of the axis of rotation)

$$\begin{aligned}\rho\left[\frac{\partial^2 u}{\partial t^2}-\Omega^2 u+2\Omega\dot{w}\right] &= (\lambda+\mu)\frac{\partial e}{\partial x}+\mu\nabla^2 u-\gamma\left(1+\nu_o\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}\\ &\quad -\mu_o H_o\frac{\partial h}{\partial x}-\varepsilon_o\mu_o^2 H_o^2\frac{\partial^2 u}{\partial t^2} \qquad (24)\\ \rho\left[\frac{\partial^2 w}{\partial t^2}-\Omega^2 w-2\Omega\dot{u}\right] &= (\lambda+\mu)\frac{\partial e}{\partial z}+\mu\nabla^2 w-\gamma\left(1+\nu_o\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z}\\ &\quad -\mu_o H_o\frac{\partial h}{\partial z}-\varepsilon_o\mu_o^2 H_o^2\frac{\partial^2 w}{\partial t^2} \qquad (25)\end{aligned}$$

We introduce the displacement potentials φ and ψ by the relations

$$u=\varphi_{,x}+\psi_{,z}, \qquad w=\varphi_{,z}-\psi_{,x} \qquad (26)$$

we can obtain from Eqs (1)–(4)

$$h=-H_o\nabla^2\varphi \qquad (27)$$

For convenience, the following non–dimensional variables are used:

$$\begin{aligned}\bar{x}_i &= \frac{x_i}{C_T\omega^*}, \quad \bar{u}_i = \frac{u_i}{C_T\omega^*}, \quad \bar{\varphi} = \frac{\varphi}{(C_T\omega^*)^2}, \quad \bar{\psi} = \frac{\psi}{(C_T\omega^*)^2},\\ \bar{t} &= \frac{t}{\omega^*}, \quad \bar{\tau}_o = \frac{\tau_o}{\omega^*}, \quad \bar{\nu}_o = \frac{\nu_o}{\omega^*}, \quad \bar{\Omega} = \omega^*\Omega, \qquad (28)\\ \bar{T} &= \frac{\gamma(T-T_o)}{\lambda+2\mu}, \quad \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\mu}, \quad \bar{h} = \frac{h}{H_o}, \quad i = 1,2.\end{aligned}$$

In terms of the non-dimensional quantities defined in Eq. (28), the above governing equations reduce to (dropping the bar for convenience)

$$\begin{aligned}\beta^2[\alpha\ddot{u}-\Omega^2 u+2\Omega\dot{w}] &= (\beta^2-1)\frac{\partial e}{\partial x}+\nabla^2 u-\beta^2\left[1+\nu_o\frac{\partial}{\partial t}\right]\frac{\partial T}{\partial x}\\ &\quad -R_H\frac{\partial h}{\partial x} \qquad (29)\\ \beta^2[\alpha\ddot{w}-\Omega^2 w-2\Omega\dot{u}] &= (\beta^2-1)\frac{\partial e}{\partial z}+\nabla^2 w-\beta^2\left[1+\nu_o\frac{\partial}{\partial t}\right]\frac{\partial T}{\partial z}\\ &\quad -R_H\frac{\partial h}{\partial z} \qquad (30)\\ \nabla^2 T &= (1+\tau_o\frac{\partial}{\partial t})\dot{T}+\varepsilon(1+\delta\tau_o\frac{\partial}{\partial t})\dot{e} \qquad (31)\end{aligned}$$

The constitutive equations reduce to

$$\sigma_{xx}=(\beta^2-2)\,e+2\,u_{,x}-\beta^2(T+\nu_o\frac{\partial T}{\partial t}), \qquad (32)$$

$$\sigma_{zz}=(\beta^2-2)\,e+2\,w_{,z}-\beta^2\,(T+\nu_o\frac{\partial T}{\partial t}), \qquad (33)$$

$$\sigma_{xz} = u_{,z} + w_{,x}. \tag{34}$$

In the subsequent analysis we are taking into consideration the case of low speed so that centrifugal stiffening effects can be neglected. By differentiating Eq.(29) with respect to x, and Eq. (30) with respect to z, then adding, we obtain

$$[\alpha \frac{\partial^2}{\partial t^2} - \Omega^2 - (1 + R_H)\nabla^2]\varphi = -2\Omega \frac{\partial \psi}{\partial t} - (1 + \nu_o \frac{\partial}{\partial t})T, \tag{35}$$

by differentiating (29) with respect to z and (30) with respect to x and subtracting we obtain

$$[\beta^2 \alpha \frac{\partial^2}{\partial \mathbf{t}^2} - \beta^2 \mathbf{\Omega}^2 - \nabla^2]\psi = -\mathbf{2}\beta^2 \mathbf{\Omega} \frac{\partial \varphi}{\partial \mathbf{t}}, \tag{36}$$

$$\nabla^2 T - (1 + \tau_o \frac{\partial}{\partial t})\dot{T} - \varepsilon (1 + \delta \tau_o \frac{\partial}{\partial t})\nabla^2 \dot{\varphi} = 0, \tag{37}$$

Eq. (27) has the form

$$\mathbf{h} = -\nabla^2 \varphi . \tag{38}$$

where R_H is the number of magnetic pressure. It is a measure of the relative importance of magnetic effects in comparison with mechanical ones. ε is the usual thermoelastic coupling parameters.

3. Solution of the problem

For a harmonic wave propagated in the direction, where the wave normal lies in the xz–plane, and makes an angle θ with the z–axis, we assume the solutions of the system of Eqs (35)–(37) in the form:

$$\{\varphi, T, h, \varphi\}(x, z, t) = [\varphi_1, T_1, h_1, \varphi_1]\, exp\{ i\xi (x \sin\theta + z \cos\theta) - \omega t\} \tag{39}$$

where is the wave number and ω is the complex in the circular frequency.

Substituting from Eq. (39) into Eqs (35)–(38), we arrive at a system of four homogeneous equations:

$$(\xi^2\beta_1 + \alpha\omega^2 - \Omega^2)\varphi_1 + \nu'_o T_1 - 2\omega\Omega\psi_1 = 0 \tag{40}$$

$$(\alpha\omega^2\beta^2 + \xi^2 - \beta^2\Omega^2)\psi_1 - 2\omega\Omega\beta^2\varphi_1 = 0 \tag{41}$$

$$(\xi^2 - \omega\tau'_o)T_1 + \omega\varepsilon\xi^2\tau'_n\varphi_1 = 0 \tag{42}$$

$$-\xi^2\varphi_1 + h_1 = 0 \tag{43}$$

in which, $\nu'_o = 1 - \omega\,\nu_o$, $\tau'_o = 1 - \omega\,\tau_o$, $\tau'_n = 1 - \omega\,\delta\,\tau_o$.

The system of Eqs (40)–(43) has non–trivial solutions if and only if the determination of the factor matrix vanishes. So

$$\begin{vmatrix} (\xi^2\beta_1 + \alpha\,\omega^2 - \Omega^2) & \nu'_o & 0 & -2\omega\,\Omega \\ -2\omega\,\Omega\beta^2 & 0 & 0 & (\beta^2\alpha\,\omega^2 - \beta^2\Omega^2 + \xi^2) \\ \omega\,\varepsilon\,\xi^2\tau'_n & (\xi^2 - \omega\,\tau'_o) & 0 & 0 \\ -\xi^2 & 0 & 1 & 0 \end{vmatrix} = 0 \tag{44}$$

This yields

$$\nu^6 + A\,\nu^4 + B\,\nu^2 + C = 0 \tag{45}$$

where

$$
\begin{aligned}
A &= \omega \, \left\{ \alpha^2\beta^2\omega^4 - \alpha\,\omega^2 \left[\tau'_o\,\omega + \beta^2 \left(\beta_1\tau'_o\,\omega + 2\,\Omega^2 + \varepsilon\,\nu'_o\tau'_n\,\omega \right) \right] \right. \\
&\quad \left. +\ \Omega^2 \left[\tau'_o\omega + \beta^2 \left(\beta_1\tau'_o\omega + \Omega^2 + \varepsilon\,\nu'_o\tau'_n\omega - 4\omega^2 \right) \right] \right\} \big/ E \\
B &= \omega^3 \left[-\varepsilon\,\nu'_o\tau'_n\omega + \alpha\,\omega^2 - \Omega^2 - \beta_1 \left(\tau'_o\omega - \alpha\,\beta^2\omega^2 + \beta^2\Omega^2 \right) \right] \big/ E \\
C &= \beta_1\omega^5 \big/ E, \\
E &= -\,\beta^2\tau'_o \left(\alpha^2\omega^4 - 4\omega^2\Omega^2 - 2\alpha\,\omega^2\Omega^2 + \Omega^4 \right)
\end{aligned}
$$

and $\nu = \omega/\xi$ is the velocity of coupled waves (the dilatational waves and rotational waves are coupled due to the existence of rotating).

The solution of Eq.(45) can be given below:

$$
v_1^2 = \frac{2^{4/3}\left(1 + i\sqrt{3}\right)\,M_1 - M_3\left[4A + 2^{2/3}\left(1 - i\sqrt{3}\right)\,M_3\right]}{12\,M_3} \tag{46}
$$

$$
v_2^2 = \frac{2^{4/3}\left(1 - i\sqrt{3}\right)\,M_1 - M_3\left[4A + 2^{2/3}\left(1 + i\sqrt{3}\right)\,M_3\right]}{12\,M_3} \tag{47}
$$

$$
v_3^2 = \frac{-2^{4/3}M_1 + M_3\left(-2\,A + 2^{2/3}M_3\right)}{6\,M_3} \tag{48}
$$

in which

$$
\begin{aligned}
M_1 &= -A^2 + 3\,B, \\
M_2 &= -2\,A^3 + 9AB - 27C, \\
M_3 &= \left(M_2 + \sqrt{4\,M_1^3 + M_2^2} \right)^{1/3}
\end{aligned}
$$

where v_1, v_2 and v_3 are the velocities of three waves. We can call them P1, P2 and P3 wave.

(1) For incident P3 wave:

Since Eq. (45) is a cubic in ν^2, there shall be three coupled waves traveling with three different velocities. So assuming that the radiation into vacuum is neglected, when a coupled wave falls on the boundary $z = 0$ from within the elastic medium, that will make an angle θ with the negative direction of the z–axis, and the two reflected waves that will make angles θ_1, θ_2 with the same direction (see Figure1). The incident wave is a coupled (rotational and dilatational) wave because of the existence of rotation. The displacement potentials φ, ψ and T will take the following forms:

$$\begin{aligned}
\varphi &= A_1 \exp\{i\xi_1(x\sin\theta_1 - z\cos\theta_1) - \omega t\} \\
&\quad + A_2 \exp\{i\xi_2(x\sin\theta_2 - z\cos\theta_2) - \omega t\} \\
&\quad + B_1 \exp\{i\xi_3(x\sin\theta + z\cos\theta) - \omega t\} \\
&\quad + B_2 \exp\{i\xi_3(x\sin\theta - z\cos\theta) - \omega t\} && (49) \\
\psi &= \eta_1 A_1 \exp\{i\xi_1(x\sin\theta_1 - z\cos\theta_1) - \omega t\} \\
&\quad + \eta_2 A_2 \exp\{i\xi_2(x\sin\theta_2 - z\cos\theta_2) - \omega t\} \\
&\quad + \eta_3 B_1 \exp\{i\xi_3(x\sin\theta + z\cos\theta) - \omega t\} \\
&\quad + \eta_3 B_2 \exp\{i\xi_3(x\sin\theta - z\cos\theta) - \omega t\} && (50) \\
T &= \gamma_1 A_1 \exp\{i\xi_1(x\sin\theta_1 - z\cos\theta_1) - \omega t\} \\
&\quad + \gamma_2 A_2 \exp\{i\xi_2(x\sin\theta_2 - z\cos\theta_2) - \omega t\} \\
&\quad + \gamma_3 B_1 \exp\{i\xi_3(x\sin\theta + z\cos\theta) - \omega t\} \\
&\quad + \gamma_3 B_2 \exp\{i\xi_3(x\sin\theta - z\cos\theta) - \omega t\} && (51)
\end{aligned}$$

in which

$$\begin{aligned}
\eta_i &= \frac{2\omega\Omega\beta^2}{\alpha\omega^2\beta^2 + \xi_i^2 - \beta^2\Omega^2} \\
\gamma_i &= \frac{2\omega\Omega\eta_i - (\xi_i^2\beta_1 + \alpha\omega^2 - \Omega^2)}{\nu_o'} && (52) \\
i &= 1, 2, 3
\end{aligned}$$

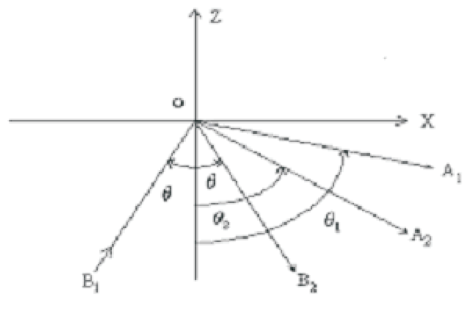

Figure 1 Relation between the incident angle and the reflect angle

The ratios of the amplitudes of the reflected waves and amplitude of the incident wave $\frac{A_1}{B_1}$, $\frac{A_2}{B_1}$, $\frac{B_2}{B_1}$ give the corresponding reflection coefficients. Also it may be noted that the angles θ, θ_1, θ_2 and the corresponding wave numbers ξ_3, ξ_1, ξ_2 are to be connected by the relations below:

$$\xi_3 \sin\theta = \xi_1 \sin\theta_1 = \xi_2 \sin\theta_2. \tag{53}$$

on the interface $z = 0$ of the medium, relation (49) may also be written as:

$$\frac{\sin\theta}{\nu_3} = \frac{\sin\theta_1}{\nu_1} = \frac{\sin\theta_2}{\nu_2} \tag{54}$$

(2) For incident P1 wave:

We consider the normal of the incident wave that makes an angle θ with the negative direction of the z–axis. There will be two reflected waves. One will make angle θ with the direction of the z–axis, and the other two reflected waves will make angles with the same direction (see Figure 1). The displacement potential φ and ψ will take the following form

$$\begin{aligned}
\varphi \;=\;& B_1 \exp\{i\xi_1(x\sin\theta + z\cos\theta) - \omega t\} \\
&+B_2 \exp\{i\xi_1(x\sin\theta - z\cos\theta) - \omega t\} \\
&+A_1 \exp\{i\xi_2(x\sin\theta_1 - z\cos\theta_1) - \omega t\} \\
&+A_2 \exp\{i\xi_3(x\sin\theta_2 - z\cos\theta_2) - \omega t\} \qquad (55)\\
\psi \;=\;& \eta_1 B_1 \exp\{i\xi_1(x\sin\theta + z\cos\theta) - \omega t\} \\
&+\eta_1 B_2 \exp\{i\xi_1(x\sin\theta - z\cos\theta) - \omega t\} \\
&+\eta_2 A_1 \exp\{i\xi_2(x\sin\theta_1 - z\cos\theta_1) - \omega t\} \\
&+\eta_3 A_2 \exp\{i\xi_3(x\sin\theta_2 - z\cos\theta_2) - \omega t\} \qquad (56)\\
T \;=\;& \gamma_1 B_1 \exp\{i\xi_1(x\sin\theta + z\cos\theta) - \omega t\} \\
&+\gamma_1 B_2 \exp\{i\xi_1(x\sin\theta - z\cos\theta) - \omega t\} \\
&+\gamma_2 A_1 \exp\{i\xi_2(x\sin\theta_1 - z\cos\theta_1) - \omega t\} \\
&+\gamma_3 A_2 \exp\{i\xi_3(x\sin\theta_2 - z\cos\theta_2) - \omega t\} \qquad (57)
\end{aligned}$$

also the angles θ , θ_1 , θ_2 and the corresponding wave numbers, $\xi_1,$ $\xi_2,$ ξ_3 are to be connected by the relations below

$$\xi_1 \sin\theta = \xi_2 \sin\theta_1 = \xi_3 \sin\theta_2 \tag{58}$$

on the interface $z = 0$ of the medium, relation (53) may also be written as:

$$\frac{\sin\theta}{\nu_1} = \frac{\sin\theta_1}{\nu_2} = \frac{\sin\theta_2}{\nu_3} \tag{59}$$

4. Boundary conditions

Since the boundary $z = 0$ is adjacent to vacuum, it is free from surface traction. So the boundary condition can be expressed as

$$\sigma_{zj} = 0 \qquad (\, j = x, y, z\,) \quad \text{on} \quad z = 0. \tag{60}$$

Assuming that the boundary $z = 0$ is thermally insulated. This means that the following relation will be

$$\frac{\partial T}{\partial z} = 0 \qquad \text{on} \quad z = 0 \tag{61}$$

5. Expressions for the reflection coefficients

(1) For incident P3 wave:

Using the conditions (55), (56) and Eqs (48)–(51) we can obtain the following relations

$$\frac{A_1}{B_1}a_{11} + \frac{A_2}{B_1}a_{12} + \frac{B_2}{B_1}a_{13} = b_1 \tag{62}$$

$$\frac{A_1}{B_1}a_{21} + \frac{A_2}{B_1}a_{22} + \frac{B_2}{B_1}a_{23} = b_2 \tag{63}$$

$$\frac{A_1}{B_1}a_{31} + \frac{A_2}{B_1}a_{32} + \frac{B_2}{B_1}a_{33} = b_3 \tag{64}$$

in which

$$\begin{aligned}
a_{1i} &= \frac{(\sin 2\,\theta_i - \eta_i \cos 2\theta_i)\,\nu_3^2}{\nu_i^2} \\
a_{2i} &= \frac{\left[\left(2-\beta^2\right) - 2\cos^2\theta_i - \eta_i \sin 2\theta_i\right] v_3^2}{\nu_i^2} - \frac{\beta^2 \gamma_i \nu_3^2}{\omega^2} \\
a_{3i} &= \frac{\cos\theta_i}{\nu_i}\left(\frac{\omega^2 \beta_1}{\nu_i^2} + \alpha\omega^2 - \Omega^2 - \frac{4\Omega^2\beta^2\omega^2}{\alpha\omega^2\beta^2 + \omega^2/\nu_i^2 - \beta^2\Omega^2}\right) \\
& \quad i = 1, 2, 3, \quad \theta_3 = \theta \\
b_1 &= \sin 2\theta + \eta_3 \cos 2\theta \\
b_2 &= -\left[\left(2-\beta^2\right) - 2\cos^2\theta + \eta_3 \sin 2\theta - \frac{\beta^2\gamma_3\nu_3^2}{\omega^2}\right] \\
b_3 &= -a_{33}
\end{aligned}$$

The solution of this system for the reflection coefficients $\frac{A_1}{B_1}$, $\frac{A_2}{B_1}$ and $\frac{B_2}{B_1}$ is

$$X1 = \frac{A_1}{B_1} = \frac{P_1}{Q_1}, \qquad X2 = \frac{A_2}{B_1} = \frac{P_2}{Q_1}, \qquad X3 = \frac{B_2}{B_1} = \frac{P_3}{Q_1} \tag{65}$$

in which

$$\begin{aligned}
P_1 &= b_1\left(a_{23}a_{32} - a_{22}a_{33}\right) + b_2\left(a_{12}a_{33} - a_{13}a_{32}\right) \\
&\quad + b_3\left(a_{13}a_{22} - a_{12}a_{23}\right) && (66) \\
P_2 &= -b_1\left(a_{23}a_{31} - a_{21}a_{33}\right) - b_2\left(a_{11}a_{33} - a_{13}a_{31}\right) \\
&\quad - b_3\left(a_{13}a_{21} - a_{11}a_{23}\right) && (67) \\
P_3 &= b_1\left(a_{23}a_{31} - a_{21}a_{32}\right) + b_2\left(a_{11}a_{32} - a_{12}a_{31}\right) \\
&\quad + b_3\left(a_{12}a_{21} - a_{11}a_{22}\right) && (68) \\
Q_1 &= a_{11}\left(a_{23}a_{32} - a_{22}a_{33}\right) + a_{12}\left(a_{21}a_{33} - a_{23}a_{31}\right) \\
&\quad + a_{13}\left(a_{31}a_{22} - a_{21}a_{32}\right) && (69)
\end{aligned}$$

(2) For incident P1 wave:

Using boundary conditions (60), (61) and Eqs (55)–(57) we can obtain the following relations

$$\frac{B_2}{B_1}c_{11}+\frac{A_1}{B_1}c_{12}+\frac{A_2}{B_1}c_{13} = d_1 \tag{70}$$

$$\frac{B_2}{B_1}c_{21}+\frac{A_1}{B_1}c_{22}+\frac{A_2}{B_1}c_{23} = d_2 \tag{71}$$

$$\frac{B_2}{B_1}c_{31}+\frac{A_1}{B_1}c_{32}+\frac{A_2}{B_1}c_{33} = d_3 \tag{72}$$

in which

$$\begin{aligned}
c_{1i} &= \frac{(\sin 2\vartheta_i-\eta_i\cos 2\vartheta_i)\,\nu_1^2}{\nu_i^2}\\
c_{2i} &= \frac{\left[\left(2-\beta^2\right)-2\cos^2\vartheta_i-\eta_i\sin 2\vartheta_i\right]v_1^2}{\nu_i^2}-\frac{\beta^2\gamma_i\nu_1^2}{\omega^2}\\
c_{3i} &= \frac{\cos\vartheta_i}{\nu_i}\left(\frac{\omega^2\beta_1}{\nu_i^2}+\alpha\,\omega^2-\Omega^2-\frac{4\Omega^2\beta^2\omega^2}{\alpha\omega^2\beta^2+\omega^2/\nu_i^2-\beta^2\Omega^2}\right)\\
&\quad i=1,2,3,\ \ \vartheta_1=\theta,\,\vartheta_{2,3}=\theta_{1,2}\\
d_1 &= \sin 2\theta+\eta_1\cos 2\theta\\
d_2 &= -\left[\left(2-\beta^2\right)-2cos^2\theta+\eta_1 sin\,2\theta-\frac{\beta^2\gamma_1\nu_1^2}{\omega^2}\right]\\
d_3 &= c_{31}
\end{aligned}$$

The solution of this system for the reflection coefficients $\frac{A_1}{B_1}$, $\frac{A_2}{B_1}$ and $\frac{A_3}{B_1}$ is

$$X1=\frac{B_2}{B_1}=\frac{R_1}{Q_2}\,,\qquad X2=\frac{A_1}{B_1}=\frac{R_2}{Q_2}\,,\qquad X3=\frac{A_2}{B_1}=\frac{R_3}{Q_2} \tag{73}$$

in which

$$\begin{aligned}
R_1 &= d_1\left(c_{23}c_{32}-c_{22}c_{33}\right)+d_2\left(c_{12}c_{33}-c_{13}c_{32}\right)\\
&\quad +d_3\left(c_{13}c_{22}-c_{12}c_{23}\right) && (74)\\
R_2 &= -d_1\left(c_{23}c_{31}-c_{21}c_{33}\right)-d_2\left(c_{11}c_{33}-c_{13}c_{31}\right)\\
&\quad -d_3\left(c_{13}c_{21}-c_{11}c_{23}\right) && (75)\\
R_3 &= d_1\left(c_{23}c_{31}-c_{21}c_{32}\right)+d_2\left(c_{11}c_{32}-c_{12}c_{31}\right)\\
&\quad +d_3\left(c_{12}c_{21}-c_{11}c_{22}\right) && (76)\\
Q_2 &= c_{11}\left(c_{23}c_{32}-c_{22}c_{33}\right)+c_{12}\left(c_{21}c_{33}-c_{23}c_{31}\right)\\
&\quad +c_{13}\left(c_{31}c_{22}-c_{21}c_{32}\right) && (77)
\end{aligned}$$

6. Special case: in the absence of rotating

In this case we put $\Omega=0$ and here

$$\nu_3^2=-\frac{1}{\alpha\beta^2}$$

Also in this case the dilatational wave and rotational wave are uncoupled.
For incident rotational wave:

Here the displacement potentials φ, ψ and T will take the following forms:

$$\begin{aligned}
\varphi &= A_1\exp\{i\xi_1(x\sin\theta_1 - z\cos\theta_1) - \omega t\} \\
&\quad + A_2\exp\{i\xi_2(x\sin\theta_2 - z\cos\theta_2) - \omega t\} && (78) \\
\psi &= B_1\exp\{i\xi_3(x\sin\theta + z\cos\theta) - \omega t\} \\
&\quad + B_2\exp\{i\xi_3(x\sin\theta - z\cos\theta) - \omega t\} && (79) \\
T &= \gamma_1 A_1\exp\{i\xi_1(x\sin\theta_1 - z\cos\theta_1) - \omega t\} \\
&\quad + \gamma_2 A_2\exp\{i\xi_2(x\sin\theta_2 - z\cos\theta_2) - \omega t\} && (80)
\end{aligned}$$

The solution of this system for the reflection coefficients $\frac{A_1}{B_1}$, $\frac{A_2}{B_1}$ and $\frac{B_2}{B_1}$ satisfy Eqs (62)–(64) and a_{ij} $(i, j = 1, 2, 3)$ are given below

$$\begin{aligned}
a_{1j} &= \frac{\nu_3^2}{\nu_j^2}\sin2\theta_j \\
a_{2j} &= \nu_3^2\left[\frac{\beta^2 - 2 - \beta^2\beta_1}{\nu_j^2} + \frac{2\cos^2\theta_j}{\nu_j^2} - \alpha\beta^2\right] \\
a_{3j} &= \frac{\cos\theta_j}{\nu_j}\left(\frac{\beta_1}{\nu_j^2} + \alpha\right), \quad j = 1, 2 \\
a_{13} &= -\cos2\theta, \qquad a_{23} = \sin2\theta, \qquad a_{33} = 0
\end{aligned}$$

For incident dilatational wave:

Here the displacement potential will take the following form

$$\begin{aligned}
\varphi &= B_1\exp\{i\xi_1(x\sin\theta + z\cos\theta) - \omega t\} \\
&\quad + B_2\exp\{i\xi_1(x\sin\theta - z\cos\theta) - \omega t\} && (81) \\
\psi &= A_2\exp\{i\xi_3(x\sin\theta_2 - z\cos\theta_2) - \omega t\} && (82) \\
T &= \gamma_1 B_1\exp\{i\xi_1(x\sin\theta + z\cos\theta) - \omega t\} \\
&\quad + \gamma_1 B_2\exp\{i\xi_1(x\sin\theta - z\cos\theta) - \omega t\} \\
&\quad + \gamma_2 A_1\exp\{i\xi_2(x\sin\theta_1 - z\cos\theta_1) - \omega t\} && (83)
\end{aligned}$$

$$[\alpha\frac{\partial^2}{\partial t^2} - (1 + R_H)\nabla^2]\varphi = -(1 + \nu_o\frac{\partial}{\partial t})T$$

The solution of this system for the reflection coefficients $\frac{A_1}{B_1}$, $\frac{A_2}{B_1}$ and $\frac{B_2}{B_1}$ satisfy

Eqs (70)–(72) and a_{ij} $(i, j = 1, 2, 3)$ are given below

$$
\begin{aligned}
c_{1j} &= \frac{\nu_1^2}{\nu_j^2}\sin 2\vartheta_j \\
c_{2j} &= \nu_1^2\left[\frac{\beta^2 - 2 - \beta^2\beta_1}{\nu_j^2} + \frac{2\cos^2\vartheta_j}{\nu_j^2} - \alpha\beta^2\right] \\
c_{3j} &= \frac{\nu_1^3\cos\vartheta_j}{\nu_j^3}\left(\beta_1 + \alpha\nu_j^2\right), \quad j = 1, 2, \quad \vartheta_1 = \theta, \quad \vartheta_2 = \theta_1 \\
c_{13} &= -\frac{\nu_1^2}{\nu_3^2}\cos 2\theta_2 \\
c_{23} &= \frac{\nu_1^2}{\nu_3^2}\sin 2\theta_2, \quad c_{33} = 0, \quad d_1 = c_{11}, \quad d_2 = -c_{21}, \quad d_3 = c_{31}
\end{aligned}
$$

7. Numerical Results

The copper material is chosen for numerical evaluations. In these solutions the circular frequency ω is expressed by a complex number, namely, $\omega = \omega_o + i\,\zeta$, where i is an imaginary unit, $e^{\omega\, t} = e^{\omega_o t}(\,cos\,\zeta\, t + i\, sin\,\zeta\, t\,)$, so the waves must attenuate. In other words, the waves cannot arrive at the region near the boundary surface. In fact we used $\omega = \omega_o$ ($\omega_o$is a real number) in this paper, so the waves can arrive at the region near the boundary surface. The other constants of the problem are taken as $\beta^2 = 3.95596$, $\nu_o = 0.03, \tau_o = 0.02$, $\varepsilon = 0.0168$,

Figure 2 gives the variation of the reflection coefficients ratios with the angle of incidence for incident P1 and P3 waves under three theories (CD, LS and GL) in the presence of rotation. Here $R_H = 0.3$,$\Omega = 0.01$, $\omega_o = 5$. We can see that in the case of incident P3 wave, the reflection coefficient ratio $|X1| = |X2| = 0$when$\theta = 0^0$, 90^0. $|X3| = 1$ when $\theta = 0^0$, 90^0. Also we observed that the effect of relaxation times acts to decrease the amplitude of reflection coefficient ratios. In the case of incident P1 wave, the reflection coefficient ratio $|X2| = |X3| = 0$ when $\theta = 0^0$, 90^0 and $|X1| = 1$ when $\theta = 0^0$, 90^0. And the same effect of relaxation times can be observed in $|X1|$, $|X2|$.

Figure 3 gives variation of the reflection coefficients ratios with the angle of incidence for incident rotational and dilatational waves under three theories (CD, LS and GL) in the absence of rotation. We can see there has great difference for reflection coefficients ratio due to essential difference for incident waves. Figure 4 gives the effect of rotation on reflection coefficient ratios for incident P3 and P1 waves. Here $R_H = 0.3$, $\omega_o = 5$, and $\Omega = 0.01$, 0.03, 0.05, respectively. The reflection coefficient ratios $|X1|$, $|X3|$ for incident P3 wave and $|X3|$ for incident P1 wave decreases with the increase of Ω. While for incident P3 and P1 waves the reflection coefficient ratio $|X2|$ increase with the increase of Ω. So we can conclude that the rotation plays an important role for different incident waves. Figures 5 gives the effect of magnetic field on the reflection coefficient ratio. Here $\omega_o = 5.0$, $\Omega = 0.01$ and $R_H = 0.0$, 0.3, 0.5, respectively. Clearly the magnetic field has a salient influence on the reflection coefficients ratio. Also we can see that for incident P3 wave, $|X1|$, $|X3|$ decrease with the increase of the intensity of magnetic field and $|X2|$ increase with the increase of the intensity of magnetic field.

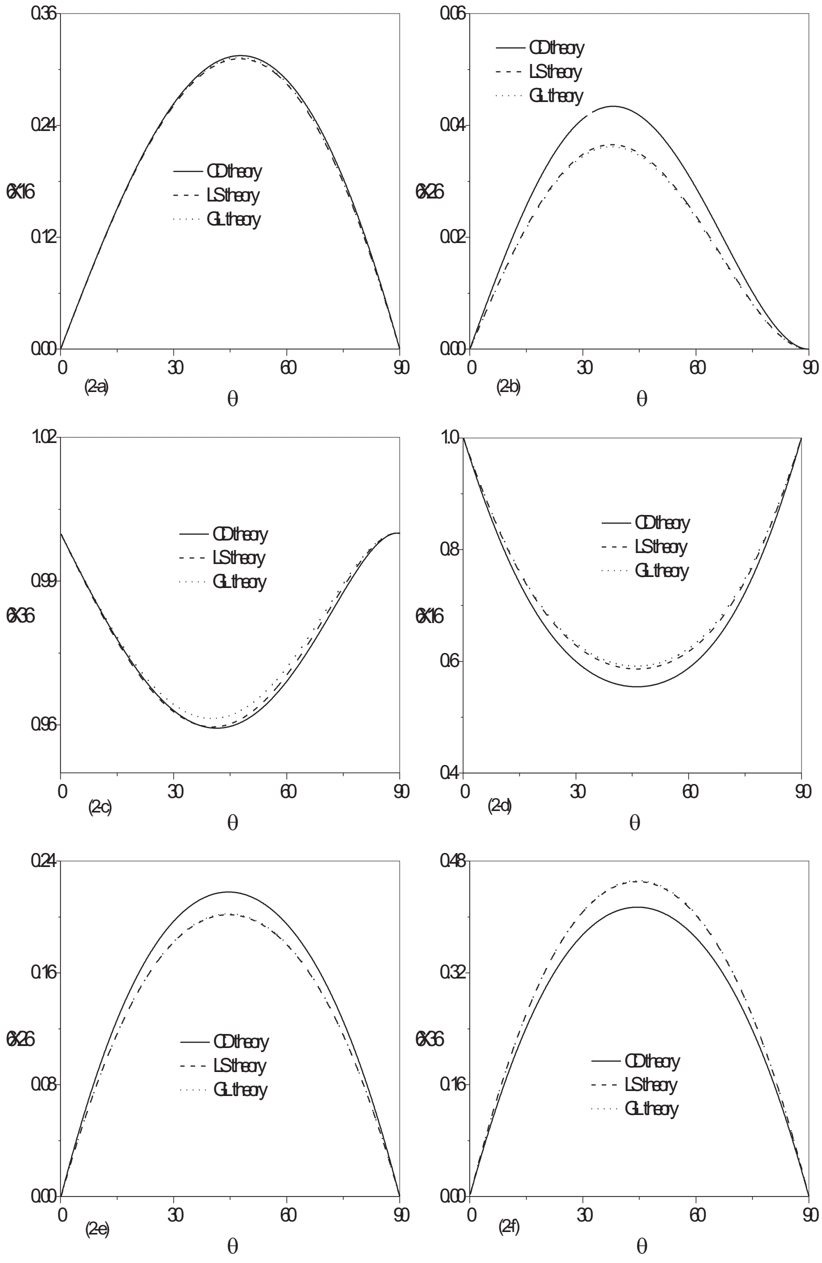

Figure 2 Variation of reflection coefficients ratio with incident angle P1 and P2 wave under three theories in presence of rotation

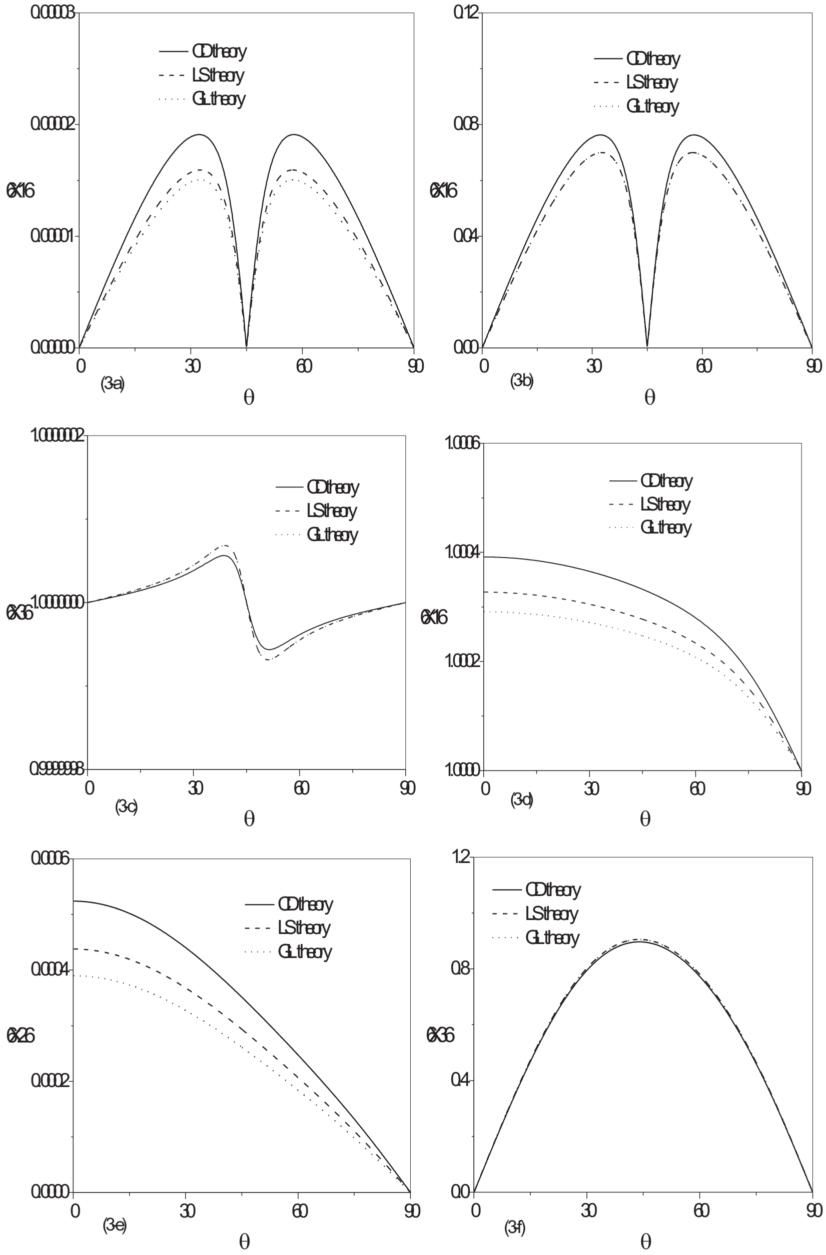

Figure 3 Variation of reflection coefficients ratio with incident angle for P1 and P3 wave under three theories in absence of rotation

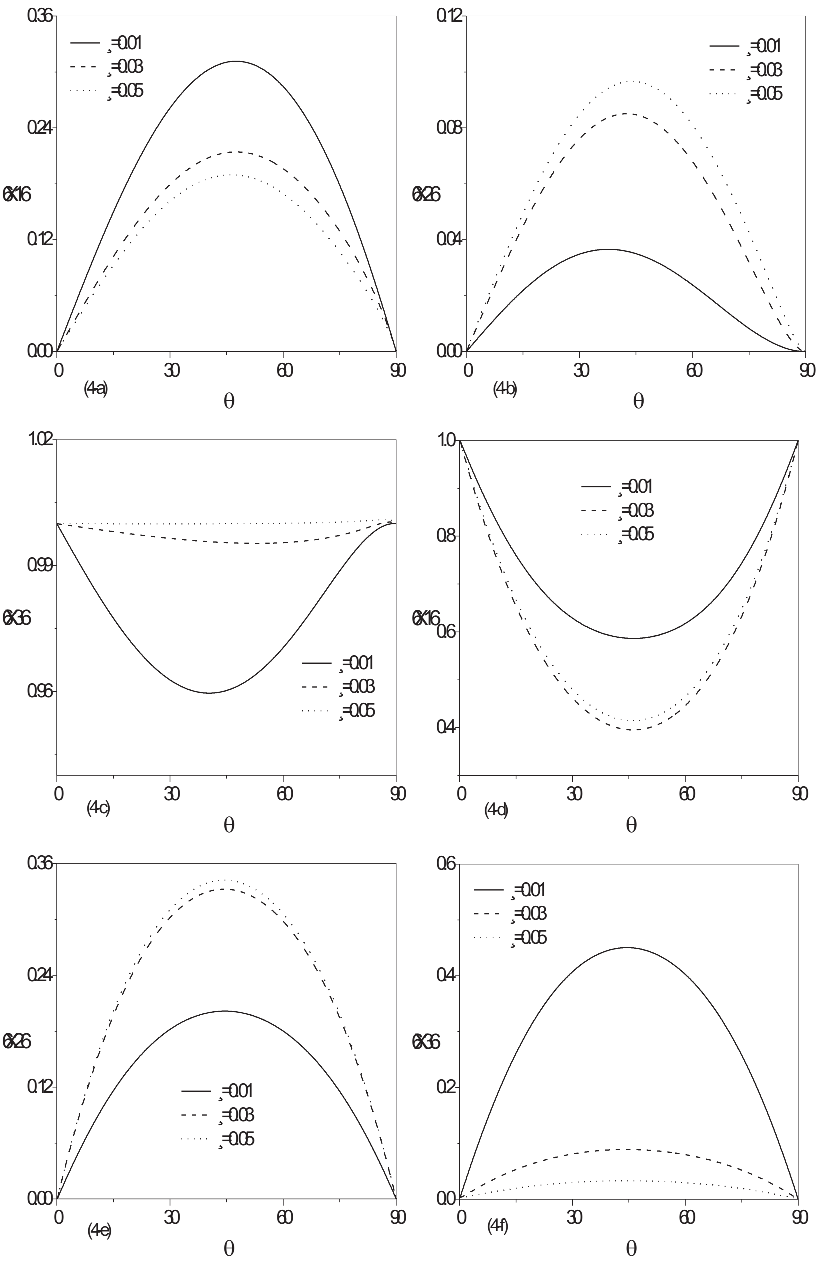

Figure 4 Effect of rotation on variation of reflection coefficients ratio for incident angle P1 and P3 wave under LS theory

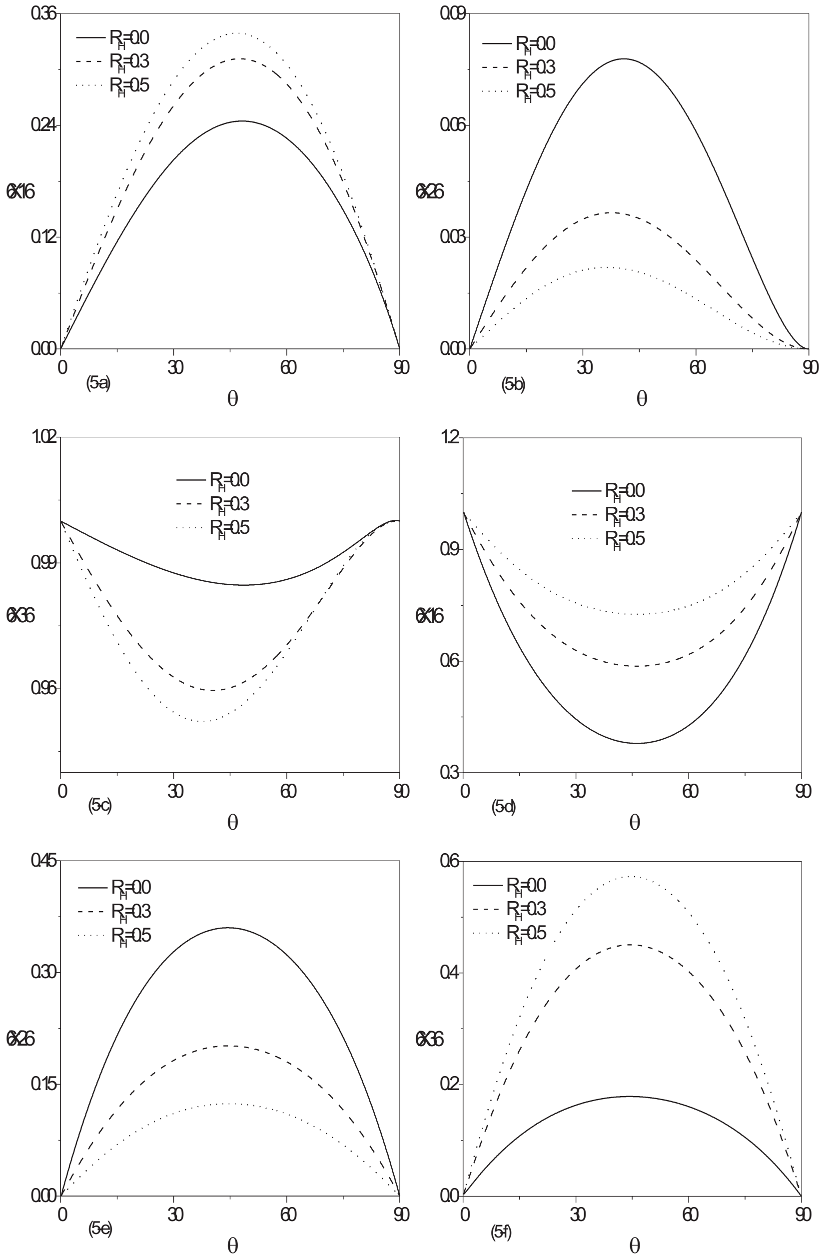

Figure 5 Effect of magnetic field on variation of reflection coefficients for incident P1 and P3 wave under LS theory

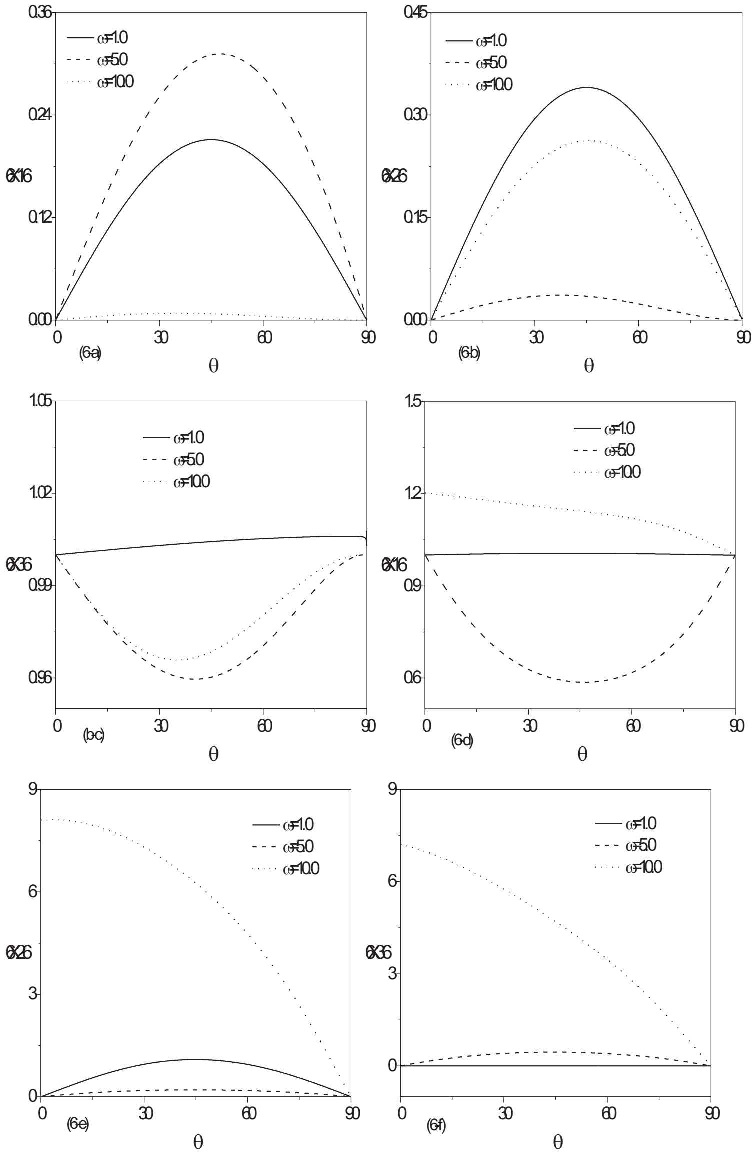

Figure 6 Variation of reflection coefficients ratio with angle of incident for incident P1 and P2 wave in different frequency under LS theory

For incident P1 wave, $|X1|$, $|X2|$ decrease with the increase of the intensity of magnetic field and $|X3|$ increase with the increase of the intensity of magnetic field. Figure 6 gives the variation of the angle of incidence with the reflection coefficient ratios for different values of frequency under LS theory. Here $R_H = 0.3$, $\Omega = 0.01$ and $\omega_o = 1.0$, 5.0, 10.0, respectively. We observed the effect of frequency is prodigious.

8. Conclusions

We can obtain the following conclusions according to the analysis above:

1. The reflection coefficient ratio depends on the angle of incidence, the nature of this dependence is different for different reflected waves.

2. The rotation and magnetic field play a significant role and the two effects have the inverse trend for reflection coefficient ratios.

3. The thermal frequency has a very great influence on the reflection coefficient ratio.

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Nomenclature

λ, μ	Lame's constants
ρ	density
C_E	specific heat at constant strain
t	time
T	absolute temperature
T_o	reference temperature chosen so that $\lvert(T - T_o)/T_o\rvert << 1$
σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
u_i	components of displacement vector
k	thermal conductivity
J	current density vector
μ_o	magnetic permeability
ε_o	electric permeability
C_T^2	$=(\lambda + 2\mu)/\rho$
C_L	$= \sqrt{\mu/\rho}$ velocity of transverse waves
c^2	$= 1/\mu_o\varepsilon_o$ sound speed
e	cubical dilatation
α_t	coefficient of linear thermal expansion
γ	$=(3\lambda + 2\mu)\alpha_t$
ε	$= \gamma^2 T_o/\rho^2 C_E C_T^2$
C_A^2	$= \mu_o H_o^2/\rho$
C_A	the Alfven speed
α	$= 1 + C_A^2/c^2$
β^2	$= C_T^2/C_L^2$
ω^*	$= k/\rho C_E C_T^2$
R_H	$= C_A^2/C_T^2$