Multilevel Optimization of the Semi–Open Impeller in a Centrifugal Pump

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Full optimization task for the case of the semi–open impeller with straight blades, requires description of its' geometry by means of, at least, eighteen design variables. In the case of constant meridional cross-section, are required at least eight design variables. Solution of the task with such great vector of design variables requires much more time. One way to manage to obtain a solutions with great variety of design variables is comprehensive approach to the task depending on the partition into minor subtasks.

After decomposing the optimization task, one should choose procedure of solving it. One of these procedures is parametric optimization, which is two-stage method of minimizing (maximizing). This optimization is carried on in two levels. On a lower level, multi–optimization of decomposed parts of the tasks, depending on design variables, is being held. The solution of the lower level is used in an upper level (coordinating level) to find optimal coordination variables.

It has been shown that result of multilevel optimization and full task optimization is the same in limits of accepted accuracy of the objective functions' calculus. Time of the calculation for the multilevel optimization task is over four times shorter than undecomposed task time.

Keywords: Semi–open impeller, multilevel optimization

1. Introduction

Design methods published in the available literature data concern machines which work in nominal point. Input data in calculus procedures are nominal point parameters: pump head H_N and capacity Q_N. The point is described by maximum value of total efficiency $\eta_N = \eta_{max}$. The reference level for calculated efficiency of designed pumps are efficiencies of pumps already existing. Empirical formulas allow to determine efficiencies in output data, however they do not answer if could reach higher values. This fact was a direct cause of inserting algorithm into design,

optimization methods describing objective function(efficiency) extreme determined on the base of numerical calculus, results of velocity and pressure fields. Oftentimes the additional demand is the accomplishment of pump of the minimal available NPSH in surrounding of its' nominal work point. Due to that the second objective function was minimal of the available NPSH of pumps' first stage.

Methodology of proceeding in the design process for engineering constructions is subordinated by practical consideration, which aim is to achieve some real solutions. Activities tending to receive the optimal solution in sense of Pareto should contain:

- Normalization of partial objective functions, which represents the essential treatment in case when partial criteria of evaluation are presented in different units or differ in scale value. Normalization is an essential procedure.

- Scalarization of normalized objective function with use of weight factors.

- Determination of compromised solutions set.

- Determination of favorite function set.

- Selection from a set of compromised solution, the subset of favorite solutions.

- Coming to a decision in choosing of the best solution.

Scalarization of a vector–valued objective function is a beneficial treatment leading to receive of practical solutions in optimization tasks. Thanks to scalarization, the multi-objective task is transformed to single-objective task.

It determines a task simplification, however it is often used in practice. Scalarization is proceeded on the base of the following formula:

$$\overline{F}(x) = \sum_{i=1}^{N} w_i \overline{\overline{F}}_i(x) \tag{1}$$

where:

w_i – weight factor of i–th partial objective function

N – number of partial objective functions

$\overline{F}_i(x)$ – normalized partial objective functions.

In engineering practice there are oftentimes met the tasks of bi–optimization (uses two factors). The choice of weight factor the most often results from primary objective function which appears as the cost of manufacturing and exploitation of construction. Giving its' real value is impossible without information about conditions in which construction has to work (it is pump in this very case): what is the working period, which material was used, etc.

Designer often gives a set of solutions for different weight factors, so called Pareto optimal set and decision is undertaken by decision-maker on the base of information, analysis and design problem definition [11].

2.   Proceeding in impellers pump design

In the article, the design method of semi–open impellers was described with special regard of these with low specific speed ($n_q < 20$). Main dimensions of impeller (see Fig. 1) are designated with method based on the one dimension flow theory, completed with empirical relationships and dependences, determined by own research.
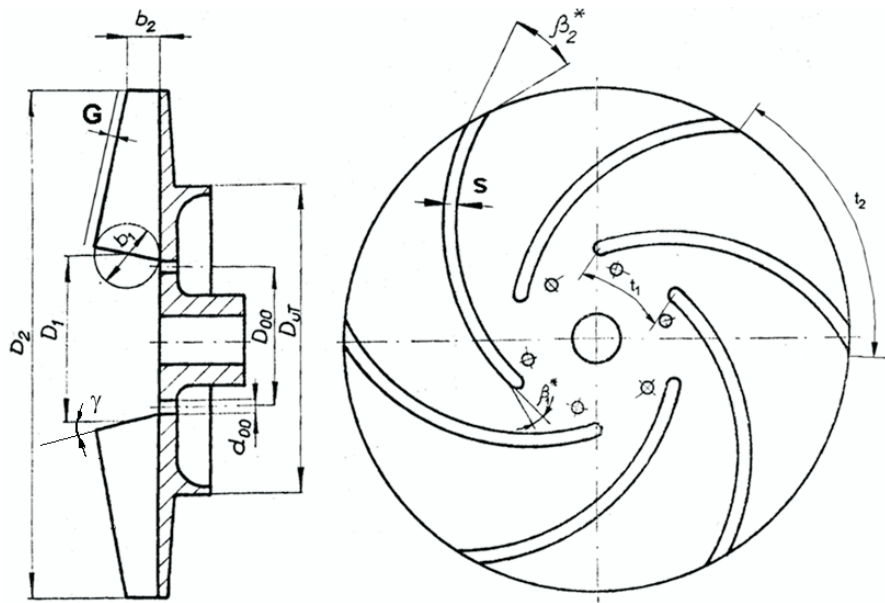

Figure 1 Geometry of semi–open impeller – head dimension

In the case of semi–open impellers, one–dimensional theory is completed with elements describing phenomena conducted to existence of non–rotating shroud and blade tip (see Fig.1).

Results of one–dimensional calculus establish basis of the next design stage based on the three–dimensional flow analysis, during which, the local fluid parameters (velocity and pressure) are calculated.

Whilst then, observed disadvantageous hydraulic phenomena e.g. vortex zone (see Fig. 2), which increments the hydraulic loss, cannot be identified by means of 1–D method, which establishes base of correction shapes in blade and meridional channel profile.

In such case, calculations are executed once more until satisfactory results of calculus are obtained, alongside with design assumptions.

Such "traditional" use of 3–D methods requires capital knowledge and engineer institution as for designer.

This does not guarantee maximal available efficiency, yet only better than obtained in calculations based on 1–D model.
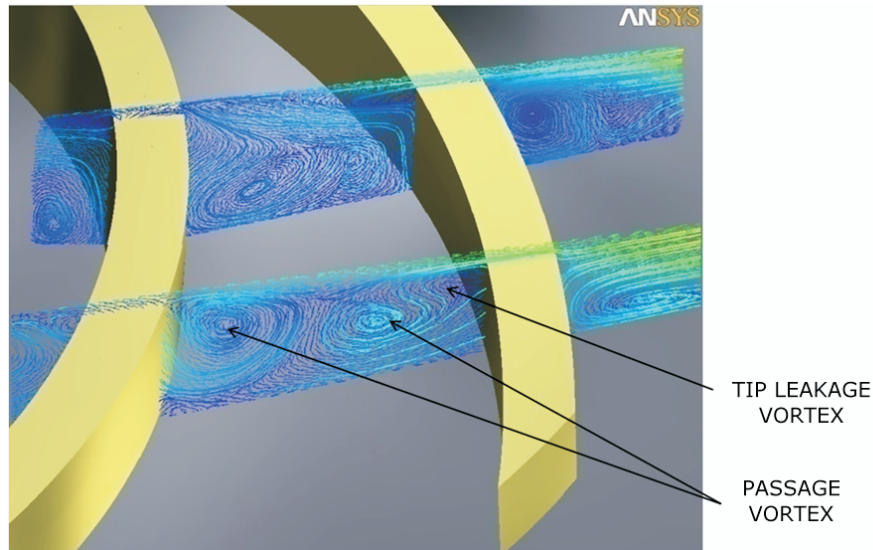
Figure 2 Secondary flows view received during RANS flow calculus

Formally, the introduction of global optimization methods by definition of objective function on the base of results, laying out 3–D flows using numerical methods for fluid mechanics, allows to obtain the highest possible efficiencies under the given limits.

Hydraulic loss in channels decides on pumps' efficiencies. The reference level for calculated efficiencies of designed pumps are efficiencies of already existing machines [7].

Empirical formulas allow to determine efficiencies bounded with output data H_N, Q_N for pump design, however they do not answer if could reach higher values. This fact was a direct cause of inserting algorithm into design, optimization methods describing objective function(efficiency) extreme determined on the base of numerical calculus, results of velocity and pressure fields.

Pumps should also be characterized by non–cavitational work in whole range of efficiency change. Impellers are usually designed with nominal parameters of pump work taken into account. It does not guarantee work of impellers without cavitation in different nominal efficiencies.

And that is why the second objective function in this method is the minimal region of the vapour phase in the inlet part of the impeller, which is the standard for NPSH. Both functions are defined on the set of the decisional parameters, limited with standardized design parameters, such as: thickness of a blade resulting from manufacturing technology and strength of a material and flowability (such as head of pump lift). Oftentimes, one of the dimensional limits are impellers' development

pump casing. In the case of modernisation, often the pump casing is a certain limit, which with economical regards, has to stay unchanged. This makes limitations in dimensions of impellers' construction.

Objective functions in this method are calculated on the base of results, obtained from solving the task for double–phase RANS flow, determined with ANSYS–CFX software.

Before undertaking the decision about one of methods choice of optimization there were investigated some characteristics for given parameters of both objective functions.

In the result of this analysis it was affirmed that both of them have a form of non–smooth functions [15], [16]. For smooth function there was a random noisy function applied.

For such objective function the most often, gradient optimization algorithms are used (ex. Broyden–Fletcher–Goldfarb–Shanno algorithm), they could not find global extreme and stop in one of many local minima.

As the tool for optimization, there were applied algorithms for evaluation of global extreme of non–smooth functions.

It was practiced a Implicit Filtering [4], [9] algorithm, among others, which appeared to be useful for a small number of decision variables and aside from chosen initial point, finds the global extreme of the objective function. In numerical verification procedure, there was used a Direct Search algorithm [9] and Genetic Algorithm [5], which is more effectible and guarantees the evaluation of global extreme, usually for bigger number of decision parameters than 10 [12].

Verification should be understood as the process defining whether implementation of model represenst exactly conceptual description and its' solution [1].

It estimates accurately, how many equations of model were solved with computer software and does not define if the model has anything to do with reality.

3. Multilevel optimization algorithms

One of the ways in solving tasks with many decision variables is a system–defined approach to the problem, which consists in partition of it into the minor subtasks.

In engineering problems, this division is being proceeded in natural way by explicit between separate subsystems.

That kind of approach is named a multilevel optimization. The Expenditure of calculations can be considerably reduced if the task is being divined into subtasks which are solved separately.

Partial tasks are connected within a certain relation between each other with one parent task. It is used to be named decomposition and coordination [6], [11].

In theoretical point of view, decomposition and coordination relies on replacement of the original problem, another problem is used to call a decomposed problem.

The specific feature is the conflict between primary system and subsystems due to the fact that sum of subsystem solutions is not the solution of the original problem. Coordination depend on the specific dependence between these subsystems.

To solve the original problem using the decomposition one should divine the x vector into two components: the vector of coordinating variables y and the vector of decomposed task decision variables z.

The vector of decomposed decision variables we divine into a set of subsystems $z_1, \ldots, z_i, \ldots, z_N$ with respect to objective functions set p_i.

$$\min_{\substack{\mathbf{x}=(\mathbf{y},\mathbf{z})\in X}} \mathrm{Q}(\mathbf{x}) = \min_{\substack{\mathbf{y}\in Y, \mathbf{z}\in Z \\ (Y\times Z)\in X}} \mathrm{Q}(\mathbf{y},\mathbf{z}) = \min_{\substack{\mathbf{y}\in Y, \mathbf{z^1}\in Z^1(\mathbf{y})}} \Phi\left[p_1(\mathbf{y},\mathbf{z^1}), \ldots, p_N(\mathbf{y},\mathbf{z^N})\right] \qquad (2)$$

where:

$\mathbf{x}$ – vector of decision variables

X – set of acceptable solutions

$\mathbf{y}$ – vector of coordination variables

Y – set of acceptable coordination task solutions

$\mathbf{z}$ – vector of decomposed decision variables

Z – set of acceptable subsystem solutions

$\mathbf{z}_1, \ldots, \mathbf{z}_i, \ldots, \mathbf{z}_N$ – separate parts of vector z

$Z_i(\mathrm{y})$ – set of vectors $\mathbf{z}_i$ dependent of vector y, i=1,2,...N

Q – objective function of decomposed task

Φ – global objective function of decomposed task

p_i – objective function of subsystem, i=1,2,...,N

After decomposing the optimization problem one should choose methodology of task solution. One of such s is the parametric optimization method [6], [11] which is the two-stage method of minimizing (maximizing).

This optimization depends on the fact that it is executed in two stages shown in Fig. 3. On the lower level there is the multiple optimization of decomposed parts of the task carried out, in relation to their decision variables. The result of optimization on the lower stage is used on the upper level (coordinative) used in finding the optimal values of coordinative variables.
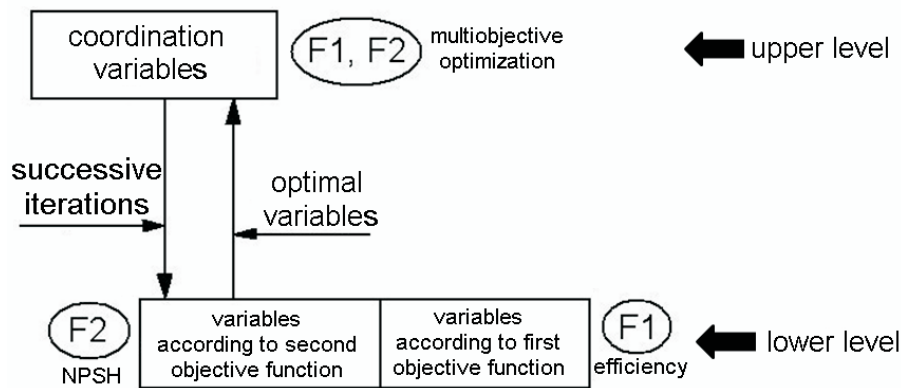

Figure 3 Scheme of multilevel parametric optimization

3.1. *Multilevel optimization in applications for pump impellers*

In the case which concerns design with pumps' impeller optimization, this division of the task into subsystems is easy to notice.

With respect to suction (cavitational) properties of impeller the most significant are the following parameters describing blades' shape (Fig. 1):

- inlet blade angle – β_1^*,

- both coordinates of third–order Bezier control point which describe blade angle – $\%M'$, β^* (see Fig. 4) ,where $\%M'$ is non–dimensional blade length in meridional section,

- shroud gap width G,

- tblade leading edge inlet diameter – D_1,

- lean blades angle in meridional section – γ,

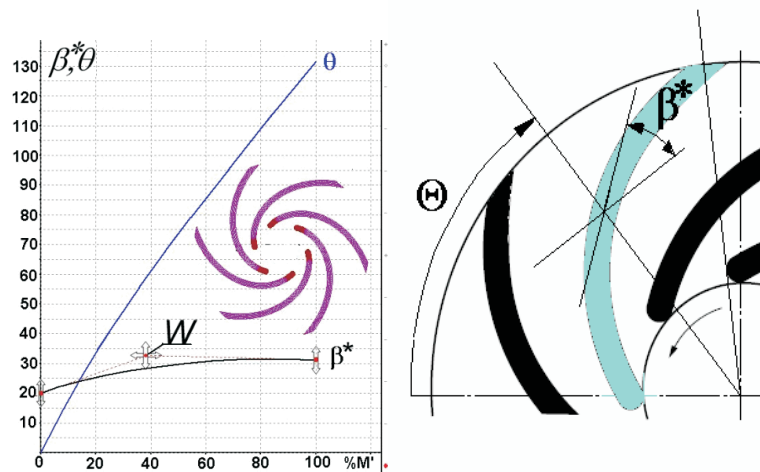- tblade leading edge width – b_1,

- number of blades – z.

Figure 4 Blades' shape described with tracing angle β^*

Due to the efficiency, the most significant meaning have the lower exchanged parameters describing the blades' shape:

- outlet blade angle – β_2^*,

- both coordinates of third-order Bezier control point which describe blade angle tracing – $\%M'$, β^*,

- shroud gap width – G,

- blade trailing edge diameter – D_2,

- blade leading edge width – b_1,

- blade trailing edge width – b_2,

- number of blades – z.

Some of the higher exchanged parameters have the significance, for efficiency, as well as for suction characteristics.

These parameters are:

- shroud gap width – G,

- both coordinates of third-order Bezier control point which describe blade angle tracing – $\%M'$, β^*,

- blade leading edge width – b_1,

- number of blades – z.

These variables function as coordinative in method of multilevel optimization.

The number of blades, due to the fact, that it could accept only the integer values and it appears the small range of change for this parameter, was subtracted from the set of decision variables and treated as the primary variable to find out if the solution for the number of blades z-1 and z+1 gives the better solution. In analyzed for this case value z=6 was the optimal value.

Rest of the values is treated as parameters and set on the base of one–dimensional method.

Obviously, during the increase of available computational power there is a possibility to increase a vector of decision variables.

Additionally, due to the later validation, the width in inlet and in outlet of the blade palisade was subtracted from the set of decision variables. It was induced with the change of meridional profile would influence on the change of the casing of measuring head and significantly increase the costs of research (validation).

This very case suites the shape optimization of the impeller, without the change of the rest pump components, especially the pump casing.

Within such described optimization task one have eight decision variables determining the shape of impellers' blade.

Taking into account, that each of the eight decision variables In the range of acceptable change (in the unit area) receive 10 values, which is the number of acceptable combinations of different geometry, that is so called the power of the acceptable set of solutions for one weight value equal to 10^8.
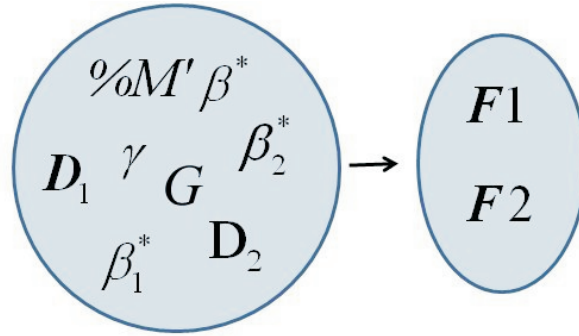
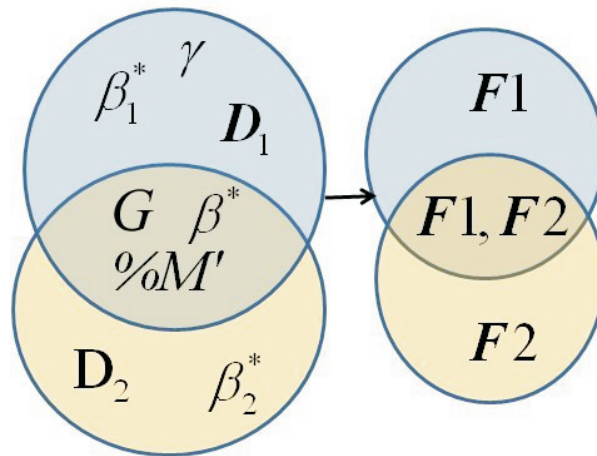**Figure 5** Onelevel optimization

Figure 6 Multilevel optimization

If one makes the decomposition (partition) of the task into two stages, depending on the influence of defined objective functions (Fig. 6):

1. Three decision variables β_1^*, D_1, γ having the highest influence on the first objective function. It is the suction criteria.

2. Two decision variables β_2^*, D_2 having significant influence on the second objective function (efficiency criteria).

3. Coordinative variables G, $\%M'$, β^* having the influence on both of objective functions.

If just like before, coordinative variables will receive only 10 values, the count of the acceptable set of solutions will evaluate $10^3 \cdot (10^2 + 10^3)$. This number is 727 times lower than count of the undecomposed set.

There was the following multilevel optimization algorithm processed (Fig. 3):

1. There was established the value of coordinative values G, β^*, $\%M'$ according to values obtained with 1–D method

2. There was established an optimization for the first objective function that is the NPSH characteristics in relation to decision variables γ, D_1, β_1^*. Decision variables D_2, β_2^* were taken from previous iterative stage (for the fist it was used the 1–D method).

3. Secondly there was done an optimization for the second objective function that is the efficiency in relation to decision variables D_2, β_2^*. Decision variables γ, D_1, β_1^* were left with values obtained In the point 2 of this very algorithm.

4. In the upper coordinative stage, it was the multi–criteria optimization established with the weight coefficient equal to 0.5.

5. Calculations were repeated from the point 2 until the moment of standardized objective function, between actual and previous iteration was smaller than assumed accuracy. The taken accuracy was 0,01. This value was determined in an experimental way.

Higher accuracy caused certain changes resulting from objective function, which is burdened with random perturbation.

The comparison between multilevel optimization and original optimization task results was shown in Pareto sense on the Fig. 7.

4.   Validation of desidn method

Received numerical solution is based on the double–phase RANS flow model. It was established on the grid compromising between accuracy and the rate of the calculations. On the research stand (see Fig. 8) there were done some measurements aiming the comparison between optimized impellers' parameters with the real values and comparison of parameters of traditional one–dimensional method, and that one using the multilevel optimization method. The result of optimization was subject for final experimental verification (validation) comparind real parameters of pump with the impeller designed on the base of one dimension method using optimization.

Validation results of the impeller net positive suction head and the efficiency of the pump with the designed impeller, based on the one–dimensional (1–D) flow theory and the optimized impeller, have been shown on the plots 9 and 10.

Results of the research are presented on the plots 11 and 12. The real efficiency of pump with optimized impellers is higher in 3,5%, yet the net positive suction head (NPSH3) is lower at about 0.65 m.
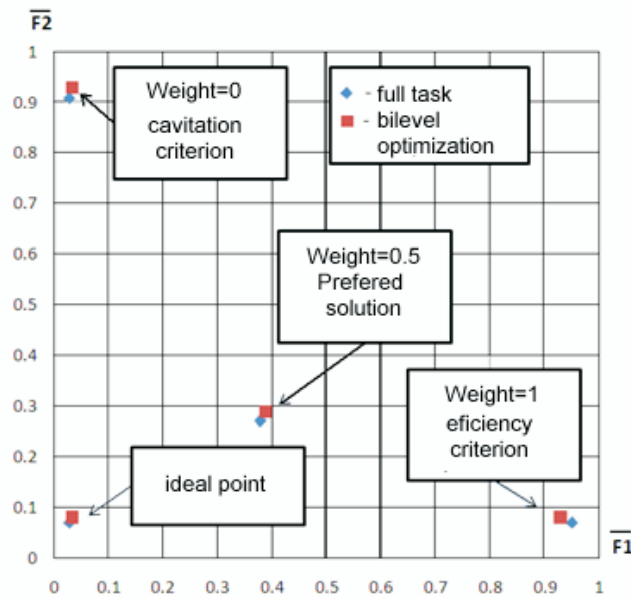
Figure 7 Value of partial objective function $\overline{F}_1$, $\overline{F}_2$ for full task optimization and multilevel optimization. Partial objective functions were normalized with min-max method

Figure 8 General view of test stand for semi–open impeller research in Institute of Turbomachinery Technilal University of Łódź
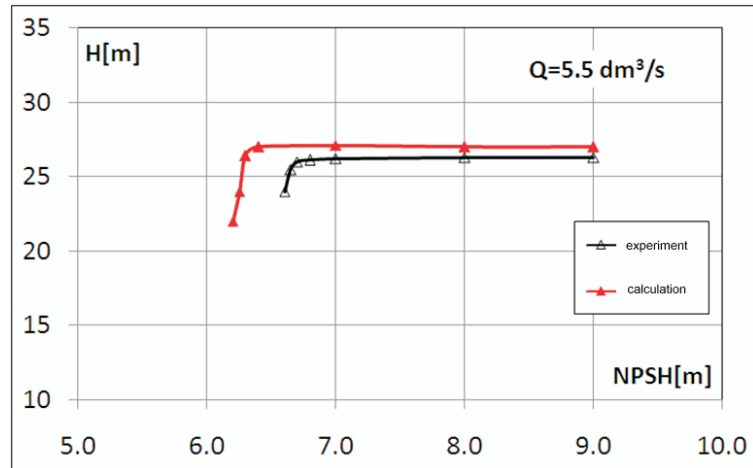
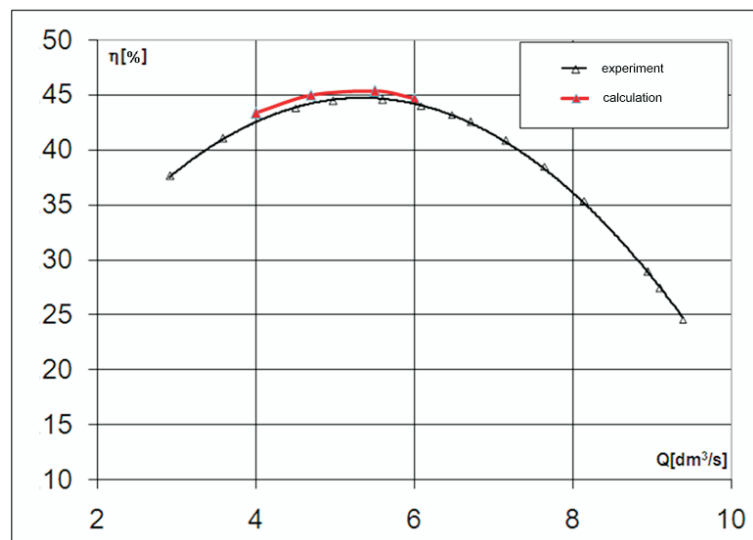**Figure 9** Comparison calculated and measurement results of NPSH characteristics

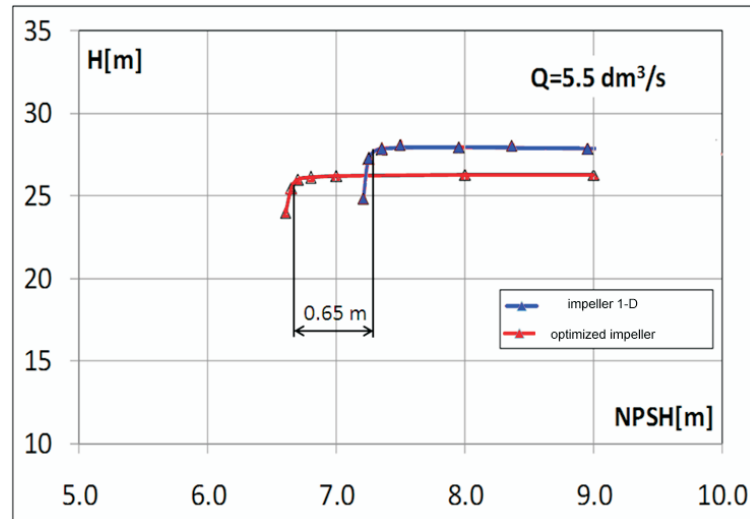**Figure 10** Comparison of calculus and measurement results of efficiency

Figure 11 Experimental comparison of impellers' NPSH properness designed by 1-D method and impeller designed with optimization method tool
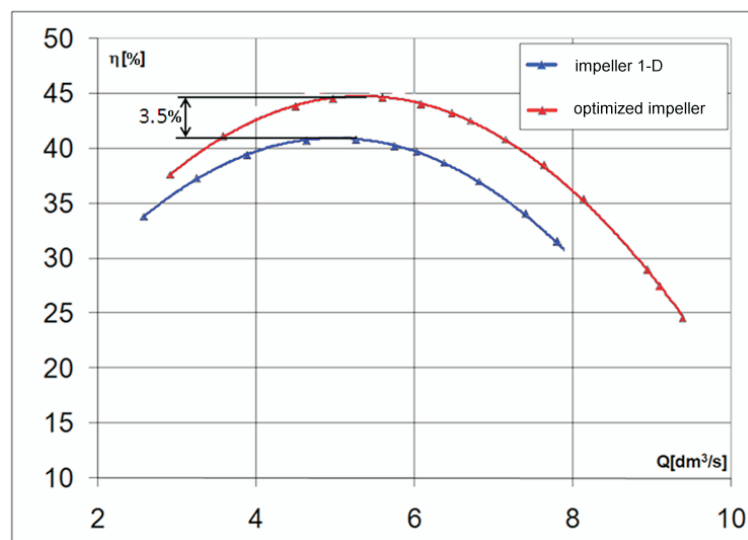

Figure 12 Experimental comparison of impeller efficiency designed using the 1–D method and impeller designed with optimization method tools

5.   Conclusions

It has been shown (Fig. 7) that result of multilevel optimization and full task optimization is the same in limits of accepted accuracy of the objective functions' calculus. Time of the calculation for the multilevel optimization task is over four times shorter then undecomposed task time period.

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