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Multistage Coupling of Eight Mistuned Bladed Discs on a Solid Shaft with 1% Mistuning

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Considered here was the effect of multistage coupling on the dynamics of a rotor consisting of eight mistuned bladed discs on a solid shaft. Free and forced vibrations were examined. In this study, the global rotating mode shapes of flexible mistuned bladed discs–shaft assemblies were calculated, taking into account rotational effects, such as centrifugal stiffening. The thus obtained natural frequencies of the blade, the shaft, the bladed disc, and the entire shaft with discs were carefully examined to discover resonance conditions and coupling effects.

Keywords: Multistage coupling, mistuning, blades, bladed disc

1. Introduction

The effects of multistage coupling and disc flexibility on mistuned bladed disc dynamics were presented for the first time by Bladh et al. [1], who studied single–stage and two–stage rotors of a very simple stage geometry. Shahab and Thomas [7] presented the coupling effects of disc flexibility on the dynamic behaviour of a multi disc–shaft system. Rzadkowski et al. [3] showed that the coupling of three identical industrial bladed discs on a shaft segment changes the mode shapes of shrouded bladed discs up to the seventh node diameter. Sharma et al. [8] analyzed a turbine rotor with 16 discs, with only one of them being bladed, under earthquake–force excitation, but they did not investigate couplings between the shaft and bladed discs. Sinha [9] made an analysis of two mistuned bladed discs which were connected using the stiffness coefficient.

Here the bladed disc was replaced by a system of masses and springs. Laxalde et al. [2] used the multistage cyclic symmetry method to show the coupling of two bladed discs, each with a different number of blades, mounted on a flexible shaft. Rzadkowski and Drewczynski [4, 5] performed an analysis of eight tuned bladed discs on a shaft, with an equal number of blades on each disc, to show that coupling between particular bladed discs was visible up to modes with two nodal diameters. Rzadkowski and Drewczynski [6] also analyzed forced vibration results for rotors with 8 tuned bladed discs, each with different numbers of blades.

Considered in this paper is the dynamic behaviour of a rotor similar to the one presented in [6] but consisting of eight mistuned bladed discs on a solid shaft with different numbers of blades on each disc (Fig. 1). The effect of shaft flexibility on the dynamic characteristics of the bladed discs and the multistage coupling effects between the shaft and the bladed disc modes were investigated. The analysis results from the rotor with tuned blades in each disc (case 1) were compared with those of a rotor with mistuned blades in each disc (case 2).

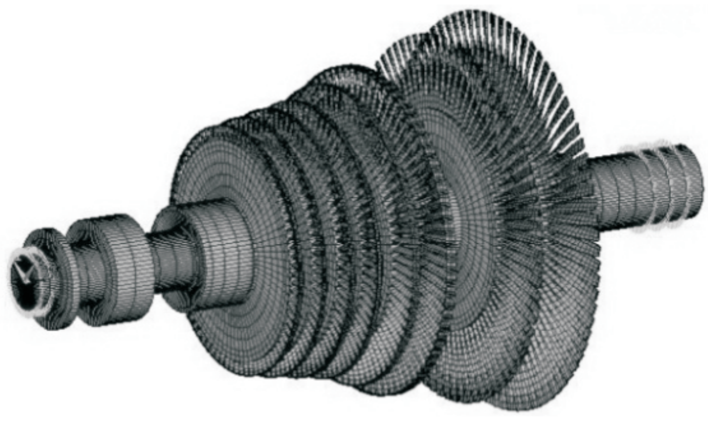

Figure 1 FEM bladed discs on the shaft

The performed free and forced vibration analysis shows that in bladed disc models, from the designer's point of view, it is important to include the shaft, since this can considerably change the spectrum of frequencies with zero, one and two nodal diameters.

2. Description of the model

An industrial shaft with eight bladed discs was analyzed. In the spin pit tests (vacuum condition), eight bladed disc was excited by an air jet, with the rotation speed being gradually increased and next decreased. There were 80 blades on the eighth bladed disc. Experimental results shown that the frequency measured on the seventh bladed disc also include the natural frequencies of the 8^{th} bladed disc from the 1^{st} and 2^{nd} series of blade–dominated modes.

Generally there is problem with publishing numerical and experimental results for industrial structures, so in this paper the geometry of the bladed discs were simplified, assuming the shaft dimensions to be unchanged thus the free and forced vibrations of eight bladed discs on the shaft were analyzed. The first six bladed discs were shrouded and the seventh and eighth were unshrouded.

Fig. 1 presents a finite element model of the bladed discs on the shaft. The number of blades on each disc is different: stage I – 176 rotor blades, stage II – 180 blades, stage III – 154, stage IV – 180, stage V – 144, stage VI – 114, stage VII – 98 and stage VIII 80 rotor blades, i.e. the configuration as it appears in real life. Isoparametric brick elements with 20 nodes and 3 degrees of freedom per node were used. FEM model in its entirety had just over a million DOF's. The natural frequencies of the single bladed discs are presented in Table 1, where "k" denotes the nodal diameters.

The tuned bladed disc model presented here is case 1, whereas the mistuned bladed disc model is case 2. 1% of mistuning was achieved for the case 2 model by modifying the Young modulus for every blade in each stage. A free and forced vibration analysis for both cases was considered. The calculation period for case 2 was almost 100 times longer than for case 1. All the calculations were made for a 3000 RPM velocity.

Table 1 Natural frequencies of the cantilever bladed discs in each stage for 3000 RPM

k	Bladed disc 1		Bladed disc 2		Bladed disc 3		Bladed disc 4	
	Series 1 [Hz]	Series 2 [Hz]	Series 1 [Hz]	Series 2 [Hz]	Series 1 [Hz]	Series 2 [Hz]	Series 1 [Hz]	Series 2 [Hz]
0	418.1	1334	322.1	1015	360.7	833.3	366.1	574.2
1	419.5	1830	323.5	1272.	361.1	1086	365.6	809.3
2	449.4	1951	361.7	1309	401.6	1190	402.5	949.7
3	559.6	1978	496.9	1342	549.6	1221.	538.1	993.5
k	Bladed disc 5		Bladed disc 6		Bladed disc 7		Bladed disc 8	
	Series 1 [Hz]	Series 2 [Hz]	Series 1 [Hz]	Series 2 [Hz]	Series 1 [Hz]	Series 2 [Hz]	Series 1 [Hz]	Series 2 [Hz]
0	349.4	384.6	295.7	353.9	152.1	288.5	129.3	280.0
1	360.5	623.1	335.0	520.7	152.4	286.7	129.5	279.5
2	384.3	746.1	349.3	718.6	152.6	291.8	129.6	281.4
3	384.3		415.5	813.06				

3. Free vibration

The natural frequencies of the rotating shrouded bladed discs were calculated using the ANSYS code. Two identical bearings were modeled as springs with the stiffness of $k_{xx} = k_{yy} = 2 * 10^{10}$ N/m and $k_{xy} = k_{yx} = 2 * 10^{10}$ N/m.

In cases 1 and 2 the natural frequencies for all bladed discs were computed. The modes of the tuned bladed disc were classified according to axisymmetric modes, which are mainly characterized by nodal lines running along the diameters of the structure and having constant angular spacing.

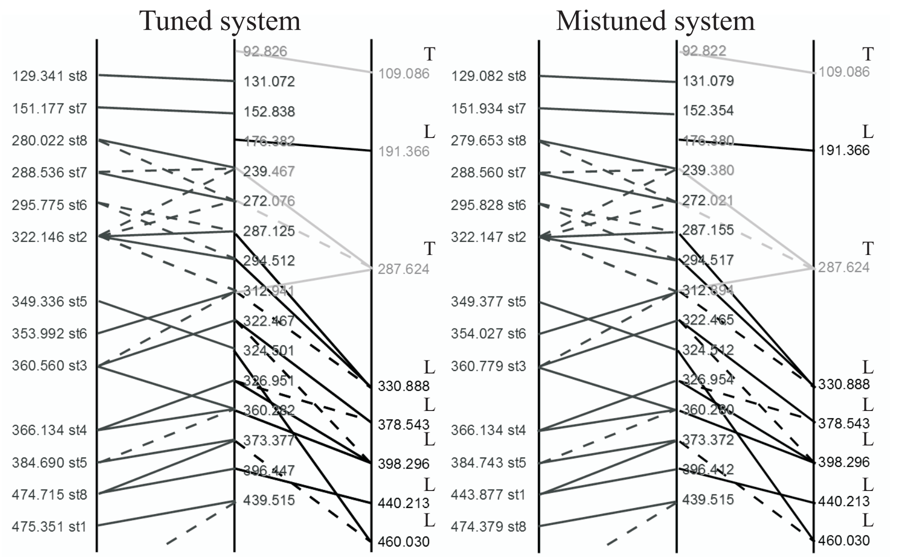

Figure 2 Natural frequencies of tuned bladed discs on the shaft corresponding to zero nodal diameter modes for both cases

There were either zero ($k = 0$), one ($k = l$), two ($k = 2$), or more ($k > 2$) nodal diameter bending or tensional modes. Series 1 was associated with the first natural frequency of the single cantilever blade; Series 2 was associated with the second natural frequency of the single cantilever blade, and so on.

Figs 2, 3 and 4 present the natural frequencies of the tuned (left side) and mistuned (right side) bladed discs on the shaft system corresponding accordingly to zero, one and two nodal diameters. The right axis indicates the natural frequencies of the eight discs (without blades) on the shaft, the middle axis shows the natural frequencies of the tuned and mistuned bladed discs on the shaft, while the left axis presents the uncoupled natural frequencies of bladed discs corresponding to an appropriate nodal diameter. The "st No." show the number of the stages of a particular bladed disc. The longitudinal are marked as L, the torsional modes as T, while the bending bladed disc modes remained unmarked. The bending shaft dominated modes are marked as B. The lines connecting the natural frequencies of the investigated systems are divided into two types: the solid lines indicate strong coupling and the dashed lines weaker coupling. A detailed description of couplings for a tuned system was presented in [6].

Natural frequencies of cantilever blades, single bladed discs and the complete shaft with eight discs were carefully examined in both cases to find resonance conditions and coupling effects. Analyses of tuned and mistuned systems showed that despite the small changes in frequencies, the couplings for zero (Fig. 3), one (Fig. 4) or two nodal diameters modes (Fig. 5) did not change. The split in the one and two nodal diameter coupling frequencies that appears in the mistuned system diagram (Fig. 4 and Fig. 5) is an effect of blade mistuning.

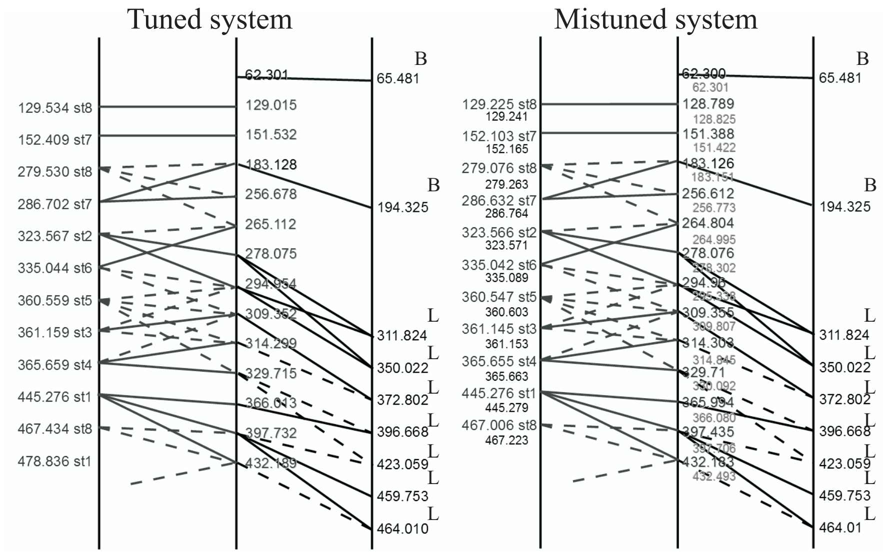

Figure 3 Natural frequencies of tuned bladed discs on the shaft corresponding to one nodal diameter modes for both cases

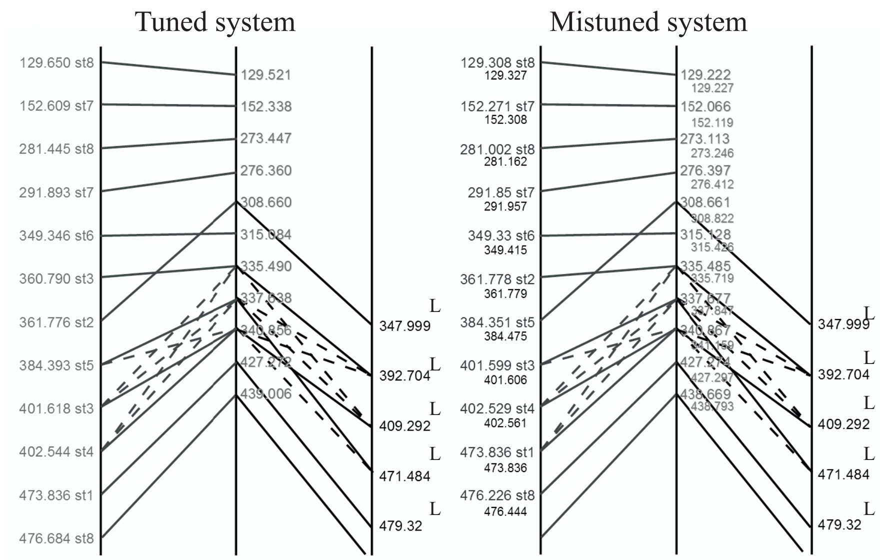

Figure 4 Natural frequencies of tuned and mistuned bladed discs on the shaft corresponding to two nodal diameter modes

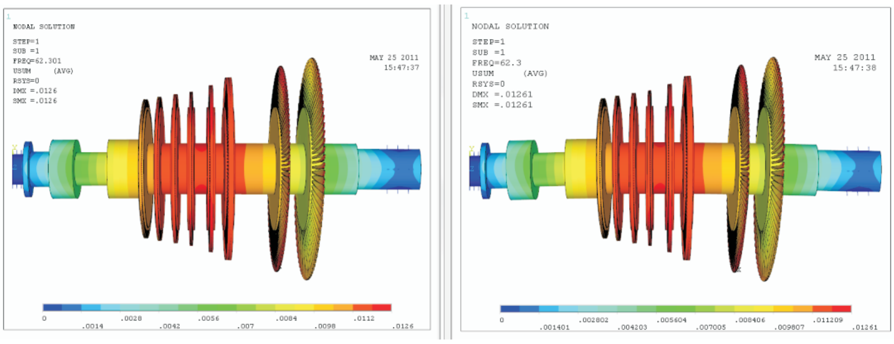

Figure 5 Comparison of mode shapes of tuned and mistuned bladed discs on shaft corresponding to the bending mode of the shaft at a frequency of 62.3 Hz

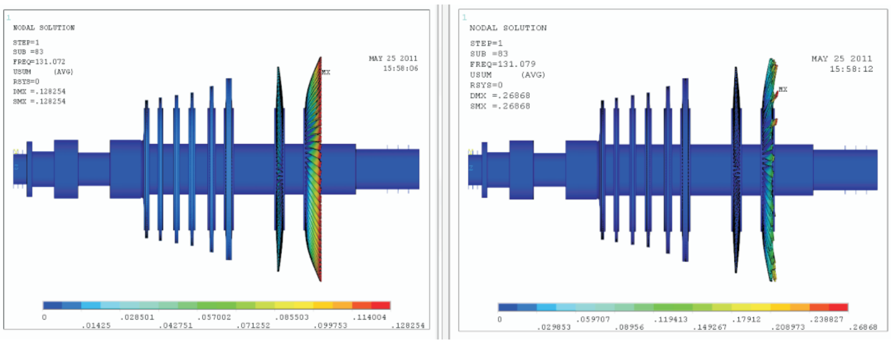

Figure 6 Comparison of mode shapes of tuned and mistuned bladed discs on shaft corresponding to the zero nodal diameter on 8th stage at a frequency of 131 Hz

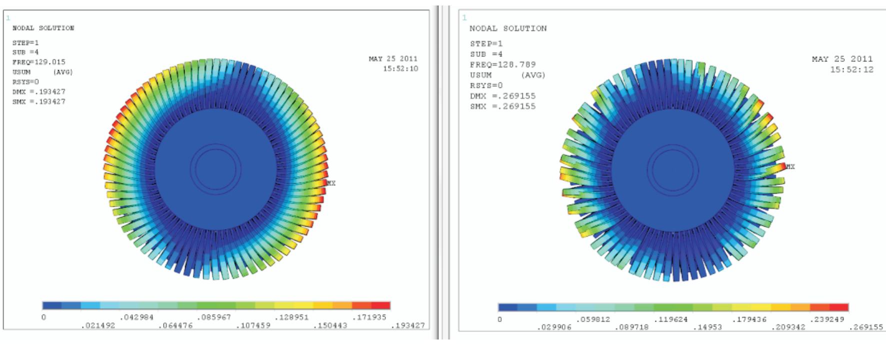

Figure 7 Comparison of mode shapes of the 8th stage bladed disc on shaft corresponding to the one nodal diameter at a frequency of 129 Hz for both cases

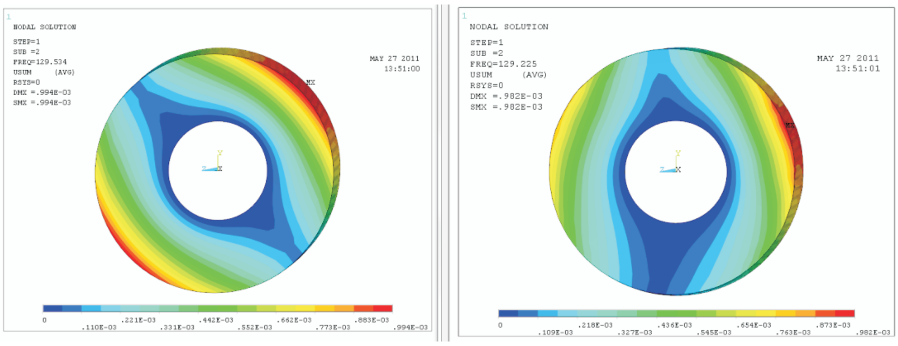

Figure 8 Comparison of mode shapes of the 8th stage tuned and mistuned bladed discs without shaft (blades are hidden) corresponding to the one nodal diameter at a frequency of 129 Hz

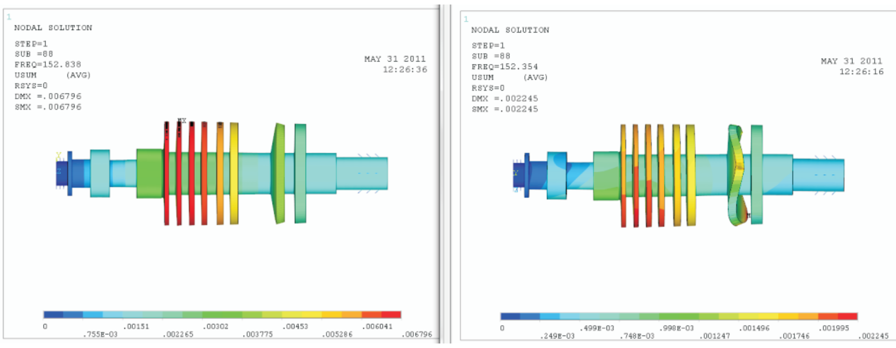

Figure 9 Comparison of mode shapes of the 7th stage discs on shaft (blades are hidden) corresponding to the zero nodal diameter at a frequency of 152 Hz for both cases

4. Forced vibration

Bladed disc eight was excited by 0EO and 1EO with an excitation frequency ranging from 50 to 450 Hz. The modal damping for all frequencies was assumed to be equal to 0.5%. For both cases the stress amplitudes were calculated for every blade on each stage.

Significant couplings that occurred in a tuned system were presented in [6]. Our calculations show that there is no striking difference in multistage coupling between tuned and mistuned bladed discs on a shaft in the case of 1% mistuning.

Fig. 10 presents a comparison of the stress amplitude results for the 8^{th} bladed disk: the thick line indicates the amplitude for tuned system, while the thin lines represent the response for the mistuned system. The amplitude response of the tuned system is the same for every blade in a given stage, reaching ∼1600 MPa for 0EO excitation (left) and ∼1800 for 1EO excitation (right). In the case of the mistuned system the differences were only significant in the exited stage. For example, at a frequency of ~131 Hz for 0EO and ~129 Hz for 1EO.

Fig. 11 focuses on the near resonance response of the 8^th stage bladed disc at a range of 128 to 133Hz. The values of stress amplitudes in the mistuned system vary from 1000MPa to 2500MPa for 0EO and 1000MPa to 2400MPa for 1EO. The observed shift of amplitude peaks was the result of blade mistuning. To achieve the 1% frequency mistuning, the blade Young modulus was modified to 2% range. Due to the coupling of the mistuned bladed discs through the shaft, the blade stress amplitude values increased by 75%.

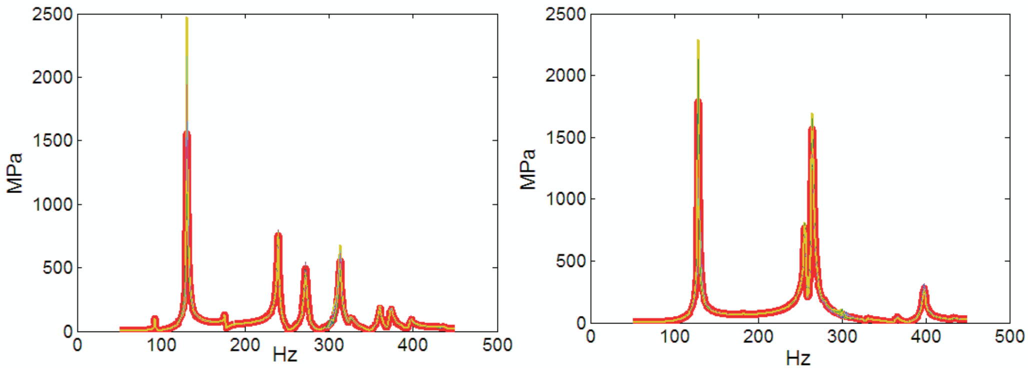

Figure 10 The response of the 8^{th} stage when the bladed disc is excited by 0EO (left) and 1EO (right) for both cases in the wide range of 50-450Hz

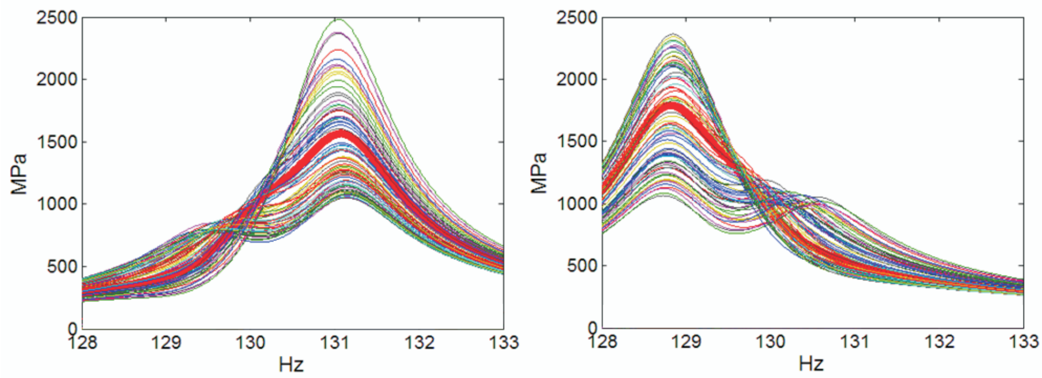

Figure 11 The response of the 8^{th} stage when the bladed disc is excited by 0EO (left) and 1EO (right diagram) for both cases in the narrow range of 128-133 Hz

5. Conclusions

The results obtained for a system with 1% mistuning clearly show that such a small mistuning does not influence the couplings between bladed discs.

Applied mistuning mainly affects the resonance stress amplitudes of the excited bladed disc. The intensity of this effect dissipates with higher EO excitation.

Applied mistuning does not change the stress amplitudes on discs other than the exited one.

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