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Postbuckling of an Imperfect Plate Subjected to the Shear Load

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The stability analysis of an imperfect plate subjected to the shear load is presented, a specialized code based on FEM has been created. The nonlinear finite element method equations are derived from the variational principle of minimum of total potential energy. To obtain the nonlinear equilibrium paths, the Newton–Raphson iteration algorithm is used. Corresponding levels of the total potential energy are defined. The peculiarities of the effects of the initial imperfections are investigated. Special attention is paid to the influence of imperfections on the post–critical buckling mode. Obtained results are compared with those gained using ANSYS system.

Keywords: stability, buckling, postbuckling, geometric nonlinear theory, initial imperfection.

1. Introduction

Solving stability of the thin plate, it is often insufficient to determine the elastic critical load from eigenvalue buckling analysis, i.e. the load, when perfect plate starts buckling. It is necessary to include initial imperfections of real plate into the solution and determine limit load level more accurately. The geometrically nonlinear theory represents a basis for the reliable description of the postbuckling behaviour of the imperfect plate.

Murray and Wilson [1] first presented idea of combining incremental (Euler) and iterative (Newton–Raphson) methods for solving nonlinear problems. Early works involving critical points and snap-through effect were written by Sharifi and Popov [2], and Sabir and Lock [3]. Using arc-length method to pass limit points on load–displacement paths introduced Riks in [4]. Getting through this problem using displacement control procedure presented Batoz and Dhatt [5]. Detection of critical points using arc–length method was introduced by Wriggers and Simo [6]. Works of Bathe [7] dominate in application of FEM to geometric nonlinear problems, Crisfield [8] incorporated problematic into pc codes. Using the hitherto obtained knowledge and picking up works of Ravinger [9], the author presents results obtained as outputs of an own code for analysis of geometrically nonlinear tasks of perfect and imperfect thin plates and shells.

2. Nonlinear Theory

Restricting to the isotropic elastic material and to the constant distribution of the residual stresses over the thickness, the total potential energy can be expressed as:

$$\begin{aligned} U \; = \; & \int\limits_A \frac{1}{2} \left(\varepsilon_m - \varepsilon_{0m}\right)^T t\,\mathbf{D} \left(\varepsilon_m - \varepsilon_{0m}\right) dA + \int\limits_A \frac{1}{2} \left(\mathbf{k} - \mathbf{k}_0\right)^T \frac{t^3}{12} \mathbf{D} \left(\mathbf{k} - \mathbf{k}_0\right) dA \\ & - \int\limits_A \mathbf{q}^T \mathbf{p}\, dA \end{aligned} \tag{1}$$

where ε_m, $\mathbf{k}$ are strains and curvatures of the neutral surface, ε_{0m}, $\mathbf{k}_0$ are initial strains and curvatures, $\mathbf{q}$, $\mathbf{p}$ are displacements of the point of the neutral surface, related load vector, $\mathbf{D}$ is the elasticity matrix.

The system of conditional equations [10] one can get from the condition of the minimum of the increment of the total potential energy $\delta\,\Delta\,U = 0$. This system can be written as:

$$\mathbf{K}_{inc}\,\Delta\,\alpha + \mathbf{F}_{int} - \mathbf{F}_{ext} - \Delta\,\mathbf{F}_{ext} = \mathbf{0} \tag{2}$$

where $\mathbf{K}_{inc}$ is the incremental stiffness matrix of the plate, $\mathbf{F}_{int}$ is the internal force of the plate, $\mathbf{F}_{ext}$ is the external load of the plate, $\Delta\,\mathbf{F}_{ext}$ is the increment of the external load of the plate. Eq. (2) represents the base for the Newton–Raphson iteration and the incremental method as well. The Gauss numerical integration (5 points) was used to evaluate the stiffness matrices and the load vectors.

In the case of the structure in equilibrium $\mathbf{F}_{int} - \mathbf{F}_{ext} = \mathbf{0}$, one can execute the incremental step $\mathbf{K}_{inc}\,\Delta\,\alpha = \Delta\,\mathbf{F}_{ext} \Rightarrow \Delta\,\alpha = \mathbf{K}_{inc}^{-1}\,\Delta\,\mathbf{F}_{ext}$ and $\alpha^{i+1} = \alpha^i + \Delta\,\alpha$. The Newton-Raphson iteration can be arranged in the following way: supposing that α^i does not represent exact solution, the residua are $\mathbf{F}_{int}^i - \mathbf{F}_{ext}^i = \mathbf{r}^i$. The corrected parameters are $\alpha^{i+1} = \alpha^i + \Delta\alpha^i$, where $\Delta\alpha^i = -\mathbf{K}_{inc}^{-1}\,\mathbf{r}^i$. The identity of the incremental stiffness matrix with the Jacobbian of the system of the non-linear algebraic equation was used. Iteration process is finished using the norm:

$$\|n\| = \frac{\left(\mathbf{q^{i+1}}\right)^T . \mathbf{q^{i+1}} - \left(\mathbf{q^{i}}\right)^T . \mathbf{q^{i}}}{\left(\mathbf{q^{i+1}}\right)^T . \mathbf{q^{i}}} \leq 0.001. \tag{3}$$

3. Finite Element Method

The FEM computer program using a 48 DOF element [11] has been created for analysis. Used FEM model consists of 8x8 finite elements. Full Newton–Raphson procedure, in which the stiffness matrix is updated at every equilibrium iteration, has been applied [8]. The fundamental path of the solution starts from the zero load level and from the initial displacement. It means that the nodal displacement parameters of the initial displacements and the small value of the load parameter have been taken as the first approximation for the iterative process. To obtain other paths of the solution, random combinations of the parameters as the first approximation have been used. Interactive change of the pivot member during calculation is necessary for obtaining required number of L–D paths.

Obtained results were compared with results of the analysis using ANSYS system, where 32x32 elements model was created. Boundary conditions (UZ on all nodes along the edges, UX and UY as shown in Fig. 1b) and loads (shear) are considered the same as in user code analysis. Element type SHELL143 (4 nodes, 6 DOF at each node) was used [12]. The arc-length method was chosen for analysis, the reference arc-length radius is calculated from the load increment. Only fundamental path of nonlinear solution has been presented. Shape of the plate in postbuckling has been also displayed.

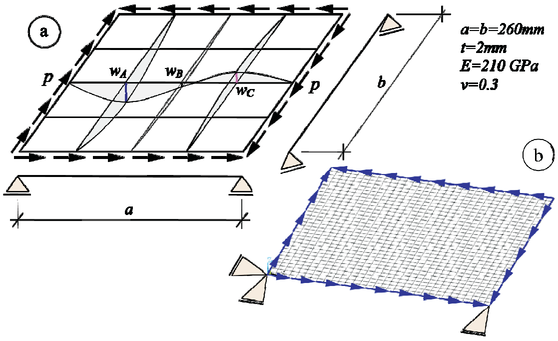


Figure 1 a) Notation of the quantities of the plate loaded in shear, b) ANSYS FEM model

4. Illustrative Examples

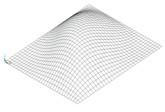
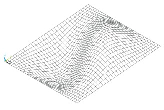
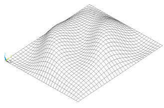
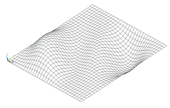
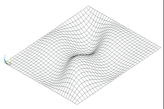
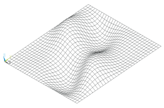
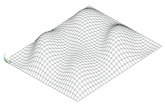
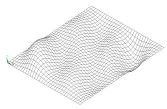
Illustrative example of steel plate loaded in shear (Fig. 1) is presented.

Results of eigenvalue buckling analysis are presented first. These serve to prepare shapes of initial geometrical imperfection [13, 14] as a linear combination of eigenvectors. Also offer an image about location of critical points of nonlinear solution, help with settings in the management of nonlinear calculation process. Results of fully nonlinear analysis follow. In order to better describe post–buckling shape of the plate, nodal displacements w_A, w_C have been taken as the reference nodes.

4.1. *Eigenvalue buckling analysis*

Eigenvalue buckling analysis predicts the theoretical buckling strength of an ideal linear elastic structure and is a problem of eigenvalues and eigenvectors. Eigenvalues define the buckling load multipliers and the corresponding eigenvectors buckling mode shapes of the structure. Results for perfect plate [15] from Fig.1 can be seen in Tab. 1.

Table 1. Buckling loads and buckling modes

209.07 [N/mm]	258.86	554.57	598.53
			
685.41	721.70	896.66	983.59
			

4.2. *Nonlinear analysis*

The geometrically nonlinear theory represents a basis for the reliable description of the postbuckling behaviour of imperfect plate [16, 17]. The result of the numerical solution of steel plate loaded in shear is presented as load – displacement paths. The initial displacements were assumed as the out of plane displacements only [18] as a combination of first three buckling modes

$$d_0 = \sum \alpha_i * MODE_i \tag{4}$$

where α_i is multiplier – appropriately selected constant (specified for each case individually), $MODE_i$ represents i-th eigenmode (dimensionless).

These presented nonlinear solutions of the postbuckling behaviour of the plate are divided into two parts. On the left side, there is load versus nodal displacement parameters relationship, on the right side the relevant level of the total potential energy is drawn [19]. (Unloaded plate represents a zero total potential energy level.)

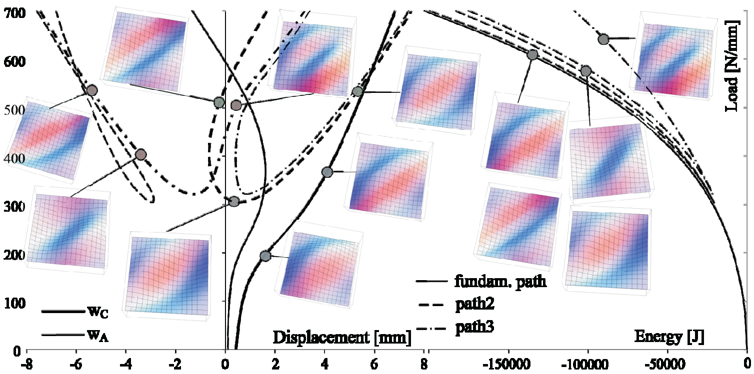


Figure 2 Results for $\alpha_1 = 0.3$ mm, $\alpha_2 = 0.2$ mm, $\alpha_3 = 0.1$ mm

Following Figures present two cases, in which the plate in a post–buckling mode buckles in the shape that is identical to a shape of initial imperfection (different from the first buckling mode obtained from eigenvalue buckling analysis). The difference consists in a fact, that while in first case the fundamental path represents the path with minimum value of the total potential energy for a given load, in the second case there exists also a path with the total potential energy level lower than that of the fundamental path [20].

Fig. 2 presents a nonlinear analysis of the plate with initial imperfection whose shape was formed from first three eigenmodes. According to Eq. (4), following parameters α were considered: $\alpha_1 = 0.3$ mm, $\alpha_2 = 0.2$ mm, $\alpha_3 = 0.1$ mm. There are presented first three loading paths representing various forms of changes between buckling shapes. Displacement w_C has been plotted by a thick line, w_A by a thin line. The Figure illustrates also shapes of the buckling area for particular paths and selected load values. In the right part, respective values of total potential energy can be seen. Fundamental path (solid line) corresponds with the minimum value of total potential energy, thus there is no presumption of a sudden snap.

For comparison with an analysis of the same plate using ANSYS software system, the fundamental path of solution is presented (see Fig. 3).

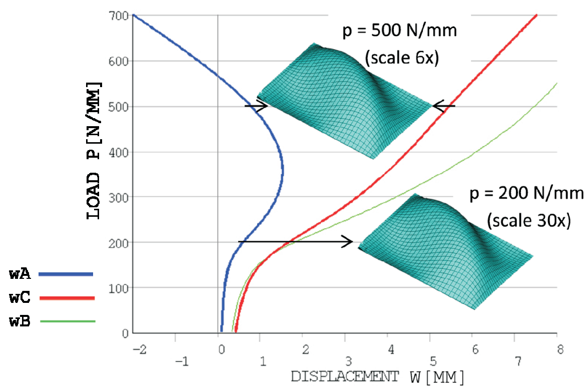


Figure 3 Fundamental path for $\alpha_1 = 0.3$ mm, $\alpha_2 = 0.2$ mm, $\alpha_3 = 0.1$ mm obtained by ANSYS

In Fig. 4 one can observe analysis of an imperfect plate with initial imperfection of a shape identical to a shape of the 2nd eigenmode. Parameter α_2 of a value 0.1mm has been considered.

Displacement w_C has been plotted by a thick line, w_A by a thin one again. Shapes of the buckling area are located next to the paths. On the right side of the Figure one can see, that the total potential energy for the fundamental path (solid line) is higher than energy for path 2 (dashed line). This path 2 represents buckling according to the 1st buckling mode, thus there is presumption of a snap effect. For comparison with an analysis of the same plate using ANSYS software system, the fundamental path of solution is presented (see Fig. 5).

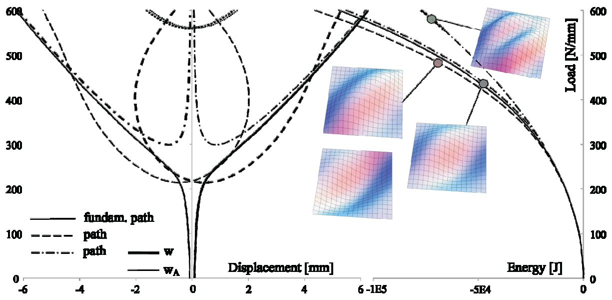


Figure 4 Results for $\alpha_2 = 0.1$ mm

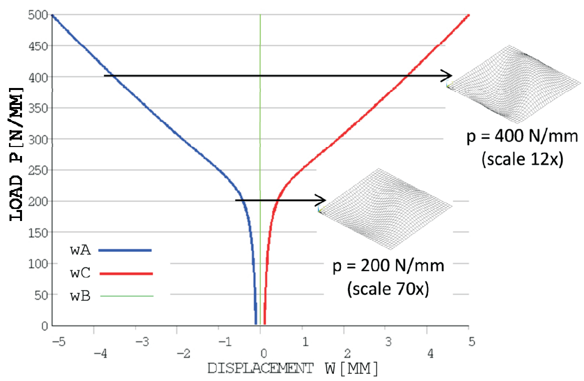


Figure 5 Fundamental path for $\alpha_2 = 0.1$ mm obtained by ANSYS

The shape of a loading path provides immediate information about the behaviour of a structure subjected to a load of a respective level. Individual paths of the presented solutions have different stability properties. This evaluation was not the subject of the paper. More detailed description of evaluation of quality of solution resulting from the analysis of the incremental stiffness matrix can be found e.g. in [20]. It should be pointed out, that $\boldsymbol{K}_{inc}$ must be in a fundamental form (load control).

5. Summary

The influence of the value of the amplitude and the mode of the initial geometrical imperfections on the postbuckling behaviour of the imperfect plate subjected to the shear load was presented. Finite elements created for special purposes of plate stability analysis, enable high accuracy and speed convergence of the solution at less density of meshing. The possibility on an interactive affecting of the calculation within the user code makes it possible to investigate all load – displacement paths of the problem.

As the important result one can note, that the level of the total potential energy of the fundamental stable path can be higher than the total potential energy of the secondary stable path. This is the assumption for the sudden change in the buckling mode of the plate. The evaluation of the level of the total potential energy for all paths of the non-linear solution is a small contribution to the investigation of the post buckling behaviour of imperfect plates. To be able to give a full answer for the mechanism of this phenomenon, more in–depth research will be required.

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