Mechanics and Mechanical Engineering
Vol. 16, No. 1 (2012) 15–24

Worm Gear's Kinematic Accuracy Measurement Methods

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Received (15 October 2012)
Revised (21 December 2012)
Accepted (3 January 2013)

The paper discusses measurements conditions of worm gear kinematic accuracy, including construction and operation principles of the kinematic deviation measuring stand. There are presented two methods of measuring gear kinematic accuracy: direct method - consisting in comparison of rotation angle of gear and worm wheel in the full range of rotation relatively to tooth elements; indirect method – consisting in measurement of worm wheel's circuital tooth scale. In addition, notions of kinematic deviation and kinematic diversion of worm gear were defined. Paper presents as well examples of kinematic errors results of A80 gear depending on adopted measuring step – a number of measurements per one full worm rotation. The result of research was determination of a minimal measuring step per full worm rotation L1 = 300, wider resolution did not cause significant changes of measurement result.

Keywords: Kinematic deviation, worm, worm wheel, worm gear

1. Introduction

Accuracy of rotation motion transmission of a gear's toothed elements is considered as little known, particularly from practical side of worm gear properties. At active gear shaft there is worm winding, whereas worm wheel is embedded at driven shaft. Accuracy of rotation motion transmission between shafts is used in gears called kinematic. Unlike to power gear, particular attention is paid to obtain accordance of actual motion with as–sumed motion [1], [2], [3]. Under the term of worm gear kinematic accuracy one should understand the correctness of mutual binding of worm wheel's active shaft motion with driven shaft motion. All errors of this motion result from execution errors and assembly errors of toothed elements as well as other elements of a gear. The Polish Norm [PN–80/M–88522.04] defines a range of performed elements deviation constituting a worm gear. According to this norm kinematic accuracy is determined on basis of a worm gear kinematic deviation.

Kinematic deviation is a difference of observed and nominal angle of worm wheel rotation in one – sided meshing with worm in assembled gear, measured as length of a divisional circular ar. It is determined at full rotation of worm wheel. The biggest algebraic difference of a gear's kinematic deviation F'ior present in a full cycle of relative position change of a worm – wheel and a worm is named kinematic deviation.

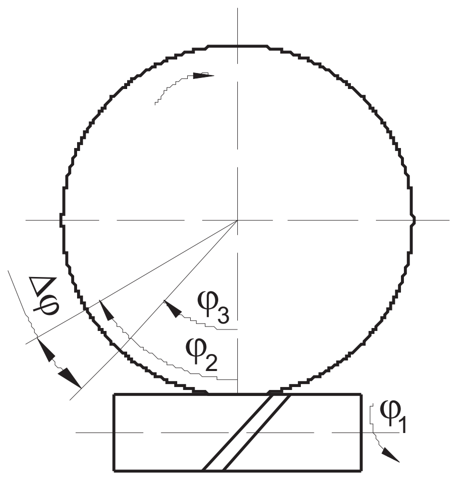

Figure 1 Kinematic deviation of worm gear

where: φ_1 – given worm rotation angle,

φ_2 – observed worm wheel rotation angle,

φ_3 – nominal rotation angle of worm wheel

$$\varphi_3 = \varphi_1 \frac{z_1}{z_2} \tag{1}$$

z_1 – number of a worm coil z_2 – number of a worm wheel tooth $\Delta\varphi$ – kinematic deviation of worm gear ["] .

$$\Delta\varphi = (\varphi_2 - \varphi_3) \tag{2}$$

This Having possibility of registering the size of kinematic deviation in a full cycle of relative position change of a worm – wheel and a worm, a graph in a form shown in Fig. 2 is the most frequently obtained.

Presenting kinematic deviation in a radar graph allows to display transmission variation between particular worm gear's tooth, however its hinders determination of a kinematic deviation F'_{ior} Fig. 3.

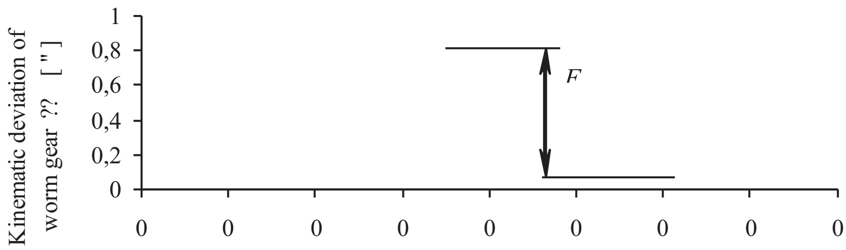

Figure 2 Exemplary variation of kinematic deviation of worm gear

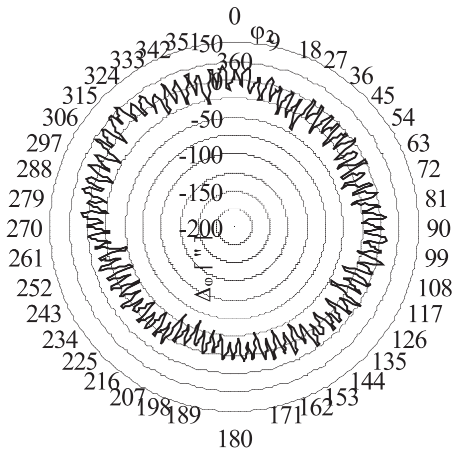

Figure 3 Exemplary variation of a worm gear kinematic deviation in a function of a worm - wheel rotation angle 2 presented in a radar graph

Kinematic accuracy is influenced by many phenomena that are generated during the manufacturing process. This process did not change in principle since many years. What only changed are the manufacturing possibilities resulting from common usage of numerically controlled machine. Apart controlling possibilities improved by measurement conducted at coordinate machine. Literature [1], [3] discusses few methods of measurement of gear hobbing machines' kinematic accuracy and parameter named position accuracy. Accuracy achieved with usage of mentioned measurement methods does not exceed however few minutes, what in present - day level of technology is insufficient. Measurement of circular pitches of worm wheel tooth is often used to determine quickly correctness of toothed elements execution

It is acknowledged that technology of worms production does not lead to generation of many serious errors, while the most serious errors are generated during worm wheel tooth execution. Basing on the measurement of worm wheel circular pitches a graph of cumulative pitch error is created. Its maximum value is used to determine kinematic deviation of worm wheel s according to the relationship:

$$\varphi_s = \arctan\left[\frac{f_{pk}}{9mz_2)/2}\right][''] \tag{3}$$

where:

f_{pk} – maximum value of cumulative error $k = z_2$ of circular pitches in [mm],

m – axis module of mesh in [mm],

z_2 – number of worm wheel tooth.

Conversion of cumulative error value of circular pitch into maximum kinematic deviation of worm wheel s is presented in a work [5], [4]. The best method currently taking into account all execution errors is the method of rotation angle comparison of worm and worm wheel in full range of relative position change of toothed elements of a gear. This method relies on measurement of worm shaft given rotation angle and observed rotation angle of worm wheel. Measurement is taken at assembled gear. Electronic converters of angle position are placed at active and driven shaft. A value of real shaft position is transmitted to measuring system – meter and further to a computer. Knowing a value of a measuring step one is able to determine periodical deviations as well as deviations at given rotation angle of worm and kinematic deviation occurring at full cycle of relative position change of a worm and a worm – wheel.

2. Experimental research

During one full rotation of a worm's wheel worm does $n = i$ rotations where i gear ratio. Defining φ_2 as observed rotation angle of a worm's wheel and φ_1 as given rotation angle during change of relative position of a worm and worm wheel it can be observed. The difference between nominal φ_3 and observed φ_2 rotation angle of worm's wheel is determined basing on the registered values of real rotation angle of a worm φ_1 and a worm wheel φ_2. The biggest algebraic difference of those indications is a kinematic deviation F'_{ior} of a worm gear.

3. A stand for measurement of a worm gear's kinematic deviation

A measuring stand based on steel body of a gear was constructed aiming to determine worm gear's kinematic deviation F'_{ior}. Figure 4 presents A80 gear with relocation $i = 80$ assembled to measuring system.

4. Description of a measuring system

Signals of angular position of worm's shaft are transferred from reading heads G.O.1 and G.O.2 to interface DSi–1. Signals of angular positions of a worm wheel's shaft are transferred from reading heads G.O.3 and G.O.4 to interface DSi–2. Furthermore signals from interfaces DSi–1 and DSi–2 are transferred to 3 axis reading apparatus LP–3. Reading apparatus is equipped with interface of serial transmission RS 232C, that transmits real value of angular position of a worm and a worm wheel to a computer. Computer has installed a program that registers value of real rotation angle φ_1 and observed angle of a worm wheel φ_2.

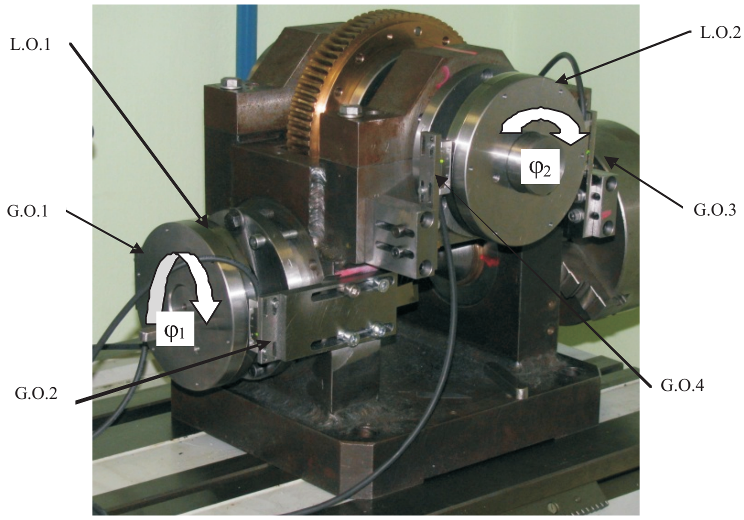

Figure 4 A gear A80 with assembled measuring system of rotation angle

Applied in a measuring system interface TONiC^{TM} of Renishaw takes reading of straight - edge scale interval division into 200 parts. Thus interpolations of signals with multiplier x200 are happening that give 11,11 impulses per 1" angular. Reading head TONiC^{TM} has a system of dynamic signal conversion, thus improvement of clarity and stability of signal is occurring ensuring ultra low cyclic error $< \pm 30$ nm. Fig. 5. Presents scheme of a measuring system.

5. Kinematic accuracy measurement with Renishaw's company device

A worm gear of division should be characterized by constant position in range of full cycle of relative position change of a worm's and worm– wheel's shaft. Constant position ensures lack of acceleration of gear's units. Accelerations of gear's elements are connected with deviation of a gear's toothed elements execution.

Making assumption that :

φ_1 – given rotation angle of a worm, φ_2 – observed rotation angle of a worm wheel

i – gear ratio,

$\Delta\varphi$ – kinematic deviation of a worm gear

For a mechanism representing a worm gear equation can be written:

$$\Delta\varphi = \frac{\varphi_1}{i} - \varphi_2 \tag{4}$$

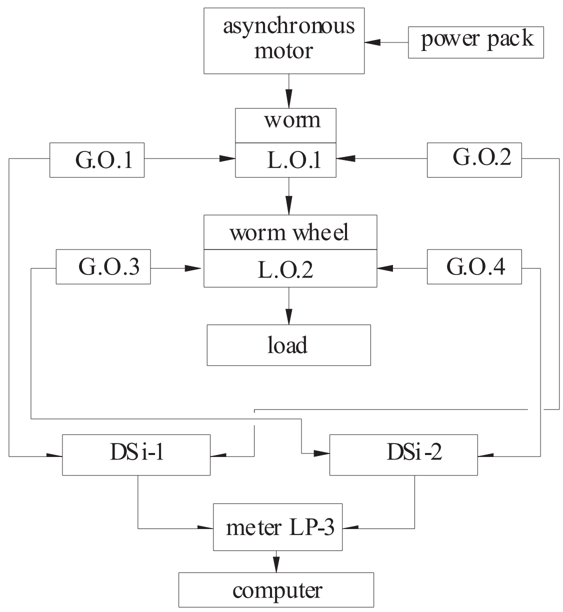

Figure 5 Scheme of a worm gear's kinematic deviation measuring system

For an ideal mechanism equation is:

$$\varphi_2 = \frac{\varphi_1}{i} \tag{5}$$

Concluding, kinematic accuracy can be determined on basis of comparison of a value of real rotation angle φ_2 of driven element (meaning worm – wheel) with a value of real rotation angle φ_1 of driven element of a worm in condition that cooperating worm is a control worm, nominal reciprocal distance of axis is maintained and there is onesided contact in mesh.

6. Measurements methodology

Worm's shaft was rotated during examination. Real value of rotation angle of worm's φ_1 and worm – wheel's φ_2 shafts was registered with a measuring system. There were three measuring steps taken for worm's shaft during system testing. The first of value $L_1 = 360^0$, the second $L_1 = 30^0$, the third $L_1 = 1^0$. Measuring step is an angular distance between subsequent measurements. Basing on it measuring system registers value of angular position of a worm and corresponding to it angular position of a worm – wheel. Tab. 2 presents number of measurements occurring per full cycle of reciprocal position change of a worm and a worm – wheel for a measuring step $L_1 = 360^0$, $L_1 = 30^0$, $L_1 = 1^0$, for a gear A 80 of ratio $i = 80$.

Measuring system registers rotation angle of a worm φ_1 every 30^0 for measuring step $L_1 = 30^0$ as shown in table 2. System takes 12 measurements in the range of full worm's rotation $0^0 \div 360^0$. Worm performs 80 rotatations for a gear A 80 that

Table 1 Features of a measuring system

<table>
<tr><td>Element of System</td><td>Ring of angle position converter [L.0.1], [L.0.2]</td><td>Reading head [G.0.1], [G.0.2]</td><td>Interface</td><td>Interface [DSi-1], [DSi-2]</td><td>Meter</td></tr>
<tr><td>Type</td><td>RESM20USA115 Diameter ∅ 115mm</td><td>TONIC READHEAD 5M RESM T2011-50A</td><td>TONIC INTERFACE x200 (4MHz) TI0200A04A</td><td>DSI Signum interface DSI-QTL4</td><td>[LP3] LABSTER</td></tr>
<tr><td rowspan="2">Features</td><td rowspan="2">Ring diameter ϕ115mm. Scale interval 20μm. There are 1800 scale interwal at ring circuit.</td><td colspan="2">Divides ring's scale interval into 200 parts
20 μm : 200 = 0,1 μm</td><td rowspan="2">Sums signals from reading heads G.0.1, G.0.2 ; G.0.3, G.0.4</td><td rowspan="2">3 – axis Display</td></tr>
<tr><td colspan="2">Cyclic error < ± 30 nm,</td></tr>
<tr><td>System resolution</td><td colspan="5">18000x200x4=14400000 impulses per one full rotation of worm wheel 14400000:(360^{o}x 60'x60")= 11,11 impulses per 1" angular</td></tr>
<tr><td colspan="6">According to Renishaw company system ensured measurement with accuracy +/- 1,74"</td></tr>
</table>

Table 2 Number of measurement per full cycle of relative position change of a worm and a worm - wheel for measuring step L_1=360^0, L_1=30^0, L_1=1^0

Measuring step at worm shaft [L_1]	Number of measurement of worm's actual rotation angle per one full rotation of a worm$\varphi_1 = 360^0$ [K]	Number of measurement of worm wheel's actual rotation angle per one full rotation of a worm wheel [L_2]
	K=$\varphi 1/L1$	$L2$=$K \cdot i$
360^0	1	80
30^0	12	960
1^0	360	2880

corresponds to one full rotation of a worm – wheel. This results in 12x80= 960 measuring points for full cycle of relative position change of a worm's and a worm – wheel's shaft. Figure 6 shows exemplary result of kinematic deviation measurement in function of rotation angle of a worm – wheel $\varphi_{2,}$ for measuring step $L_1 = 360^0$, $L_1 = 30^0$, $L_1 = 1^0$ for a gear A 80.

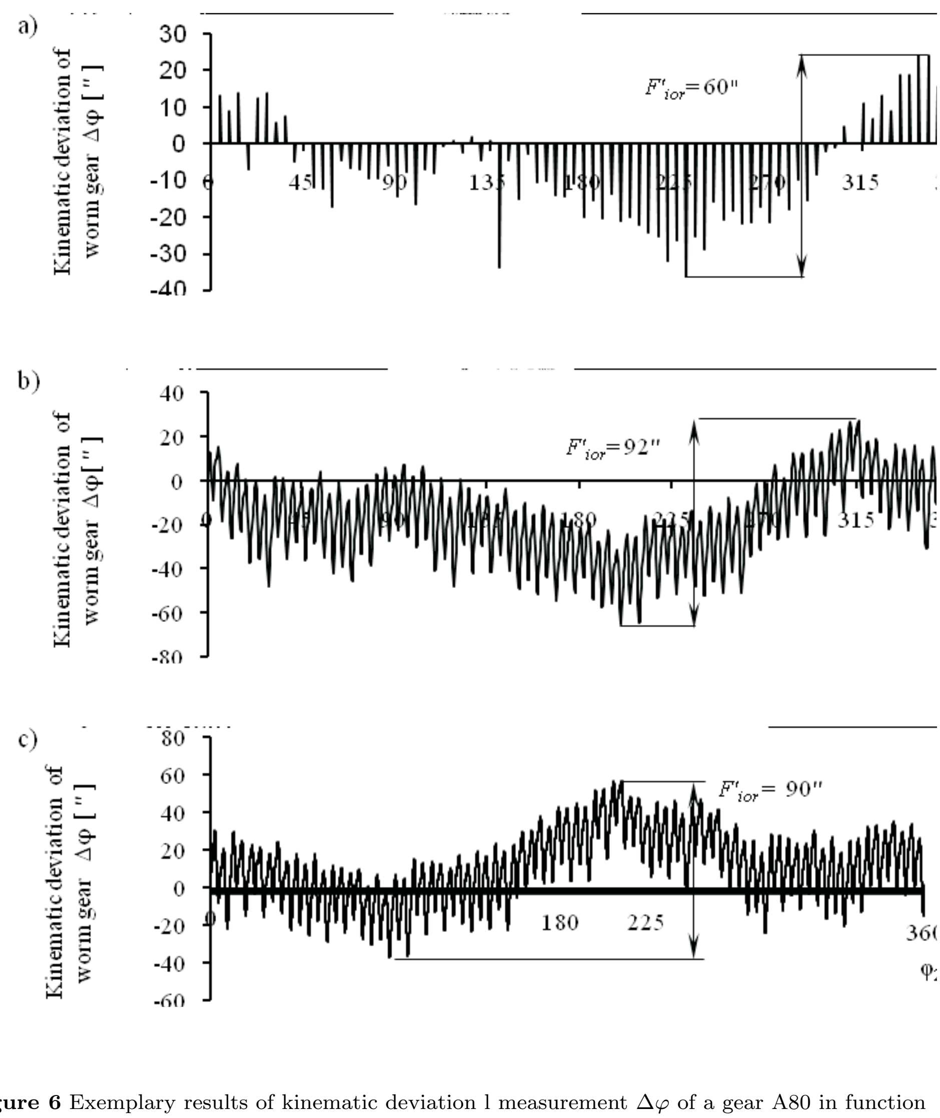

Figure 6 Exemplary results of kinematic deviation l measurement $\Delta\varphi$ of a gear A80 in function of worm wheel's rotation angle φ_2 for measuring step : a) $L_1 = 360^o$, b) $L_1 = 30^o$, c) $L_1 = 1^o$

After several attempts it appeared that change of measuring step L_1 value influence the resulting kinematic deviation. Limiting value in this case resulted to be step $L_1 = 30^0$. Measurements taken for values lower than $L_1 = 30^0$ did not change significantly the value of kinematic deviation. These results are presented in Tab. 3.

Table 3 Value of kinematic deviation of an A80 gear for different steps L_1.

Step[L_1]	Measuring range	Number of measuring points [L_2]	Value of a gear's kinematic deviation F'_{ior}
360^0	$0^0 \div 360^0$	80	60"
30^0	$0^0 \div 360^0$	960	92"
1^0	$0^0 \div 360^0$	2880	90"

As a next step measurement of kinematic deviation of a gear $A80$ was repeated in the range of one full cycle of relative position change of a worm and a worm – wheel and then two full cycles of relative position change for a measuring step $L_1 = 30^0$. Results are presented in Tab. 4.

Table 4 Measurement of kinematic deviation of a gear A80 for measuring step $L_1 = 30^0$.

Measurement no.	Measuring range	Number of measuring points [L_2]	Value of a gear's kinematic deviation of a gear F'_{ior}
1	$0^0 \div 360^0$	960	92"
2	$0^0 \div 720^0$	1920	91"
3	$360^0 \div 720^0$	960	90"

Basing on achieved results from Tab. 4, it was finally decided that for a gear A 80 kinematics deviation of worm gear measurements should be taken with a measuring step $L_1 = 30^0$.

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