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Postbuckling of an Imperfect Plate Loaded in Compression

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The stability analysis of a plate loaded in compression is presented. The non–linear FEM equations are derived from the minimum total potential energy principle. The peculiarities of the effects of the initial imperfections are investigated using the user computer code. Special attention is paid to the influence of imperfections on the post–critical buckling mode. The FEM computer program using a 48 DOF element has been used for analysis. FEM model consists of 4x4 finite elements. Full Newton-Raphson procedure has been applied.

Keywords: Initial imperfections, postbuckling, finite element method, Newton–Raphson iteration.

1. Introduction

In the presented paper behaviour of rectangular plate loaded in compression has been explained [1, 2]. The geometrically nonlinear theory represents a basis for the reliable description of the postbuckling behaviour of the plate [3]. Influence of initial imperfection on the load–displacement path is investigated. The result of the numerical solution represents a lot of the load versus displacement paths. Solution from the user code (created by authors) is compared with results gained using ANSYS system.

2. Theory

Let us assume a rectangular plate simply supported along the edges (Fig. 1) with the thickness t. The displacements of the point of the neutral surface are denoted $\mathbf{q} = [u, v, w]^T$ and the related load vector is $\mathbf{p} = [p_x,\ 0,\ 0]^T$.

By formulation of the strains, non-linear terms have to be taken into account [4]. Then it can be written as

$$\varepsilon = \varepsilon_m + \varepsilon_b, \varepsilon_m = \varepsilon_l + \varepsilon_n, \tag{1}$$

where

$\varepsilon_l = [u_{,x}\ ,\ v_{,y}\ ,\ u_{,y} + v_{,x}]^T,$

$\varepsilon_n = \frac{1}{2}\left[w_{,x}^2\ ,\ w_{,y}^2\ ,\ 2w_{,x}w_{,y}\right]^T,$

$\varepsilon_b = -z \cdot [w_{,xx}\ ,\ w_{,yy}\ ,\ 2w_{,xy}]^T$, $\varepsilon_b = -z \cdot \mathbf{k}$,

the indexes denote the partial derivations and w represents the global displacement.

The initial displacements have been assumed as the out of plane displacements only and so it yields

$$\varepsilon_0 = \varepsilon_{0n} + \varepsilon_{0b}, \tag{2}$$

where

$\varepsilon_{0n} = \frac{1}{2}\left[w_{0,x}^2\ ,\ w_{0,y}^2\ ,\ 2w_{0,x}w_{0,y}\right]^T,$

$\varepsilon_{0b} = -z \cdot \mathbf{k}_0 = -z \cdot [w_{0,xx}\ ,\ w_{0,yy}\ ,\ 2w_{0,xy}]^T$

and w_0 is the part related to the initial displacement.

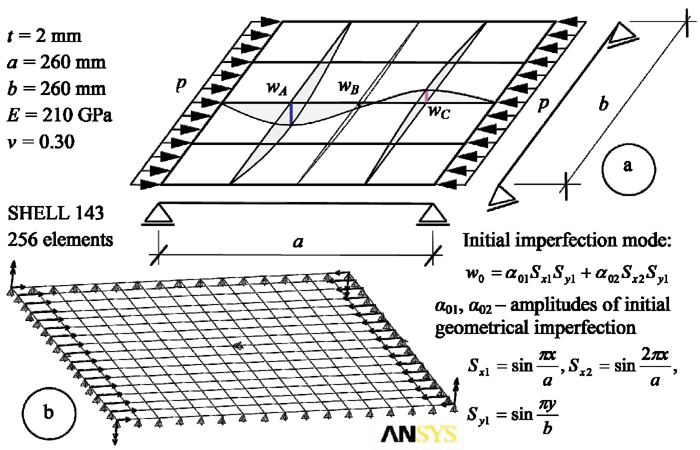

Figure 1 Thin plate a) Notation of quantities, b) FEM model

The linear elastic material has been assumed

$$\sigma = \mathbf{D} \cdot (\varepsilon - \varepsilon_0), \tag{3}$$

where

$$\mathbf{D} = \tfrac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

E, ν are the Young's modulus and Poisson's ratio.

The total potential energy can be expressed as

$$U = U_i + U_e = \int\limits_V \frac{1}{2} (\varepsilon - \varepsilon_0)^T \sigma \, dV - \int\limits_A \mathbf{q}^T \mathbf{p} dA. \tag{4}$$

After modification Eq. (**??**) can be written as

$$\begin{aligned} U \quad &= \quad \int\limits_A \frac{1}{2} (\varepsilon_m - \varepsilon_{0n})^T \, t \mathbf{D} \, (\varepsilon_m - \varepsilon_{0n}) \, dA \\ &+ \int\limits_A \frac{1}{2} (\mathbf{k} - \mathbf{k}_0)^T \, \frac{t^3}{12} \mathbf{D} \, (\mathbf{k} - \mathbf{k}_0) \, dA - \int\limits_A \mathbf{q}^T \mathbf{p} \, dA \end{aligned} \tag{5}$$

where ε, $\mathbf{k}$ are strains and curvatures of the neutral surface, ε_0, $\mathbf{k}_0$ are initial strains and curvatures, $\mathbf{q}$, $\mathbf{p}$ are displacements of the point of the neutral surface, related load vector.

The system of conditional equations can be obtained from the condition of the minimum of the increment of the total potential energy

$$\delta \, \Delta \, U = 0. \tag{6}$$

This system [5] can be written as

$$\mathbf{K}_{inc} \, \Delta \, \alpha + \mathbf{F}_{int} - \mathbf{F}_{ext} - \Delta \, \mathbf{F}_{ext} = \mathbf{0}, \tag{7}$$

where

$\mathbf{K}_{inc} = \begin{bmatrix} \mathbf{K}_{incD} & \mathbf{K}_{incDS} \\ \mathbf{K}_{incSD} & \mathbf{K}_{incS} \end{bmatrix}$ is the incremental stiffness matrix

$\mathbf{F}_{int} = \begin{Bmatrix} \mathbf{F}_{intD} \\ \mathbf{F}_{intS} \end{Bmatrix}$ is the vector of the internal forces

$\mathbf{F}_{ext} = \begin{Bmatrix} \mathbf{F}_{extD} \\ \mathbf{F}_{extS} \end{Bmatrix}$ is the vector of the external load of the plate

$\Delta \, \mathbf{F}_{ext} = \begin{Bmatrix} \Delta \, \mathbf{F}_{extD} \\ \Delta \, \mathbf{F}_{extS} \end{Bmatrix}$ is the increment of the external load of the plate

$\mathbf{q} = \mathbf{B} \cdot \, \alpha = \begin{bmatrix} \mathbf{B}_D & \\ & \mathbf{B}_S \end{bmatrix} \begin{Bmatrix} \alpha_D \\ \alpha_S \end{Bmatrix}$ and $\Delta \, \mathbf{q} = \mathbf{B} \, \cdot \Delta \, \alpha$

Index D means part related to the bending parameters, index S – part related to the axial parameters, DS – part related to the bending – axial parameters, α is vector of variational coefficients and $\Delta\alpha$ is increment of variational coefficients.

In the case of the structure in equilibrium $\mathbf{F}_{int} - \mathbf{F}_{ext} = \mathbf{0}$, one can do the incremental step $\mathbf{K}_{inc}\,\Delta\,\alpha = \Delta\,\mathbf{F}_{ext} \Rightarrow \Delta\,\alpha = \mathbf{K}_{inc}^{-1}\,\Delta\,\mathbf{F}_{ext}$ and $\alpha^{i+1} = \alpha^i + \Delta\,\alpha$. The Newton-Raphson iteration can be arranged in the following way: supposing that α^idoes not represent the exact solution, the residua are $\mathbf{F}_{int}^i - \mathbf{F}_{ext}^i = \mathbf{r}^i$. The corrected parameters are $\alpha^{i+1} = \alpha^i + \Delta\alpha^i$, where $\Delta\alpha^i = -\mathbf{K}_{inc}^{-1}\,\mathbf{r}^i$ ($\mathbf{r}^i$ – vector of residuum).

The identity of the incremental stiffness matrix with the Jacobian of the system of the nonlinear algebraic equation $\mathbf{J} \equiv \mathbf{K}_{inc}$ has been used in analysis.

To be able to evaluate the different paths of the solution, the pivot term of the Newton–Raphson iteration has to be changed during the solution.

3. Numerical Results

Illustrative examples of compressed steel plate from Fig. 1 are presented as load – displacement paths for different amplitudes of initial geometrical imperfection. From Figs. 2 and 4 it is obvious that two almost identical modes of initial imperfection at the beginning of the loading process offer two different solutions in postbuckling mode. Due to the mode of the initial imperfection the nodal displacements denoted w_A and w_C have been taken as the reference values (see Fig. 1a).

The aim of this paper was to try to give an answer to the problem of the threat of collapse of the plate loaded in compression in the second mode of buckling.

Fig. 2 show the solution for the amplitudes of initial geometrical imperfection $\alpha_{01} = 0.05mm$ and $\alpha_{02} = 0.33mm$. One can see that the fundamental path is in the postbuckling phase in 1^{st} mode of buckling. The thick line in Fig. 2 represents displacement of node A and the thin line represents displacement of node C. Shape of the plate in buckling and in postbuckling is also displayed.

Stable and unstable paths are shown in Fig. 3 (the thick lines represent stable load – displacement paths). For the stable path the incremental stiffness matrix $\mathbf{K}_{inc}$ must be positively defined; all minors must be positive as well; and the incremental stiffness matrix must be evaluated for the load as the pivotal term.

Increasing the effect of the 2^{nd} mode in the shape of the initial displacement ($\alpha_{01} = 0.05mm\ and\ \alpha_{02} = 0.35mm$) the postbuckling mode of the plate is 2^{nd} mode (Fig. 4).

Stable and unstable paths are shown in Fig. 5. Thick lines represent stable load – displacement paths, thin lines represent unstable load – displacement paths. Limit points are denoted with dots. Continuous lines represent displacement w_A and dashed lines represent displacement w_C. It is obvious that in the post-buckling the shape of buckling surface remains identical with the shape of the initial imperfection.

The FEM computer program using a 48 DOF element (4 nodes, 12 DOF at each node) [6] has been used for analysis. FEM model consists of 4x4 finite elements (Fig. 1a). Full Newton–Raphson procedure, in which the stiffness matrix is updated at every equilibrium iteration, has been applied.

Obtained results were compared with results of the analysis using ANSYS, where 16x16 elements model was created (Fig. 1b). Element type SHELL143 (4 nodes, 6 DOF at each node) was used.

4. Summary

The influence of the value of the amplitude and the mode of the initial geometrical imperfections on the postbuckling behaviour of the plate is presented. Finite elements created for special purposes of thin plates stability analysis, enable high accuracy and speed convergence of the solution at less density of meshing. The possibility on an interactive affecting of the calculation within the user code makes it possible to investigate all load – displacement paths of the problem. From the comparison of results shown both in Figs 2 – 3 and in Figs 4 – 5 a good coincidence of presented load–displacement paths is evident.

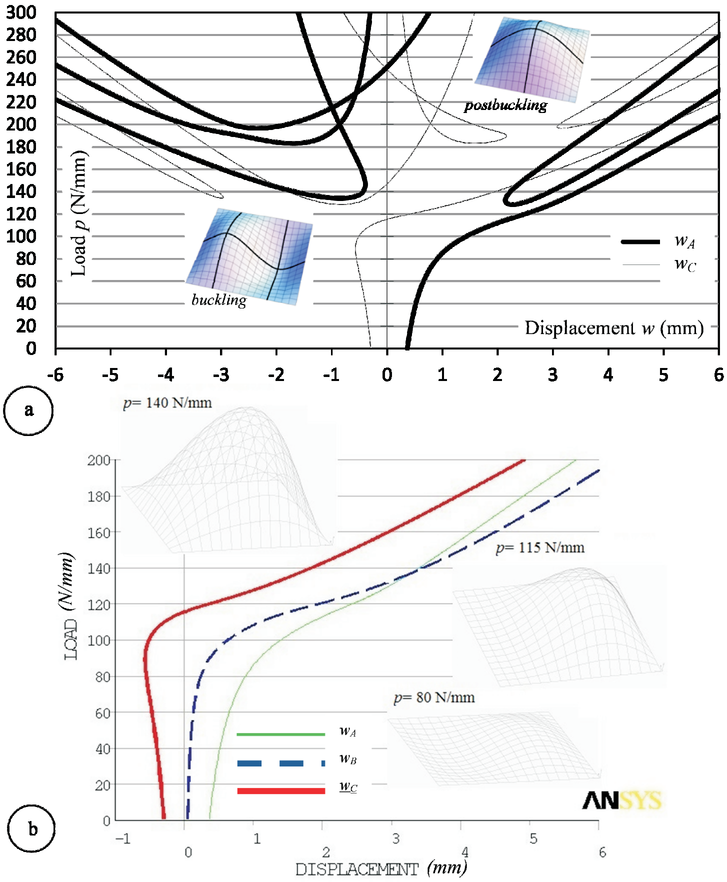

Figure 2 The postbuckling of the thin plate with initial displacement: $w_0 = 0.05 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$ $+0.33 \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b}$, a) user code [4, 5], b) ANSYS system

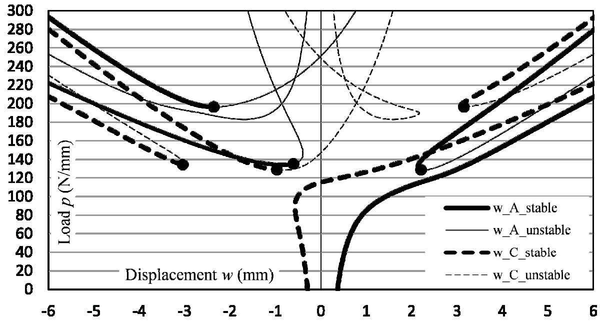

Figure 3 Stable and unstable load – displacement paths. Thin plate with initial displacement $w_0 = 0.05 \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} + 0.33 \sin\frac{2\pi x}{a} \sin\frac{\pi y}{b}$

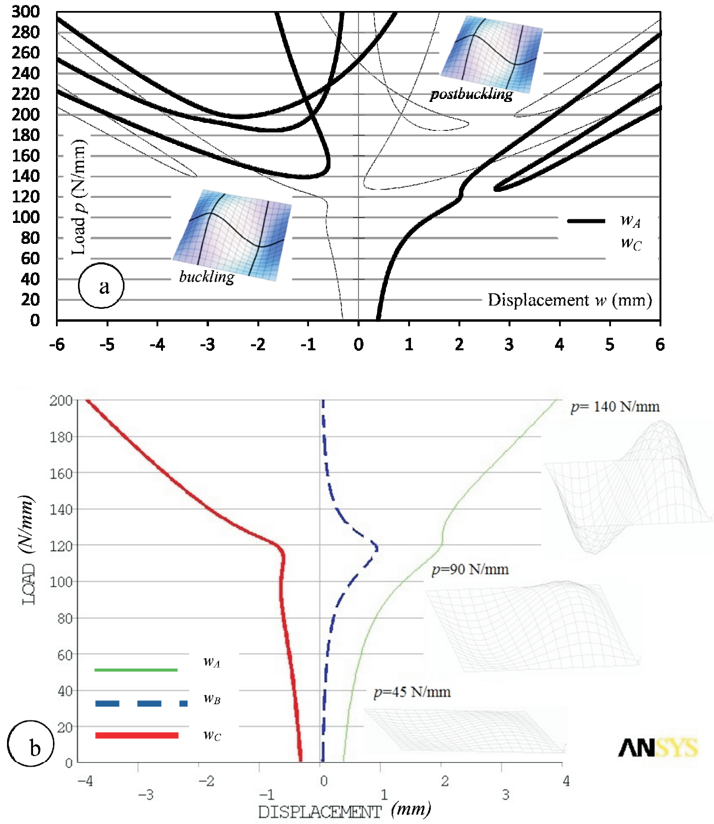

Figure 4 The postbuckling of the thin plate with initial displacement: $w_0 = 0.05 \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} + 0.35 \sin\frac{2\pi x}{a} \sin\frac{\pi y}{b}$, a) user code [4, 5], b) ANSYS system

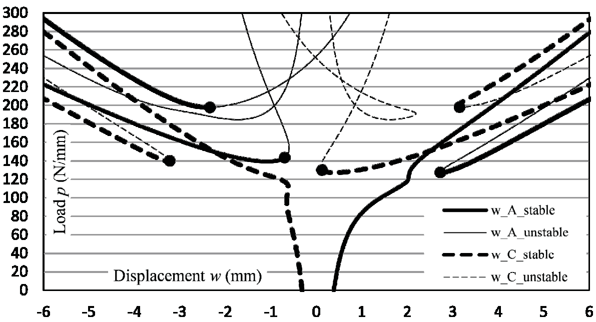

Figure 5 Stable and unstable load – displacement paths. Thin plate with initial displacement $w_0 = 0.05 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + 0.33 \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b}$

Capabilities of the user code in the analysis of post–bucking effect of thin plates were presented. Using the presented user code provides and advantage of obtaining all desired paths in the post–buckling and making their qualitative analysis. The commercial software enables one to obtain the fundamental path only. Another benefit of the user code (compared to the commercial software), is the simple detection of critical points.

Objectives specified at the beginning were accomplished. The accuracy of the results was verified by commercial computer program. For complex analysis of postbuckling of an imperfect plate it should be used more convenient user code.

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