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Experimental and Numerical Studies on the Static Deflection of the Composite Beam with the MFC Element

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In this paper the FE model of multi–layers composite beam with added Macro Fiber Composite (MFC) active element is presented. At the first step of study the model of the MFC element was prepared. The experimental validation was made. Next, a static deflection of the composite beam with the actuator was calculated. The piezoelectric effects was analyzed. The results of the FEM simulations were compared with the experimental results. A very good agreement was achieved.

Keywords: Finite Element Method, structure, laminate, actuator, PZT, MFC.

1. Introduction

The active structure consists of load–bearing elements and active elements. These sensors track the real–time behavior of the structure. On the other hand, the actuators can change their behavior. There are a lot of different type of the active elements. The shape memory alloys (SMA), magnetorheological materials (MR), piezoelectric (PZT) and Macro Fiber Composite (MFC) elements can be used. The basic problem is selection of the active element for analyzed structure and verification of the effectiveness of its actions. The aim of this paper was to build numerical model of multi-layered composite beam with embedded MFC active element using the finite element method (FEM). The static deflection of the composite cantilever beam with MFC element for different voltage applied to the terminal transducer was studied. Results of the FEM analysis has been verified in experimental study. Detailed simulations were performed using the commercial software package Abaqus.

In this software there are special elements to describe the electromechanical effect. The prepared numerical model of the composite beam with the piezoelectric element was used for further dynamic simulations in order to determine rightly application of the transducer for damping of vibrations of thin-walled composite structures.

In the literature one can find many works dealing with the application of active elements and their use for analysis structures as well as the active structured behavior in real time. However, there is a lack of works in which the behavior change of the composite structure was caused by the MFC active element.

The results of research were performed so far and published by the authors [1–13] confirm a high suitability of piezoelectric and MFC type elements for structural health monitoring and control in real time. In the paper [4] we presented our static analysis for cantilever beam made of two layers of PVDF. Some different applications of these active elements were presented by other authors, for example: vibration suppression, energy harvesting etc. [14–24]. Macro Fiber Composite actuators are characterized by large deformation and large forces generated there. The interest of this type of actuators is due to their low weight, resistance to temperature change, high flexibility, short time operation and high conversion efficiency of the electric energy into the mechanical one. This effect in MFC type element causes change in the electric field, next it has influence on the deformation, which change the density of charge and the electric field.

In order to determine the behavior of intelligent element a finite element method seems to be a good choice [25–28].

2. Numerical Model of Active Element

Detailed calculations were made for a cantilever beam made of glass-epoxy unidirectional composite prepreg with Macro Fiber Composite (MFC) active element. The M8528-P1 element made by Smart Material Corp., USA [29] was used. This was the transducer of the d_{33} effect type. This PZT effect corresponds to the deformation of a specimen in the direction of the driving electric field.

The constitutive equations describing the piezoelectric property are the following:

$$\{\varepsilon\} = [S]\{\sigma\} + [d]\{E\} \tag{1}$$

where: $\{\varepsilon\}$ – the strain vector, $\{\sigma\}$ – the stress vector, {E} – components of the electric field strength, [S] – the compliance matrix, [d] – the matrix of piezoelectric coefficients.

In this case, it was assumed that the intensity of the electric field E_3 acts only along the x_3 axis (denote as z axis). The piezoelectric effect d_{33} is clarified in Fig. 1.

The extension Δa of the free element in axis x_3 can be written as:

$$\Delta a = d_{33} E_3 a \tag{2}$$

where: d_{33} – the electromechanical coupling coefficient in the direction 3, a – the length of the free active element, E_3 – the electric field.

The strain in direction x_3 equals to:

$$\varepsilon_{33} = d_{33} \frac{U}{a} \tag{3}$$

The geometrical dimensions of the transducer were: the length – 103 mm (85 mm), the width – 35 mm (28 mm), the thickness – 0.3 mm ±10%. Dimensions of the active area of this element were given in brackets. The mechanical properties of the M8528-P1 element are presented in Tab. 1 [29]. It is orthotropic linear elastic material. The shear modulus was estimated using the rule–of–mixtures.

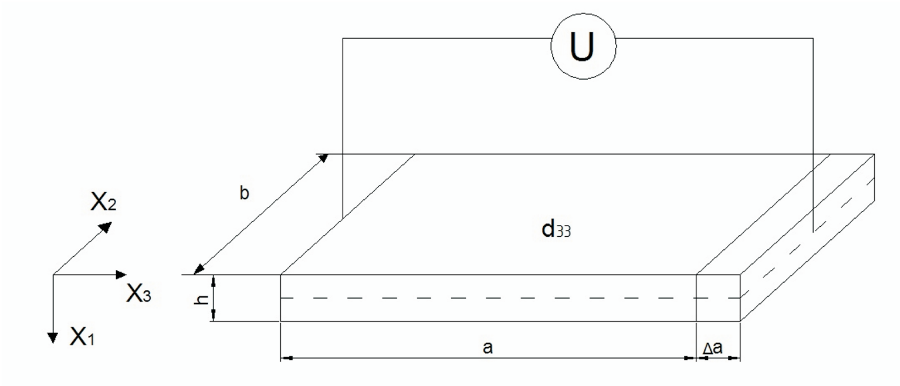

Figure 1 The d_{33} effect for piezoelectric material

Table 1 The mechanical properties of M8528-P1 element [29]

Parameter	**Value**
Young modulus [MPa]	
E_3	30 336
E_2	15 857
Poisson's ratio [-]	
ν_{23}	0.31
Shear modulus [MPa]	
G_{23}	5 515
Density [kg/m^3]	
ρ	5 440

The MFC active element consist of PZT fibers isolated from each other (170 sections of electrocouples distant by 0.5mm one from another) embedded in the epoxy resin (see Fig. 2). Therefore, the necessity arose to build a model of continuous supplementary element, which electrical will be identical to the one of the real element.

FEM modeling assumed that there is a piezoelectric material on the whole area of element. The numerical model of actuator was constructed using the C3D20E type solid elements. They made it possible to model effect of electromechanical coupling. They were 20–node 2nd order elements, having three translational degrees of freedom at each node and one extra degree of freedom associated with the piezoelectric properties. The simulations were performed by using the finite element method in the Abaqus software environment [30].

According to the data provided by the manufacturer the value of d_{33} coefficient in the M8528-P1 element depends on the electric field. In case of a single couple of electrodes the parameter d_{33} equals 4.6 10^{-9} m/V for E >1000 V/mm. Following this information the provided d_{33} values had to be multiplied by the number of sections in the MFC element and this way the initial value of the d_{33} coefficient was accepted in the numerical model.

Verification of this model was done in two tests given in documentation – the free strain test and the blocking force test [29]. Mechanical and electrical boundary conditions for both tests are presented in Tab. 2.

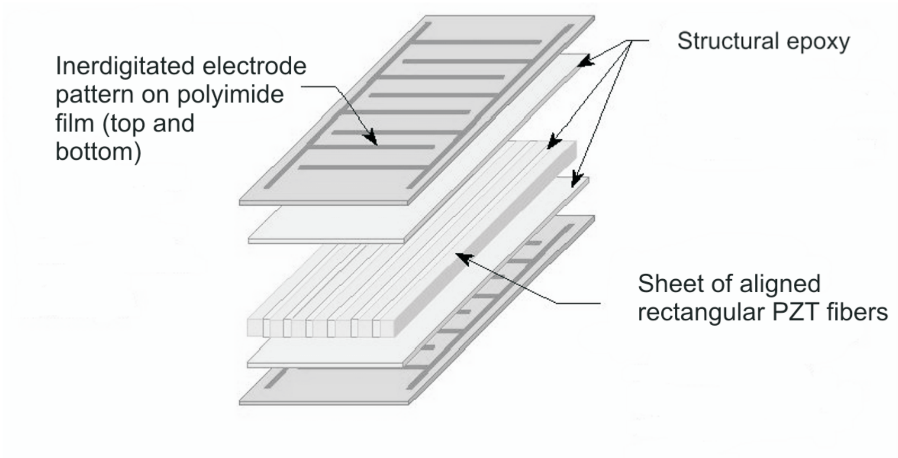

Figure 2 MFC active element made by Smart Material Corp., USA [29]

Results from initial numerical tests showed a significant relative difference to the catalogue data. In the first test the relative error was 23.3% ($\varepsilon_3 = 1380\,\mu$ m/m with respect to reference value 1800 μ m/m) and in the second test the relative error was 21.7% (R_z = 355.7 N with respect to 454 N). In order to achieve better agreement of the results, was necessary to correct the d_{33} coefficient. The optimal value of d_{33} parameter for the M8528-P1 element was equal 1.01 10-7 m/V, what gave correction in strain equal $\varepsilon_3 = 1782\ \mu$ m/m and the blocking force R_z = 459.4 N. The obtained results prove that the modeling technique for the considered element was correct. More specific discussion of these studies was presented in paper [3].

3. The FEM Analysis of the Composite Beam with MFC Actuator

The verified model of the transducer was used to build the model of composite cantilever beam with M8528-P1 active element. The beam was made of 6–layers of

Table 2 The boundary conditions in numerical model validation

Boundary conditions	Free strain	Blocking force
One surface in XY plane supported in Oz direction	X	X
The bottom surface in YZ plane supported in Ox direction	X	X
One whole edge in XZ plane supported in Oy direction	X	X
Charge potential 0 V is applied on one surface in XY plane	X	X
Charge potential 1500 V is applied on second surface in XY plane	X	X
On the second surface in XY plane supported in Oz direction	-	X

glass–epoxy unidirectional composite prepreg with configuration of composite plies $[45/\text{-}45/90]_s$. The material data for the laminate was determined in accordance with the standard ISO tests and was presented in Table 3. Additionally, a theoretical relationship for the material constants can be written:

$$E_2\nu_{32} = E_3\nu_{23} \tag{4}$$

Active element was adhered on one face of the beam only, directly at the clamped end. The geometrical dimensions of the beam are presented in Fig. 3.

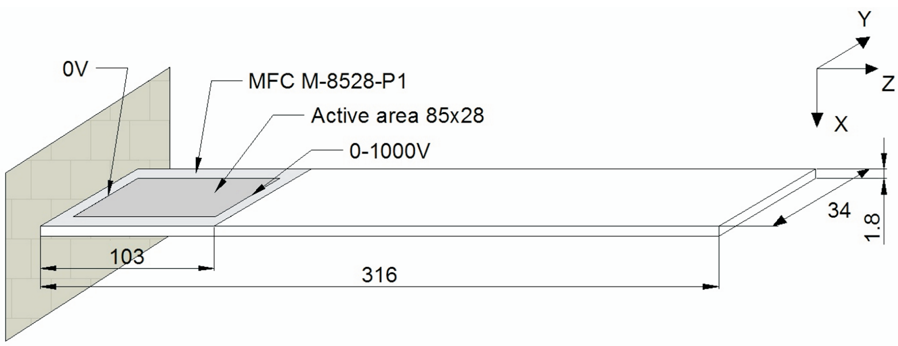

Figure 3 Schematic of the composite cantilever beam (dimensions in mm)

The model of the cantilever beam (Fig. 4) was constructed with the C3D20RE type solid elements. They are 20–node 2nd order (with a square shape function) elements, having three translational degrees of freedom at each node. All the elements used a reduced integration method. Individual layers of the laminate had

Table 3 Mechanical properties of glass–epoxy composite

Parameter	**Value**
Young modulus [MPa]	
E_3	46 430
E_2	14 926
Poisson's ratio [-]	
ν_{23}	0.27
Shear modulus [MPa]	
G_{23}	5 233
Density [kg/m^3]	
ρ	2 032

been modeled according to Layup–ply technique. The mechanical boundary conditions for the model of the beam were realized by restraining the nodes located at one end of the beam all degrees of freedom. The combination of all parts was realized by defining interactions as "TIE", what resulted in linking the degrees of freedom of the nodes in contact on the appropriate surfaces of the model [30].

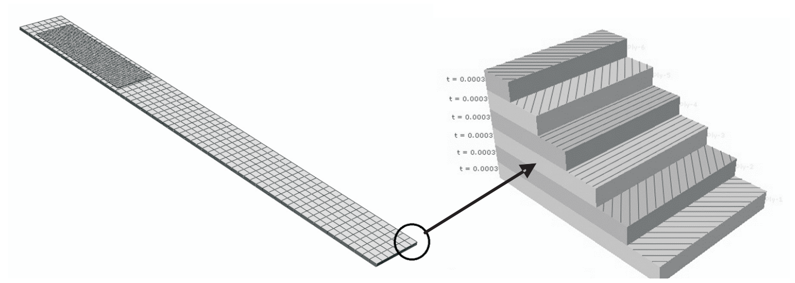

Figure 4 FEM model of the cantilever beam with the composite stacking sequence $[45/-45/90]_s$

During numerical analysis, the static deflection of the composite cantilever beam with the MFC element was determined. Subsequent loadings within the range of 0-1000 V were applied to the positive terminal transducer, keeping the constant value of 0 V at a negative one. Deflections of the beam were calculated by reporting the vertical displacement (axis Ox) of the free end of the beam.

4. Experimental Study

Experimental study was performed on the special experimental setup. Diagram of the measuring system was shown in Fig.5; it consists of several subsystems. The first–being the main object is the multilayer composite beam with the MFC actuator. It was installed in a holder. The second subsystem was used to supply the piezoelectric actuator. It had two elements: a high–voltage amplifier and a signal generator. Both devices together provided a change of voltage applied to control of the MFC in an open loop (without feedback). The vibration of the free end of the beam can be monitored by the measuring subsystem. To measure the response of the mechanical part the laser triangulation sensor was used. The signals from the sensor and the generator were transmitted to the recorder. In the applied experimental setup for recording and saving data the MGC+ device was used, connected to a PC computer.

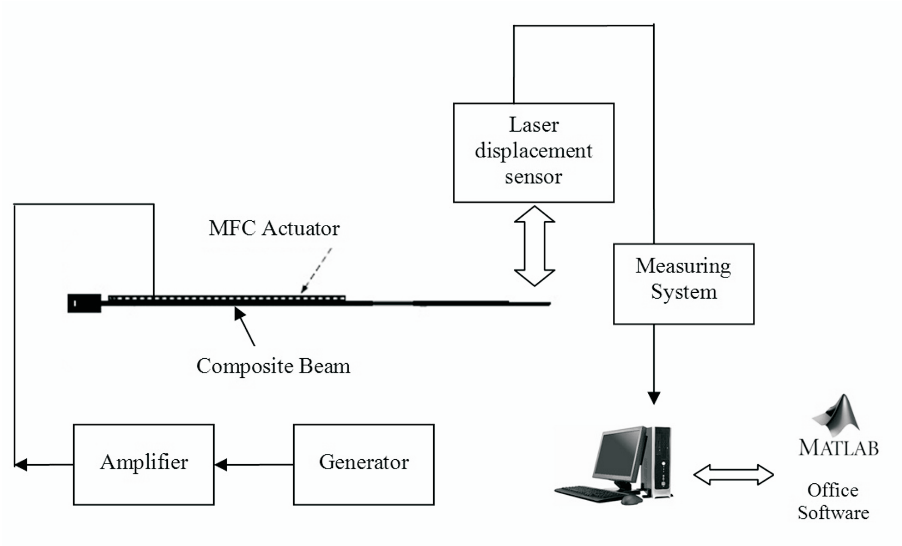

Figure 5 Diagram of measuring system

During the experimental study the responses of the beam were obtained. Full test cycle $U(t)$ can be described mathematically using the Heaviside's step function H(...):

$$U(t) = A \cdot H(s(t)), \tag{5}$$

where: A is the step level and $s(t)$ is a new coordinate depending on time

$$s(t) = 300 - \mathrm{mod}(t, 600), \tag{6}$$

where the function mod$(\dots)$ is a modulo operation. Interpretation of the exampling test cycle was performed in the Matlab software and is shown in Fig. 6.

Experimental tests were made at the levels A from 100 V to 1000 V (Eq. 5). In this paper two cases are presented: for the voltage 700 V and 1000 V (Figs. 7–8). These pictures show only one cycle of the step function change. One can observe two effects. The first, free damped vibrations oscillations after jump values of the voltage from 0 to A and vice versa. The vibrations quickly disappear. The second effect is a creep of piezoelectric elements, which is seen as a slow change in displacement of the free beam end.

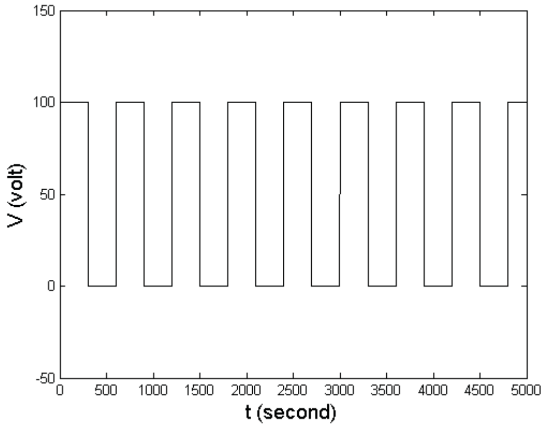

Figure 6 Step excitation of the MFC actuator power supply (A = 100 V)

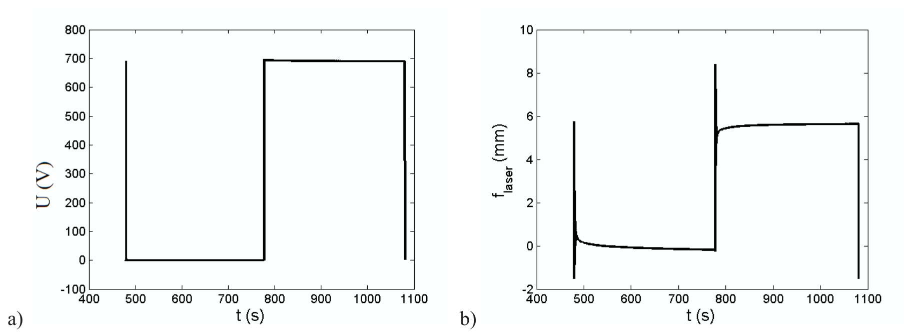

Figure 7 Voltage signal of power supply (a) and vertical deflection of the free end of the beam for voltage 700 V (b)

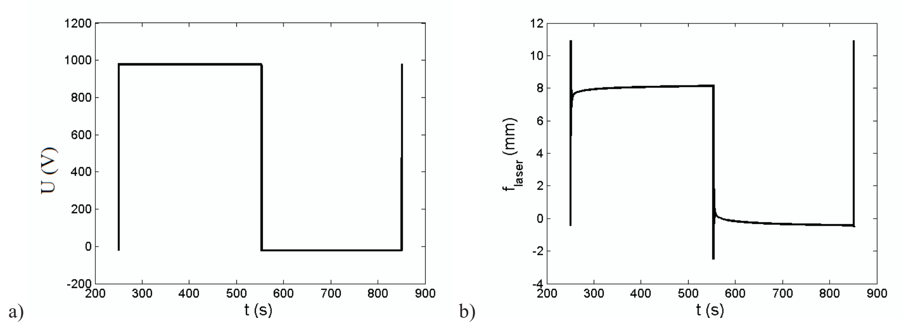

Figure 8 Voltage signal of power supply (a) and vertical deflection of the free end of the beam for voltage 1000V (b)

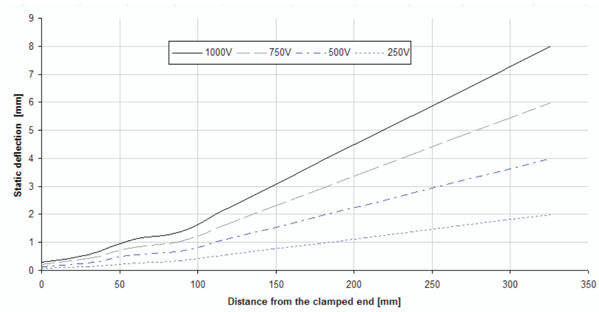

Figure 9 Static deflection of the beam for four cases of applied voltage

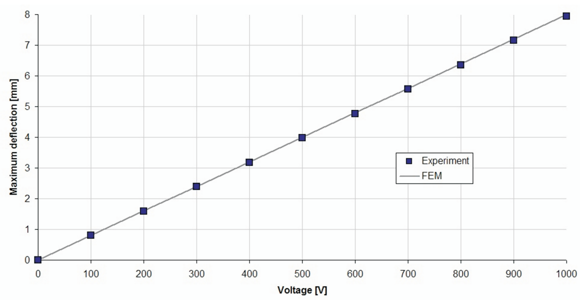

Figure 10 Maximum deflection of the composite beam for both studies

5. Comparison of the results

As a result of static numerical analysis the static deflection of the beam (Fig. 9) for four cases of the applied voltage: 250 V, 500 V, 750 V and 1000 V was obtained. The results of experimental analysis were shown in Figs. 7–8. These pictures presented vertical deflection of the free end of the beam for voltage 700 V and 1000 V.

Comparison of static deflections obtained with the FEM simulations and the experimental studies was presented in graph (Fig. 10). Specific results of both analyses was shown in Tab. 4.

Table 4 Comparison of deflections obtained with the FEM simulations and the experimental studies

U[V]	Experiment [mm]	FEM [mm]	Relative Difference [%]
100	0.795	0.799	0.5
200	1.590	1.599	0.6
300	2.385	2.398	0.5
400	3.180	3.197	0.5
500	3.975	4.000	0.6
600	4.770	4.799	0.6
700	5.565	5.598	0.6
800	6.360	6.397	0.6
900	7.155	7.196	0.6
1000	7.950	7.995	0.6

A very good agreement between the FEM and the experimental results was achieved. For the examined points the calculated differences did not exceed 1%.

6. Conclusion

The paper presents numerical and experimental studies of static deflection of the composite beam with the MFC active element. The static deflection of the composite cantilever beam with the MFC element for different voltage applied to the terminal transducer was determined. A very good agreement between the FEM and the experimental results was achieved. For the examined points the calculated differences did not exceed 1%, which confirmed the correctness of the numerical modeling of the MFC active element. The numerical simulations were performed with the commercial FE system Abaqus. The prepared numerical model of the composite beam with piezoelectric element will be used in further simulations, for example: modal analysis, reducing vibration or damping of different characteristics. In the second stage a similar analysis for cantilever beam with the M8528-P2 active element will be done. This tape of active element uses d_{31} effect. A comparison of both analyses will show the effectiveness and the properties of active elements.

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