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Numerical Representation of Bifurcation in the Process of Determining Stress Distributions in Post–Critical Deformation States of Aviation Load–Bearing Structures

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The study presents the results of research on the problem of obtaining credible results of nonlinear FEM analyses of thin–walled load–bearing structures subjected to post–critical loads. The similarity of numerical simulations results and actual stress distributions state depends on the correct numerical reproduction of bifurcations that occur during advanced deformations process.

Keywords: Thin–walled structures, nonlinear analysis, post–critical deformations, equilibrium path, Finite Elements Method.

1. Introduction

Modern aviation structures are characterised by widespread application of thin–shell load–bearing systems. The strict requirements with regard to the levels of transferred loads and the need to minimise a structure mass often become causes for accepting physical phenomena that in case of other structures are considered as inadmissible. An example of such an phenomenon is the loss of stability of shells that are parts of load–bearing structures, within the range of admissible loads.

Thus, an important stage in design work on an aircraft load-bearing structure is to determine stress distribution in the post–critical deformation state. One of the tools used to achieve this aim is nonlinear finite elements method analysis. The assessment of the reliability of the results thus obtained is based on the solution uniqueness rule, according to which a specific deformation form can correspond to one and only one stress state. In order to apply this rule it is required to obtain numerical model's displacements distribution fully corresponding to actual deformations of the analysed structure.

An element deciding about a structure's deformation state is the effect of a rapid change of the structure's shape occurring when the critical load levels are crossed. From the numerical point of view, this phenomenon is interpreted as a change of the relation between state parameters corresponding to particular degrees of freedom of the system and the control parameter related to the load. This relation, defined as the equilibrium path, in case of an occurrence of mentioned phenomenon, has an alternative character, defined as bifurcation. Therefore, the fact of taking a new deformation form by the structure corresponds to a sudden change to the alternative branch of the equilibrium path.

Therefore, a prerequisite condition for obtaining a proper form of the numerical model deformation is to retain the conformity between numerical bifurcations and bifurcations in the actual structure. In order to determine such conformity it is required to verify the results obtained by an appropriate model experiment or by using the data obtained during the tests of the actual object. It is often troublesome to obtain reliable results of nonlinear numerical analyses and it requires an appropriate choice of numerical methods dependent upon the type of the analysed structure and precise determination of parameters controlling the course of procedures.

Due to the number of state parameters, the full equilibrium path should be interpreted as hyper–surface in state hyperspace, satisfying the matrix equation for residual forces:

$$\mathbf{r}(\mathbf{u}, \Lambda) = \mathbf{0} \tag{1}$$

where $\mathbf{u}$ is the state vector containing structure nodes' displacement components corresponding to current geometrical configuration, Λ is a matrix composed of control parameters corresponding to current load state, and $\mathbf{r}$ is the residual vector containing uncompensated components of forces related to current system deformation state. The set of control parameters may be expressed by a single parameter that is a function of the load. Eq. (1) takes then the following form:

$$\mathbf{r}(\mathbf{u}, \lambda) = \mathbf{0} \tag{2}$$

called a monoparametric equation of residual forces.

The prediction-correction methods of determining the consecutive points of the equilibrium path used in modern programs contain also a correction phase based on the satisfaction of an additional equation by the system, called an increment control equation or constraints equation:

$$c(\Delta\mathbf{u}_n,\, \Delta\lambda_n) = 0 \tag{3}$$

where the increments:

$$\Delta\mathbf{u}_n = \mathbf{u}_{n+1} - \mathbf{u}_n \quad \text{and} \quad \Delta\lambda_n = \lambda_{n+1} - \lambda_n \tag{4}$$

correspond to the transition from n–th state to $n+1$–th state.

In order to find out whether there is full conformity between the character of actual deformations and their numerical representation it would be required to compare the combinations of the relevant state parameters in all the phases of the course of the phenomenon considered herein. Because of the complication of such a comparative system, the deformation processes are represented in practice by applying substitute characteristics called representative equilibrium paths. They define the relations between a control parameter related to load and a selected, characteristic geometric value related to a structure's deformation, an increment of which corresponds to a change of the value of all or some state parameters.

In case of a large number of state parameters it is not possible at all to represent the character of bifurcation by applying a representative equilibrium path. Sometimes, changes of state parameters resulting from local bifurcation may show the lack of perceptible influence on the representative value, which results in non–occurrence of any characteristic points on the representative path. In general, however, these changes cause a temporary drop in the control parameter value.

So both the experiment itself and nonlinear numerical analyses may result only in a representative equilibrium path. In that case, the problem of the numerical representation of bifurcation comes down to the preservation of conformity of the representative equilibrium path obtained by a numerical method with the one obtained experimentally, where a sine qua non for the application of the solution uniqueness rule is to recognise the similarity of the post–critical deformation forms of the experimental and numerical models as sufficient.

An additional problem, occurring during the experimental determination of the equilibrium path, results from the lack of possibilities of recording the said temporary, little drops in load, arising from local bifurcations, causing changes of the values of a part of state parameters. In the majority of experiments, the load of the tested model is achieved by force control, e.g. using a gravitational system, or displacement, by means of various types of load-applying devices (Fig. 1).

Figure 1 A stand for testing thin–shell structures subject to torsion: left – a version with a loading system controlling the displacement (turnbuckle), right – a version with a system controlling by force (gravitational)

However, even in case of devices with high level of technical advancement, in general it is not possible to register precisely short–lasting force changes, occurring from the beginning of a bifurcation phenomenon to the moment of reaching the consecutive deformation form by the model. Therefore, the representative equilibrium path obtained as a result of the experiment is of smooth characteristics, and its formation is based on measuring points corresponding to the consecutive deformation states determined.

In case of nonlinear numerical analysis in the finite element approach, the accuracy of the obtaining of the representative equilibrium path may be much more accurate. The existing commercial programs usually offer the results of all the increment steps, followed during the calculation process, and thus they also allow to observe slight fluctuations of the control parameter. The only limitation here is exclusively the value of the incremental step itself. In spite of this, due to the lack of possibilities of relating the results obtained to the relevant detailed changes of the experimental characteristics, it seems appropriate to determine the numerical representative equilibrium path of the same level of simplification as in the case of the experiment.

2. Analyses of example structures

The comparative analysis of such representative equilibrium paths is not, however, a method that allows a complete enough verification of the reliability of the results of numerical calculations.

An example of a problem in which the calculated results may have been deemed incorrect despite the seeming full conformity of the representative equilibrium paths is an open cylindrical shell, which was subjected to a cycle of tests, during which it was assumed that stringers are characterised by a sufficient margin of stiffness and they do not lose stability (Fig. 2).

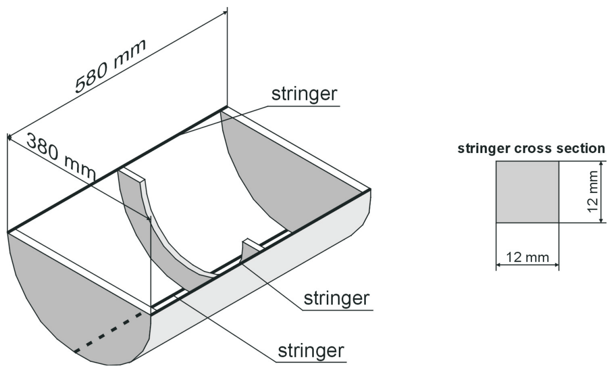

Figure 2 Geometry of the model

So the models made of the polycarbonate were strengthened with longitudinal members with large, rectangular cross–section and relatively high values of geometric moments of inertia. Each of the examined systems was subjected to constrained torsion, using a test stand presented in Fig. 1. The tests aimed at developing the methodology of determining stress distributions in a structure's shell in post–critical deformation states.

In the conditions of torsion the stress state created in the shell of such system, may be interpreted as an incomplete tension field. As a result, even if there are no geometrical imperfections, the shell loses its stability. On the other hand, the post–critical deformation increment causes a significant stress redistribution. The experiments repeated a number of times showed that the final form of post–critical deformations of such systems, occurring at sufficiently high load values, is always the same in spite of the alternative character of the course of the structure state changing process (Fig. 4).

The fact that local bifurcations following increases in loads occur with some scatter of locations and stress levels makes the nonlinear numerical analysis particularly troublesome in this case. It is practically impossible to develop a FEM model allowing to reproduce accurately the entire process of the structure's state changes, using commercial software, due to the nature of the functioning of algorithms for choosing the variants of the equilibrium path at the bifurcation points and the impossibility of the user's interference in the form of those algorithms. In this situation it seems appropriate to focus only on obtaining a numerical solution consistent with the experiment results at assumed load values.

The selection of an appropriate combination of numerical methods and parameters controlling the course of the analysis seems particularly vital in this case, likewise the proper representation of the model's stiffness. Even small mistakes in this respect result in the occurrence of incorrect forms of deformation (Fig. 3).

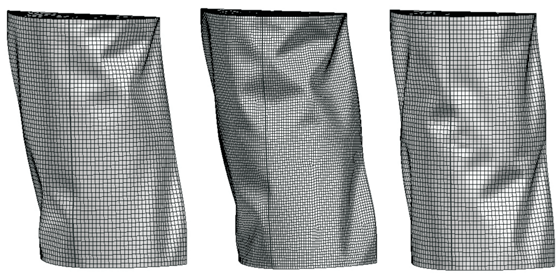

Figure 3 Incorrect forms of shell stability loss, showing a member's buckling not revealed in the experiment

It should be emphasised that it seems very risky to rely in the design process on the results of nonlinear numerical analysis of similar structures without appropriate verification in an experiment, if only a relatively cheap model experiment. In practice, multiple repetition of the analysis and systematic comparison of its results with the results of the experiment are required to obtain correct results of the numerical representation of a structure's state in the conditions of post–critical loads (Fig. 6).

The research results of various load–bearing structures confirm that the difficulty related to carrying out an appropriate nonlinear numerical analysis results from the nature of bifurcation. If the change in a structure's form is gentle in nature and it occurs in a small area, then the bifurcations related to it occur gradually, in relatively small subsets of state parameters.

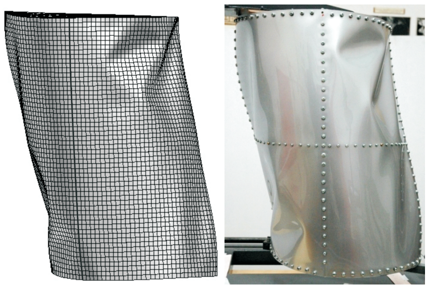

Figure 4 Accepted as satisfactory form of post–critical deformation obtained as a result of nonlinear analysis (left) and a deformation form obtained as a result of experimental tests at identical load (right)

The numerical simulation of the process is then easy to perform and it may take place when using prediction–correction methods with simple correction based on state control. But if the deformation occurs in a larger area, and the change of the form is violent in nature, then the bifurcation corresponds to the simultaneous change of a great number of state parameters, and the determination through a numerical procedure of their appropriate combination, corresponding to the new state of static equilibrium, may be hindered or even impossible. In such a case it is necessary to apply matchings of prediction methods with correction strategies based on arc length control methods, such as the Riks correction or the Crisfield hyperspherical correction.

Bifurcation changes of the forms of load–bearing structures, containing shells of considerable curvature, occur more violently if there is a higher relation of the square of the smaller of dimensions of the shell segment area limited by the adjacent members frames to the value of the local radius of its curvature. Thus, semi–monocoque structures of relatively low number of the framing elements are especially troublesome in nonlinear numerical representations.

An example of such a structure is a closed cylindrical shell presented in Fig. 5.

The structure's framing consists of a minimum number of crosswise elements, i.e. two closing frames and four longitudinal members. The type of the structure itself corresponds to solutions commonly used in the aviation technology, e.g. the construction of a fuselage of an aircraft. It should be emphasised, however, the model subjected to examinations constitutes a special instance of a structure of purposefully minimised number of longitudinal members. The actual solutions are usually based on much more extended framings. The structure described corresponds to an isolated phase of a wider cycle of examinations aiming at determining direct dependences between the number of framing elements and post–critical deformation distributions.

The examined structure was subjected to constrained torsion using a modified version of the stand presented in Fig. 1.

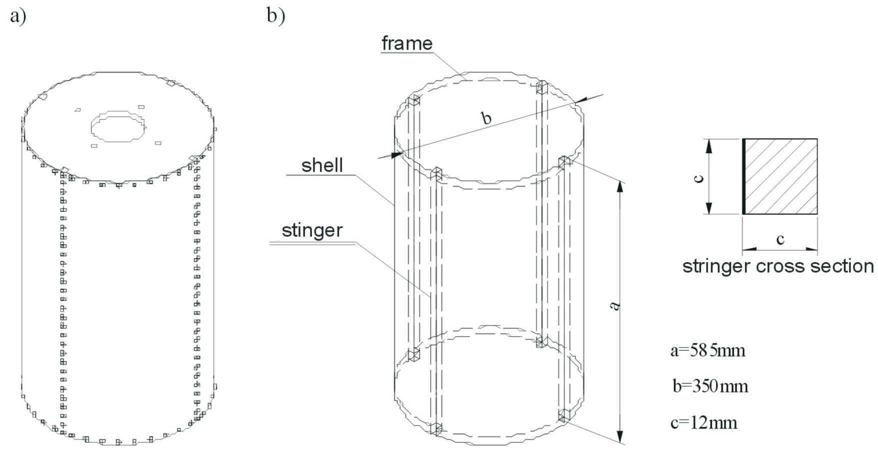

Figure 5 A schematic view of a complete cylindrical shell reinforced by four members (a) and a schematic view of a structure including dimensions (b)

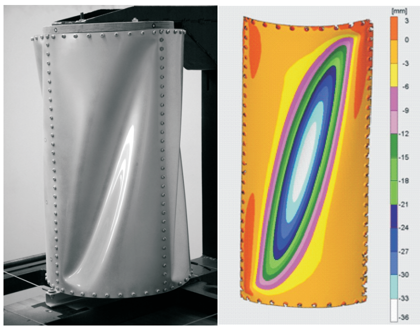

Figure 6 The advanced post-critical deformation of the examined structure (left) and the distribution of contour lines representing the size of the deformation, made using the projection moiré method (right)

According to the expectations, post–critical deformations occurred in a violent way. Due to the gravitational way of load application, the measurement of the relation between torsion angle and the torque moment, assumed as the representative equilibrium path, corresponded to the steady states (Fig. 10).

Using this mode of taking measurements, the representative characteristics does not reflect bifurcation points in an overt way, but attention should be drawn to the occurrence of its horizontal section. It corresponds to this phase of the experiment in which a sudden change occurred in the structure state with the simultaneous constant load level.

With regard to the symmetry, the deformed structure possessed four characteristic grooves in all the shell segments (Fig. 6).

During the experiment the surface geometry was registered using the projection moiré method. Atos scanner manufactured by a German company, GOM Optical Measuring Techniques was used as a registering device.

The problem discussed belongs to one of the most troublesome from the point of view of a FEM nonlinear numerical simulation. A number of tests performed using the MSC MARC software revealed the lack of effectiveness of its procedures in case of this problem, with regard to determining the appropriate post-buckling state of a structure. The algorithms used in those procedures are characterised by inability to represent the symmetry of the phenomenon. With the idealised geometric form of the model, the obtaining of the new form of the structure after crossing the critical load value occurs only in one of the segments, in spite of the apparently correct, symmetrical initiation of stability loss. This proves the faults in the algorithms for choosing the appropriate variants of the equilibrium path in case of the appearance of changes in the state parameters combination in several of their independent subsets.

The situation was improved when shell imperfections were implemented, by applying normal forces to the skin, in the central points of particular skin (Fig. 9).

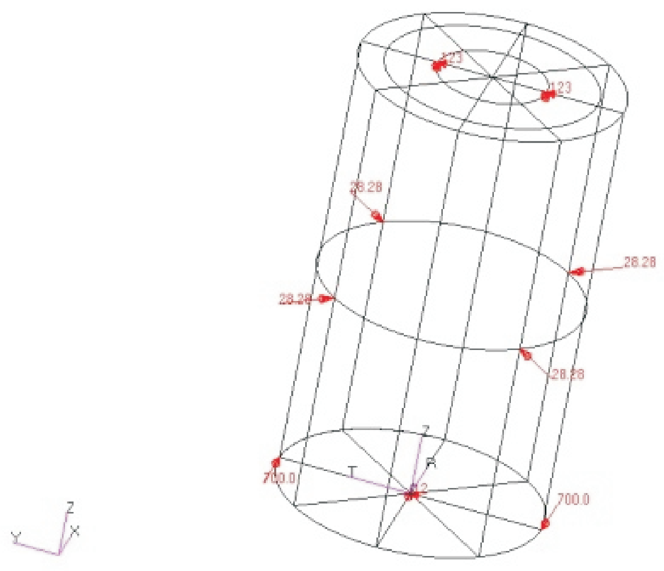

Figure 7 A geometrical model of a structure made in MSC PATRAN environment with boundary conditions and loading

However, even in the case of applying this type of forcing a form change, it was very difficult to obtain results that would fully correspond to the experimental results. Assuming the use of skin elements with linear shape functions, the appropriate density of the mesh turned out to be the key factor, but its excessive density caused incorrect forms of post–critical deformations (Fig. 8).

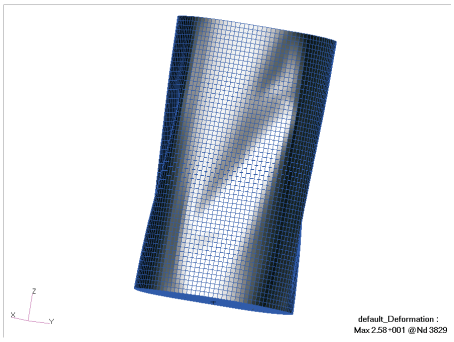

Figure 8 The incorrect form of deformation, obtained in case of too many elements

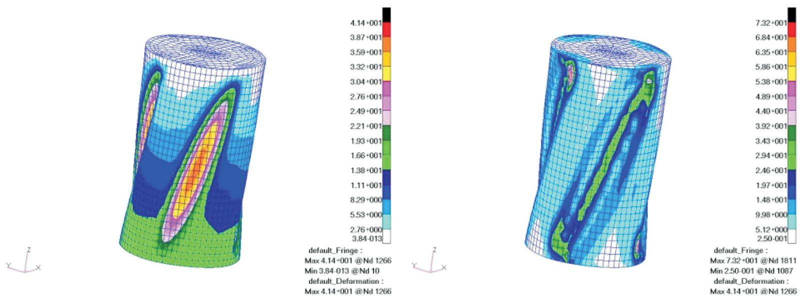

Figure 9 The deformation distribution (left) and reduced stress distribution acc. to Huber–Mises hypothesis (right) for 100% of the maximum load (stringers modelled with thick shell bilinear element)

The better result, in case of application of beam elements as a representation of stringers, was obtained with the use of a relatively low density of mesh. This proves the rightness of the thesis, proved a number of times in many studies, pursuant to which the decrease in the general number of degrees of freedom, corresponding to the number of state parameters, in case of nonlinear procedures used in the available commercial programs, often brings benefits that considerably exceed the deficiencies of a mathematical description resulting from the decrease in the number of elements.

The best result was obtained only after the fundamental change of the concept of FEM model, when the different kind of finite elements was applied as a representation of stringers (thick shell element was used instead of a recommended beam element). However this solution, from the point of view of mathematical description is much less correct, it turned out much more effective in case of relatively low values of the total torsion angle of the structure.

The results of analysis of this FEM model version, obtained using secant prediction method and strain-correction strategy, are presented in Fig. 9.

The strain–correction strategy turned out most effective in case of significant, violent change of the form of deformations, when the representative equilibrium path contains relatively long "horizontal section".

The relation between the representative equilibrium paths is presented in Fig. 10.

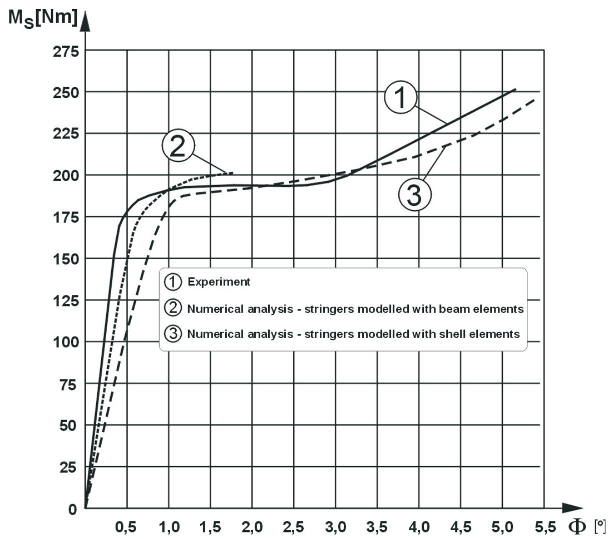

Figure 10 The presentation of the representative equilibrium paths

3. Conclusions

The presented examples of load–bearing structures represent only some of those many used in the modern aviation technology. But the criterion applied while selecting them as objects of experimental and numerical analyses was its representativeness for the most commonly met elements of constructions, in case of which the occurrence of a local stability loss is acceptable in the conditions of service load.

The fundamental conclusion that can be drawn from the presented results of the research is the absolute need for using experimental verifications with regard to FEM nonlinear numerical analyses of this type of structures. The more so that even in the cases in which the correctness of the results obtained seems unquestionable, they may be in fact burdened with errors resulting from the very limited reliability of the numerical procedures used in commercial programs.

Based on the nonlinear numerical analyses, related to the presented structures, frequently repeated many times, a general recommendation may also be formulated for the maximum possible limitation of the size of a task. Striving for increasing the accuracy of the calculations by increasing the density of finite elements mesh, applied successfully in linear analyses, may turn out ineffective in case of a nonlinear analysis and may lead to incorrect results or the lack of convergence of calculations.

The numerical representation of bifurcation, by virtue of the mere idea of the discrete representation of continuous systems, must be simplified in case of the finite elements method. In such a situation, based on the quoted examples, the need must be emphasised for obtaining the indispensable convergence of the experimental and obtained numerically relations between a selected geometric parameter characterising the essence of a structure's deformation and a selected value relating to the load, recognised as representative equilibrium paths. This convergence, in combination with the accepted as sufficient similarity of post–critical deformation forms, constitutes the grounds for accepting the reliability of stress distributions determined by means of numerical methods.

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