

## Numerical and Experimental Analysis of Compression Plate with Cut-Out

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A buckling and post-buckling behavior analysis of thin-walled composite plate under a compressive force is presented. The plate with central notch was made of a carbon-epoxy composite – a laminate consisting of eight symmetrically oriented plies. The one layer arrangement was taken into consideration. The main objective of the study was to investigate the behavior of the considered plate under quasi-static compression. A numerical analysis was conducted with the Abaqus commercial FEM software package. The experimental test was performed under standard conditions on a universal Zwick/Roell Z050 testing machine. The results obtained in the numerical methods are compared to these obtained experimentally. The experimental results were then used to develop new FEM models that allowed one to describe the post-buckling behavior and to estimate the ultimate load-carrying capacity of the composite plates under investigation.

*Keywords:* laminates, buckling, Finite Element Analysis (FEA), postbuckling, experimental investigation, plate with cut-out.

### 1. Introduction

A thin-walled structures made of different composite materials, thanks to the high strength and low weight are used in the sports industry, automotive, aerospace and civil engineering. The disadvantage of these thin-walled structures is the possibility of loss of stability when operating: compressive loads and shear, among others [1–7]. However, in the case of thin-walled structural elements it is possible to operate even after the loss of stability, provided that they work in the field of elastic [8–13]. Therefore, to improve the carrying capacity of this type of construction, and even be tempted to use of this type thin-walled structures, not only as load-bearing

elements but also as elastic elements, it is proposed to improve the load capacity, which is based on forced for the construction according to the higher form of buckling (flexural–torsional). To improve the work of the plate in that form it is necessary to perform cut-out and a slight displacement of the vertical stripes in the opposite direction. The plate thus achieved the target figure characterized by sedate work in the postcritical range. Issues of stability, critical behavior and limit load capacity of plate with holes are described, among others, in [14–19].

To be sure that proposed method of calculations and prepared numerical model allow to obtain proper, close to reality, results the experimental investigations are necessary. In the design process of complex composite materials a sequence of plies plays an important role, having a decisive influence on load carrying abilities of particular components of the stress state. This applies to thin-walled composite structures and stability as well, in which a specific ply sequence can have an essential influence on a value of critical load or a structure's stiffness in postcritical behaviour [20–23].

The work dealt with the original conception of a thin-walled plate element with a central notch for use as a spring element. The research included linear and non-linear numerical analysis of stability of the structure (FEM) and the experimental validation of the results.

## 2. Subject and Scope of Research

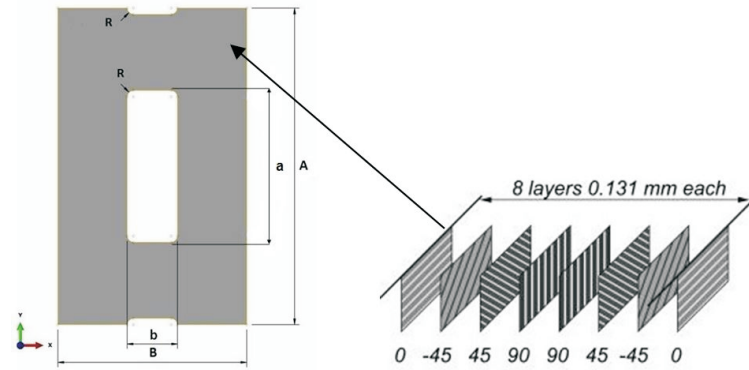
The subject of research was the thin-walled rectangular plate with overall dimensions  $A \times B = 160 \times 80$  mm and a thickness  $g = 1.048$  mm (laminate structure was composed of 8 layers of the same thickness after 0.131 mm in a symmetric arrangement of layers with respect to the median plane of the package), made of carbon–epoxy composite. Mechanical properties of the carbon–epoxy laminate used in the tests were collected in Tab. 1.

**Table 1** Mechanical properties of the carbon–epoxy laminate used in the tests

Tensile Strength [MPa]		Young Modulus [GPa]		Poisson Ratio [-]	Shearing Strength [MPa]	Shear Modulus [GPa]	Compressive Strength [MPa]	
$F_{TU}$		$E_1$	$E_2$	$\nu_{12}$	$F_{SU}$	$G_{12}$	$F_{CU}$	
$0^\circ$	$90^\circ$	$0^\circ$	$90^\circ$	$0^\circ$	$\pm 45^\circ$	$\pm 45^\circ$	$0^\circ$	$90^\circ$
187	26	131.71	6.36	0.32	100.15	4.18	1531	214

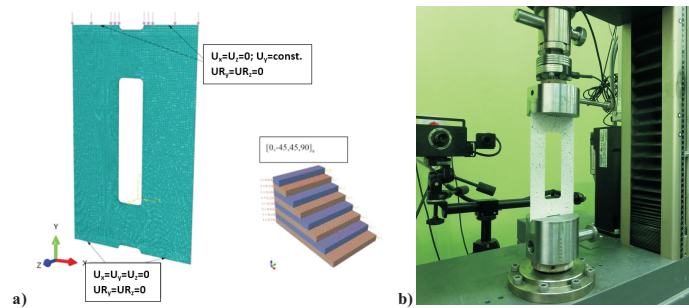
The analyzed plate had a symmetrical, centrally located cut-out (Fig. 1), with rectangular shape about dimensions  $a$  and  $b$  in the present case:  $a = 100$  mm  $b = 30$  mm. Studies were conducted on a composite plate with a  $[0 -45 45 90]_s$  layer configuration.

The plate element was supported by articulately on the top and bottom edges and loaded uniformly distributed compressive force on the upper edge of the plate.



**Figure 1** Composite profile dimensions and exemplary ply sequence

For the mapping of the real experimental conditions for nodes located on the upper edge of the plate, the conditions of an equal displacements in the direction of the  $Y$  axis was imposed. (Fig. 2a). The research was related with critical and postcritical behaviour of forced higher, flexural–torsional buckling form of plate. In order to provide an articulated support of horizontal edge of the plate, the researches have been carried out in specially made grips, allowing the free rotation of the edge and securing a fixed position of plate in the grip during the process of loading (Fig. 2b).



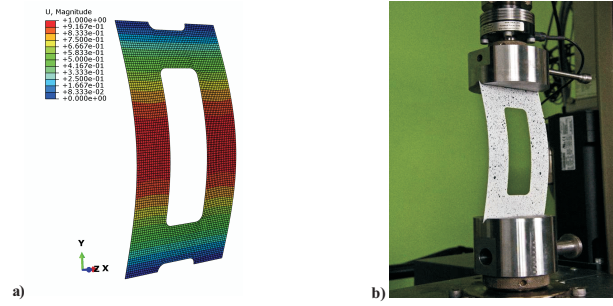
**Figure 2** a) FEM model of the tested composite plate, b) stand test with grips for compressive loading of composite plate

The scope of the research included the numerical analysis of FEM (ABAQUS software) and experimental studies conducted on a structure physical model. Numerical calculations related to the own issues and statical analysis including non-linear stability, conducted on models with initiated geometric imperfection corresponding to flexural–torsional buckling form [24, 25]. Carried out in parallel experimental studies have enabled a current verification of developed computational models. The adopted in numerical calculations method of defining boundary conditions and load of model, mapping out of the conditions for the implementation of experimental research on the testing machine.

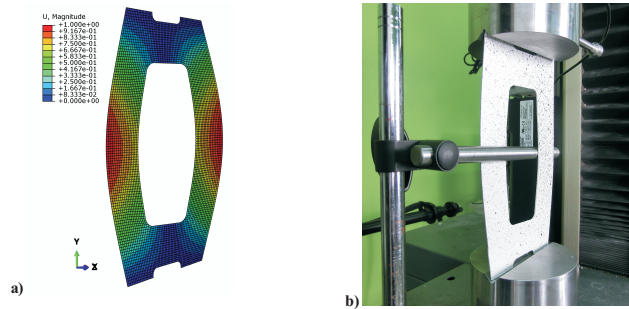
The experimental tests were carried out on the Zwick/Roell Z050 universal testing machine. All tests were conducted in standard conditions, at 23 °C with constant velocity of the cross-bar equal to 2 mm/min. During the study, the measurement of force, cross beams displacement, deflection plate at the point of maximum deformation (in half of the height plate) was recorded, using the optical system ARAMIS. Registration of these parameters has enabled adequately describe of the critical and postcritical behaviour of construction.

### 3. Results and discussion

The critical buckling mode obtained from FEM and experiment are compared in Fig. 3. From both method the same mode have been obtained. The buckling load obtained with experiment is  $P_{cr}^{EXP} = 140$  N and  $P_{cr}^{FEM} = 156.82$  N using Abaqus software correspondingly.



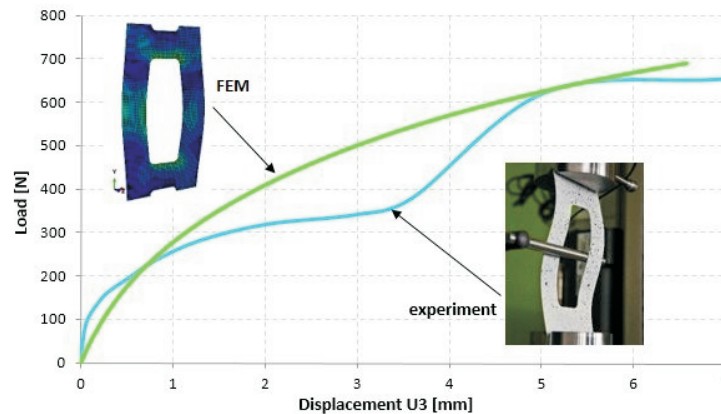
**Figure 3** First buckling modes of the composite plates [0,-45,45,90]s: a) FEM, b) experiment



**Figure 4** Higher flexural-torsional modes of the composite plates [0,-45,45,90]s: a) FEM, b) experiment



The lowest resulting buckling mode of structure with flexural form is characterized by a low critical force and a high increase of the deflection as a function of compression load, which may consequently lead to rapid its destruction. In order to ensure the greater rigidity plate in the postcritical range and the use of its as a spring element, it is necessary the extortion her work by higher flexural–torsional buckling form, which was characterized by a stable work in the postcritical range. Therefore, in the experimental study, a extortion of higher buckling form by the introduction of the initial deflection by means of a steel bar placed in the mid-plate height was used – Fig. 4.

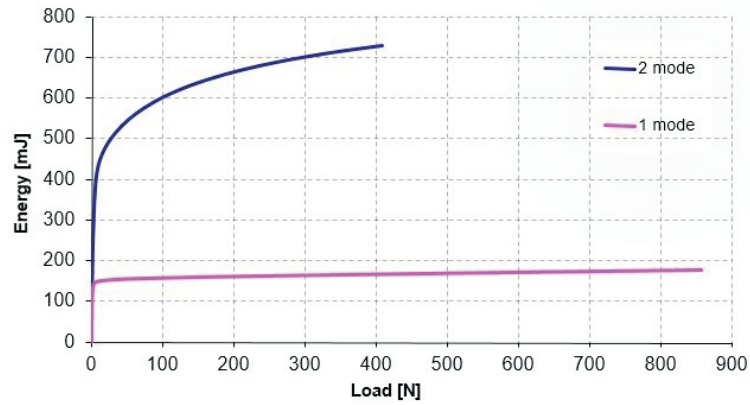


**Figure 5** Comparison of post-buckling paths appointed with FEM and experiment

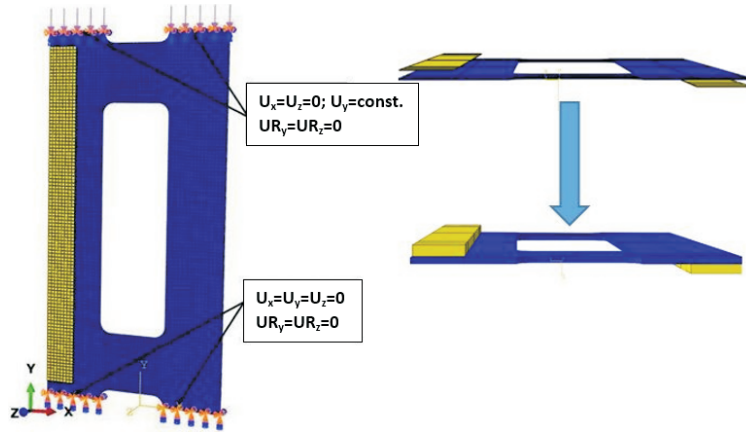
The experimental studies of compression plate with forced, flexural–torsional form of buckling, led ultimately to jumped the construction to form of postcritical deformation corresponding to the lowest (flexural) form of buckling. The different behavior of the structure showed the carried out of numerical analysis, where observed the stable postcritical works of plate by higher forced, flexural–torsional buckling form. Comparison of post–buckling paths described by dependency of the compression force to deflection of the plate in the middle of its height (P-U3), appointed with FEM and experiment, presents Fig. 5. Postcritical equilibrium path received on the basis of FEM analysis has a steady course, which exhibits a stately work of plate model according to a higher form of buckling. Whereas the resulting experimental curve exhibits a unstable of postcritical equilibrium paths for forced higher form of buckling, for which can be observed a clear curve inflection corresponding to the skipping structures at the lowest, flexural form of buckling. Observed on the chart further increase the stiffness of the physical model is caused by lateral support plates about element forced the initial deflection (rod), preventing its free deformation.

In order to explain the phenomenon of the jump of real structure from a higher forced buckling mode to the lowest mode of buckling, to the comparative analysis were subjected the received in numerical calculations charts of structure internal energy for plate with implemented the lowest (flexural) and higher (flexural–torsional)

form of loss of stability. Fig. 6 presents a graph of the internal energy as a function of loading force. A comparative analysis of internal energy for both cases showed a very high differences of energy level between the work of the construction according to the lowest and higher form of buckling. The results show more than 4-times difference in energy levels (411%), which explains the tendency of physical model to skip on a definitely lower energy level.



**Figure 6** Dependens of internal energy on compression load

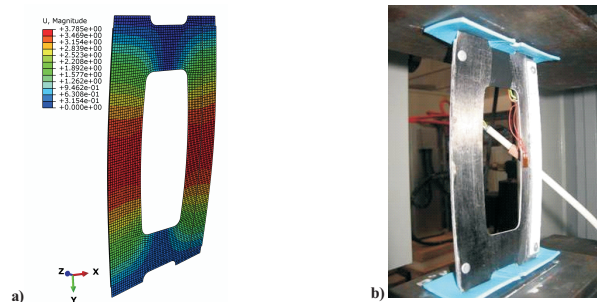


**Figure 7** FEM model of the tested composite plate with stripes

The obtained results showed that the classical approach of FEM to analysis of postcritical behaviour with implemented a higher form of buckling is not compatible with the actual behaviour of the structure. Therefore, in order to use the plate

element with cut-out as elastic element, must be sought a solution in which the flexural-torsional buckling mode will be the lowest, natural buckling form of structure. The research in this direction was carried out by adopting the disorders of symmetry of the layers of the composite idea. For this purpose, near of the outer vertical plate edges, two vertical straps made of an isotropic material (aluminum), one on each side was pasted. Numerical model of the solution with the same as the previous boundary conditions and load was shown in Fig. 7a. Discretization of vertical stiffeners was carried out analogously to discretization of plate, using membranes *Shell* elements. The combination of stiffening elements with the plate surface, was carried out by assigning suitable interface (*Tie*), enabling rigid connection of all the degrees of freedom between the plate nodes and stiffeners, while maintaining the suitable distance to accommodate the thickness of shell elements (Fig. 7b).

The performed numerical analysis showed that the accepted concept of plate with longitudinal reinforcements, the lowest form of buckling is a flexural-torsional mode, shown in Fig. 8a. The results of the calculations confirmed the experimental tests, the physical model plate with reinforcements received the loss of stability also as the flexural-torsional mode – Fig. 8b.



**Figure 8** Higher (stable) flexural-torsional modes of the composite plates  $[0,-45,45,90]_s$ : a) FEM, b) experiment

#### 4. Conclusions

The article presents the analysis of thin-walled composite plate with a central cut-out subjected to uniform compression. In order to use the plate element as a spring element, attempt to force of plate working by higher, flexural-torsional buckling form was made. The results showed that the classical approach of FEM to the analysis of postcritical behaviour of plate with implemented a higher form of buckling is incompatible with the actual behavior of the structure. In the present case FEM model showed the stately postcritical equilibrium path of constructions with implemented a higher form of buckling, while in the real model was observed the skip of construction to the lowest form of buckling. The skip to a lower energy level explains the high difference of internal energy of the structure, which in the present

case amounted to approx. 411% between of the structure working by the lowest and higher, flexural–torsional buckling form.

The applied concept of plate with longitudinal reinforcements made of an isotropic material, placed asymmetrically relative to the median plane of plate, has allowed to naturally get the lowest form of buckling plate as flexural–torsional form. This enables to the work of structure in the expected manner, without risk of jump phenomenon in postcritical behaviour. Worth the further continuation seems to be a problems related to the analysis of postcritical behaviour and the process of construction damage on the experimental and numerical way.

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