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Reflection of Plane Waves from a Rotating Magneto–Thermoelastic Medium with Two–Temperature and Initial Srtress Under Three Theories

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The model of the equations of generalized magneto-thermoelasticity in an isotropic elastic medium with two–temperature under the effect initial stress is established. The entire elastic medium is rotated with a uniform angular velocity. The formulation is applied under three theories of generalized thermoelasticity: Lord–Shulman, Green–Lindsay, as well as the coupled theory. The Harmonic function is used to obtain the exact expressions for the considered variables. Some particular cases are also discussed in the context of the problem. We introduce the equations of the velocity of p–wave, T–wave and SV-wave. The boundary conditions for mechanical and Maxwell's stresses and thermal insulated or isothermal are applied to determine the reflection coefficients for p–wave, T–wave and SV–wave. Some new aspects are obtained of the reflection coefficients and displayed graphically and the new conclusions are presented. Comparisons are also made with the results predicted by different theories (CT, L–S, G–L) in the presence of rotation, initial stress, magnetic field, as well as, the two-temperature parameter on the reflection of generalized thermo–elastic waves.

Keywords: magnetic field, rotation, iInitial stress, reflection, p–wave, T–wave.

1. Introduction

The generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity [1]. The first generalization involving one thermal relaxation time is discussed by Lord and Shulman [2]. The temperature rate-dependent thermoelasticity is developed where includes two thermal relaxation times is discussed by Green and Lindsay [3]. One can review and presentation of generalized theories of thermoelasticity is discussed by Hetnarski and Ignaczak [4]. The wave propagation in anisotropic solids in generalized theories of thermoelasticity has studied [5-7].

Propagation of plane waves at the interface of an elastic solid half–space and a microstretch thermoelastic diffusion, solid half–space was investigated by Kumar et al. [8]. The problem related to reflection and refraction of elastic waves from the boundaries of different media has been discussed in the famous book by Achenbach [9]. Mathematical work has been performed for the propagation of elastic waves in a dissipative medium; see for instance, have been studied by many authors [10–13]. By using the Biot's theory [14] several researchers have studied extensively the propagation of elastic waves [15–18].

Kaur and Sharma [19] discussed reflection and transmission of thermoelastic plane waves at liquid–solid interface. Borejko [20] introduced the reflection and transmission coefficients for three dimensional plane waves in elastic media. Singh and Bala [21] purposed the reflection of p– and SV–waves from the free surface of a two–temperature thermoelastic solid half–space. Singh [22] and Abd–alla et al. [23] studied the reflection of magneto–thermo–viscoelastic waves in different approaches. Abo–Dahab and Mohamed [24] and Abo–Dahab [25] studied the reflection of magneto–thermoelastic p- and SV-waves under different conditions. Singh and Yadav [26] discussed the reflection of plane waves in a rotating transversely isotropic magneto-thermoelastic solid half–space. Some researchers have investigated different problems of rotating media. Schoenberg and Censor [27] studied the propagation of plane harmonic waves in a rotating elastic medium without a thermal field. The effect of rotation on elastic waves was discussed [28, 29]. Abo–Dahab and Abbas [30] discussed the thermal shock problem of generalized magneto–thermoelasticity with variable thermal conductivity. Abo–Dahab and Singh [31] studied the influence of magnetic field on wave propagation in a generalized thermoelastic solid with diffusion. Also, an investigation of the distribution of deformation, stresses and magnetic field in a uniformly rotating, homogeneous, isotropic, thermally and electrically conducting elastic half–space was presented by Chand et al. [32]. The effect of rotation, initial stress and temperature dependent elastic moduli in a magneto–thermoelastic medium has been studied by Othman and Song [33–35]. Chakraborty [36] discussed reflection of plane elastic waves in half–space subjected to temperature and initial stress.

The present paper is concerned with the investigations related to the effect of rotation and initial stress with two–temperature on a generalized thermoelastic medium under three theories by applying the normal mode analysis, also, the effect of rotation and initial stress on the physical quantities. The results are plotted with MATLAB software to show the effect of temperature, magnetic field, relaxation time and initial stresses on the reflection.

2. Formulation of the problem and basic equations

We consider a homogeneous thermoelastic half–space with two–temperature under the initial stress and magnetic field. All quantities are considered are functions of the time variable t and of the coordinates x and y. We consider the normal source acting on the plane surface of generalized thermoelastic half–space under the influence of

gravity. The system of governing equations of a linear thermoelasticity with initial stress and without body forces consists of:
The stress–strain relation written as:

$$\sigma_{ij} = -p(\delta_{ij} + \omega_{ij}) + 2\mu e_{ij} + \lambda e\delta_{ij} - \gamma(1 + \upsilon_0 \frac{\partial}{\partial t})T\delta_{ij} \quad (1)$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j})i, j = 1, 2 \quad (2)$$

The heat conduction equation:

$$K\Theta_{,ii} = \rho C_E(1 + \tau_0 \frac{\partial}{\partial t})\dot{T} + \gamma T_0(1 + n_0\tau_0 \frac{\partial}{\partial t})\dot{e} \quad (3)$$

$$T = \Theta - a^* \Theta_{,ii} \quad (4)$$

where σ_{ij} are the stress components, λ, μ are the Lame' constants, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t is the thermal expansion coefficient, δ_{ij} is the Kronecker delta, T is the temperature above the reference temperature T_0, k is the thermal conductivity, n_0 is a parameter, τ_0, υ_0 are the relaxation times, ρ is the density, C_E is the specific heat at constant strain, a^* is a constant, Θ is the conductive temperature and p is an initial stress.
The equations of motion:
Since the medium is rotating uniformly with an angular velocity $\Omega = \Omega n$ where n is a unit vector representing the direction of the axis of the rotation, the equation of motion in the rotating frame of reference has two additional terms (Schoenberg and Censor [27]): centripetal acceleration $\Omega \times (\Omega \times u)$ due to time varying motion only and Corioli's acceleration $2\,\Omega \times \dot{u}$, then the equation of motion in a rotating frame of reference is:

$$\rho[\ddot{u}_i + \{\Omega \times (\Omega \times u)\}_i + 2(\Omega \times \dot{u})_i] = \sigma_{ij,j} + F_i, \quad i, j = 1, 2, 3 \quad (5)$$

Where $F_i$is the Lorentz force and is given by:

$$F_i = \mu_0(J \times H)_{\mathrm{i}} \quad (6)$$

The variation of the magnetic and electric fields are perfectly conducting slowly moving medium and are given by Maxwell's equations:

$$\mathrm{curl} h = J + \varepsilon_0 \dot{E} \quad (7)$$

$$\mathrm{curl} E = -\,\mu_0 \dot{h} \quad (8)$$

$$E = -\,\mu_0(\dot{u} \times H \quad (9)$$

$$\mathrm{div}\ h = 0 \quad (10)$$

Where μ_0 is the magnetic permeability, ε_0 is the electric permeability, J is the current density vector, E is the induced electric field vector and h is the induced magnetic field vector.

Expressing components of the vector $J=(J_1, J_2, J_3)$ in terms of the displacement by eliminating the quantities h and E from Eq. (7), thus yields:

$$J_1=\mathrm{h}_{,\mathrm{y}}+\varepsilon_0\mu_0\mathrm{H}_0\ddot{\mathrm{v}} \qquad J_2=-\ \mathrm{h}_{,\mathrm{x}}-\varepsilon_0\mu_0\mathrm{H}_0\ddot{\mathrm{u}} \qquad J_3=0 \tag{11}$$

Substituting from equation (11) in equation (6), we get:

$$F_1=-\ \mu_0\mathrm{H}_0\mathrm{h}_{,\mathrm{x}}-\varepsilon_0\mu_0^2\mathrm{H}_0^2\ddot{\mathrm{u}} \qquad F_2=-\ \mu_0\mathrm{H}_0\mathrm{h}_{,\mathrm{y}}-\varepsilon_0\mu_0^2\mathrm{H}_0^2\ddot{\mathrm{v}} \qquad F_3=0 \tag{12}$$

From Eqs. (2), (12) in Eq. (5), we get:

$$\rho(\ddot{u}-\Omega^2 u-2\Omega\,\dot{v})=(\mu-p/2)\nabla^2 u+(\lambda+\mu+p/2)e_{,x}-\gamma(1+\upsilon_0\frac{\partial}{\partial t})T_{,x}$$
$$-\mu_0 H_0 h_{,x}-\varepsilon_0\mu_0^2 H_0^2\ddot{u} \tag{13}$$

$$\rho(\ddot{v}-\Omega^2 v+2\Omega\,\dot{u})=(\mu-p/2)\nabla^2 v+(\lambda+\mu+p/2)e_{,y}-\gamma(1+\upsilon_0\frac{\partial}{\partial t})T_{,y}$$
$$-\mu_0 H_0 h_{,y}-\varepsilon_0\mu_0^2 H_0^2\ddot{v} \tag{14}$$

From Eqs. (7)-(10), we can obtain that:

$$h=-\ H_0\ e \tag{15}$$

Eq. (3) and Eqs. (13), (14) are the field equations of the generalized linear magneto–thermoelasticity for a rotating media, applicable to the coupled theory, three generalizations, as follows:

- The equations of the coupled (CT) theory, when:

$$\tau_0=\upsilon_0=0$$

- Lord-Shulman (L-S) theory, when:

$$n_0=1,\quad \upsilon_0=0,\ \ \tau_0>0$$

- Green-Lindsay (G-L) theory, when:

$$n_0=0,\quad \upsilon_0\geq\tau_0>0$$

The constitutive relations, using Eq. (2), can be written as:

$$\sigma_{\mathrm{xx}}=(\lambda+2\mu)\mathrm{u}_{,\mathrm{x}}+\lambda\mathrm{v}_{,\mathrm{y}}-\gamma(1+\upsilon_0\frac{\partial}{\partial t})T-p \tag{16}$$

$$\sigma_{\mathrm{yy}}=(\lambda+2\mu)\mathrm{v}_{,\mathrm{y}}+\lambda\mathrm{u}_{,\mathrm{x}}-\gamma(1+\upsilon_0\frac{\partial}{\partial t})T-p \tag{17}$$

$$\sigma_{\mathrm{xy}}=\mu(\mathrm{u}_{,\mathrm{y}}+\mathrm{v}_{,\mathrm{x}})-(p/2)(\mathrm{v}_{,\mathrm{x}}-\mathrm{u}_{,\mathrm{y}}) \tag{18}$$

For simplification, the following non–dimensional variables are used:

$$\begin{aligned}
&x_i'=\frac{\omega^*}{c_0}x_i \qquad u_i'=\frac{\rho c_0\omega^*}{\gamma T_0}u_i \qquad \{t',\tau_0',\upsilon_0'\}=\omega^*\{t,\tau_0,\upsilon_0\}\\
&\{T',\Theta'\}=\frac{\{T,\ \Theta\}}{T_0} \qquad \sigma_{ij}'=\frac{\sigma_{ij}}{\gamma T_0} \qquad h'=\frac{\mathrm{h}}{\mathrm{H}_0}\\
&\Omega'=\frac{\Omega}{\omega^*} \qquad p'=\frac{p}{\lambda+2\mu} \qquad \mathrm{i,j}=1,2,3
\end{aligned} \tag{19}$$

where: $\omega^* = \rho C_E c_0^2/K$ and $c_0^2 = (\lambda + 2\mu)/\rho$.
In terms of the non–dimensional quantities defined in (19) and using (15), the above governing equations take the form (dropping the primes over the non–dimensional variables for convenience):

$$\alpha\ddot{u} - \Omega^2 u - 2\Omega\dot{v} = \beta\nabla^2 u + h_2\frac{\partial e}{\partial x} - (1 - a\nabla^2)(1 + \upsilon_0\frac{\partial}{\partial t})\Theta_{,x} \tag{20}$$

$$\alpha\ddot{v} - \Omega^2 v + 2\Omega\dot{u} = \beta\nabla^2 v + h_2\frac{\partial e}{\partial y} - (1 - a\nabla^2)(1 + \upsilon_0\frac{\partial}{\partial t})\Theta_{,y} \tag{21}$$

$$\nabla^2\Theta = (1 + \tau_0\frac{\partial}{\partial t})\,\dot{\Theta} + \varepsilon(1 + n_0\tau_0\frac{\partial}{\partial t})\dot{e} \tag{22}$$

$$T = (1 - a\nabla^2)\Theta \tag{23}$$

where: $\alpha = 1 + \frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}$, $\beta = \frac{\mu}{\rho c_0^2} - \frac{p}{2}$, $h_2 = h_1 + h_0$, $h_1 = \frac{\lambda+\mu}{\rho c_0^2} + \frac{p}{2}$, $h_0 = \frac{\mu_0 H_0^2}{\rho c_0^2}$, $\varepsilon = \frac{\gamma^2 T_0}{\rho^2 C_E c_0^2}$, $a = \frac{a^*\omega^{*2}}{c_0^2}$. Also, the constitutive relations (16)–(18) reduces to:

$$\sigma_{\mathrm{xx}} = \mathrm{u}_{,\mathrm{x}} + \mathrm{a}_1\mathrm{v}_{,\mathrm{y}} - (1 + \upsilon_0\frac{\partial}{\partial t})T - p_1 \tag{24}$$

$$\sigma_{\mathrm{yy}} = \mathrm{v}_{,\mathrm{y}} + \mathrm{a}_1\mathrm{u}_{,\mathrm{x}} - (1 + \upsilon_0\frac{\partial}{\partial t})T - P_1 \tag{25}$$

$$\sigma_{\mathrm{xy}} = a_3\mathrm{u}_{,\mathrm{y}} + a_4\mathrm{v}_{,\mathrm{x}} \tag{26}$$

$$a_1 = \frac{\lambda}{\rho c_0^2} \qquad P_1 = a_2 P \qquad a_2 = \frac{\lambda + 2\mu}{\gamma T_0} \qquad a_3 = \frac{\mu + p/2}{\rho c_0^2} \qquad a_4 = \frac{\mu - p/2}{\rho c_0^2}$$

We shall consider only the two-dimensional problem. Assuming that all variables are functions of space coordinates x, y and time t and independent of coordinate z. So the displacement components are $u_x = u(x, y, t)$, $u_y = \nu(x, y, t)$, $u_z = 0$.
We introduce the scalar potential $\Phi(x, y, t)$ and the vector potential $\Psi(x, y, t)$ which related to displacement components by the relations:

$$u = \Phi_{,x} + \Psi_{,y}, \nu = \Phi_{,y} - \Psi_{,x} \tag{27}$$

Using Eqs. (1)–(3), the two-dimensional equations of motion and the heat-conduction equation become, respectively:

$$[\alpha\tfrac{\partial^2}{\partial t^2} - \Omega^2 - (\beta + h_2)\nabla^2]\Phi + 2\Omega\dot{\Psi} + (1 - a\nabla^2)(1 + \upsilon_0\frac{\partial}{\partial t})\Theta = 0 \tag{28}$$

$$(\alpha\tfrac{\partial^2}{\partial t^2} - \Omega^2 - \beta\,\nabla^2)\,\Psi - 2\Omega\,\dot{\Phi} = 0 \tag{29}$$

$$\nabla^2\Theta = (1 + \tau_0\frac{\partial}{\partial t})(1 - a\nabla^2)\dot{\Theta} + \varepsilon(1 + n_0\tau_0\frac{\partial}{\partial t})\nabla^2\dot{\Phi} \tag{30}$$

We assume now the solution of Eqs. (28)–(30) takes the following form:

$$\{\Phi, \Psi, \Theta\} = \left\{\bar{\Phi}, \bar{\Psi}, \bar{\Theta}\right\}\,\exp[i\xi(x\sin\theta + y\cos\theta) - i\omega t] \tag{31}$$

Substitute from Eq. (31) in Eqs. (28)–(30), we get:

$$(\xi^2 - b_1)\bar{\Phi} - b_2\bar{\Psi} + (b_3 + b_4\xi^2)\Theta = 0 \tag{32}$$

$$(\xi^2 - b_5)\bar{\Psi} + b_6\bar{\Phi} = 0 \tag{33}$$

$$(\xi^2 - b_7)\Theta + b_8\xi^2\bar{\Phi} = 0 \tag{34}$$

$$\begin{array}{lll} b_1 = (\alpha\omega^2 + \Omega^2)/(\beta + h_2) & b_2 = 2i\omega\Omega/(\beta + h_2) & b_3 = (1 - i\omega\upsilon_0)/(\beta + h_2) \\ b_4 = a(1 - i\omega\upsilon_0)/(\beta + h_2) & b_5 = (\alpha\omega^2 + \Omega^2)/\beta & b_6 = 2i\omega\Omega/\beta \\ b_7 = \frac{i\omega(1-i\omega\tau_0)}{[1-i\omega a(1-i\omega\tau_0)]} & b_8 = \frac{i\omega\varepsilon(1-i\omega n_0\tau_0)}{[1-i\omega a(1-i\omega\tau_0)]} & \end{array}$$

Equations (32)–(34) have a nontrivial solution if and only if the determinant vanished, so:

$$\begin{vmatrix} \xi^2 - b_1 & -b_2 & b_3 + b_4\xi^2 \\ b_6 & \xi^2 - b_5 & 0 \\ b_8\xi^2 & 0 & \xi^2 - b_7 \end{vmatrix} = 0$$

$$A\xi^6 + B\xi^4 + C\xi^2 + D = 0 \tag{35}$$

$$A = 1 - b_4b_8 \qquad B = -\,b_1 - (b_5 + b_7) - b_8b_3 + b_5b_8b_4$$

$$C = b_5b_7 + b_1(b_5 + b_7) + b_2b_6 + b_5b_8b_3 \qquad D = -\,b_1b_5b_7 - b_2b_6b_7$$

3. Solution of the problem

Where, Eq. (32) has three roots in ξ^2, there are three coupled waves T–wave, p–wave and SV–wave with three different velocities. Assuming that the radiation in vacuum is neglected, when a coupled wave falls on the boundary $z = 0$ from within the thermoelastic medium, it will make an angle θ with the negative direction of the z-axis, and three reflected waves that will make angles θ and θ_i $(i = 1, 2, 3)$ with the same direction as shown in Fig. 1.

The displacement potentials, Φ and Ψ, the conductive temperature Θ, will take the following forms:

$$\begin{aligned} \Phi &= A_0 \exp[i\xi_0(x\sin\theta_0 + y\cos\theta_0) - i\omega t] \\ &+ \sum_{j=1}^{3} A_j \exp[i\xi_j(x\sin\theta_j - y\cos\theta_j) - i\omega t] \end{aligned} \tag{36}$$

$$\begin{aligned} \bar{\Psi} &= B_0 \exp[i\xi_0(x\sin\theta_0 + y\cos\theta_0) - i\omega t] \\ &+ \sum_{j=1}^{3} B_j \exp[i\xi_j(x\sin\theta_j - y\cos\theta_j) - i\omega t] \end{aligned} \tag{37}$$

$$\begin{aligned}\Theta = \ & C_0 \exp[i\xi_0(x\sin\theta_0 + y\cos\theta_0) - i\omega\, t] \\ & + \sum_{j=1}^{3} C_j \exp[i\xi_j(x\sin\theta_j - y\cos\theta_j) - i\omega\, t]\end{aligned} \tag{38}$$

where: $B_j = \eta_j A_j$, $C_j = D_j A_j$, $\eta_j = \frac{-\,b_6}{\xi_j^2 - b_5}$, $D_j = \frac{-\,b_8\xi_j^2}{\xi_j^2 - b_7}$, $j = 0, 1, 2, 3$

Substitute from Eqs. (36)–(38) in Eq. (27), we get:

$$u = i\xi_0(A_0\sin\theta_0 + B_0\cos\theta_0)\exp[i\xi_0(x\sin\theta_0 + y\cos\theta_0) - i\omega\, t]$$

$$+ \sum_{j=1}^{3} i\xi_j(A_j\sin\theta_j - B_j\cos\theta_j)\exp[i\xi_j(x\sin\theta_j - y\cos\theta_j) - i\omega\, t] \tag{39}$$

$$v = i\xi_0(A_0\cos\theta_0 - B_0\sin\theta_0)\exp[i\xi_0(x\sin\theta_0 + y\cos\theta_0) - i\omega\, t]$$

$$- \sum_{j=1}^{3} i\xi_j(A_j\cos\theta_j - B_j\sin\theta_j)\exp[i\xi_j(x\sin\theta_j - y\cos\theta_j) - i\omega\, t] \tag{40}$$

Substitute from Eqs. (39), (40), (23), (38) in Eq. (24)–(26), we get:

$$\begin{aligned}&\sigma_{\mathrm{xx}} = [-\sin^2\theta_0\xi_0^2 A_0 + (a_1 - 1)\tfrac{B_j}{2}\sin 2\theta_0\xi_0^2 - a_1\cos^2\theta_0\xi_0^2 A_0 \\ &+(1 - i\omega\, v_0 + \xi_0^2 a - i\omega\, v_0 a\xi_0^2)C_0]\exp[i\xi_0(x\sin\theta_0 + y\cos\theta_0) - i\omega\, t] \\ &+\sum_{j=1}^{3}[A_j\sin^2\theta_j\xi_j^2 + (1 - a_1)\tfrac{B_j}{2}\sin 2\theta_j\xi_j^2 + A_j a_1\cos^2\theta_j\xi_j^2 \\ &+(1 - i\omega\, v_0 + \xi_j^2 a - i\omega\, v_0 a\xi_j^2)C_j]\exp[i\xi_j(x\sin\theta_j - y\cos\theta_j) - i\omega\, t] - p_1\end{aligned} \tag{41}$$

$$\begin{aligned}&\sigma_{yy} = [\,-\,\xi_0^2(a_1(A_0\sin\theta_0 + B_0\cos\theta_0)\sin\theta_0 + (A_0\cos\theta_0 - B_0\sin\theta_0)\cos\theta_0] \\ &+(1 - i\omega\, v_0 + \xi_0^2 a - i\omega\, v_0 a\xi_0^2)C_0]\exp[i\xi_0(x\sin\theta_0 + y\cos\theta_0) - i\omega\, t] \\ &+\sum_{j=1}^{3}\{\xi_j^2[(A_j\cos\theta_j - B_j\sin\theta_j)\cos\theta_j - a_1(A_j\cos\theta_j - B_j\sin\theta_j)\sin\theta_j)] \\ &+(1 - i\omega\, v_0 + \xi_j^2 a - i\omega\, v_0 a\xi_j^2)C_j\}\exp[i\xi_j(x\sin\theta_j - y\cos\theta_j) - i\omega\, t] - p_1\end{aligned} \tag{42}$$

$$\begin{aligned}&\sigma_{\mathrm{xy}} = [-(a_3 + a_4)A_0\frac{\xi_0^2}{2}\sin 2\theta_0 + (-a_3\cos^2\theta_0 + a_4\sin^2\theta_0)\xi_0^2 B_0] \\ &\exp[i\xi_0(x\sin\theta_0 + y\cos\theta_0) - i\omega\, t] + \sum_{j=1}^{3}[A_j(a_3 + a_4)\frac{\xi_j^2}{2}\sin 2\theta_j \\ &+(-\,a_3\cos^2\theta_j + a_4\sin^2\theta_j)\xi_j^2 B_j]\exp[i\xi_j(x\sin\theta_j - y\cos\theta_j) - i\omega\, t]\end{aligned} \tag{43}$$

3. Boundary conditions

On the plane surface $y=0$, are:

$$\sigma_{xx}+\tau_{xx}=-p_1 \qquad \sigma_{xy}+\tau_{xy}=0 \qquad \frac{\partial\Theta}{\partial y}=0 \tag{44}$$

where, Maxwell's stresses are as follows:

$$\tau_{ij}=\mu_e[H_i h_j+H_j h_i-H_k h_k\delta_{ij}]$$

For the reflected waves, the wave numbers and the reflected angles may be written as:

$$\xi_0\sin\theta_0=\xi_1\sin\theta_1=\xi_2\sin\theta_2=\xi_3\sin\theta_3 \tag{45}$$

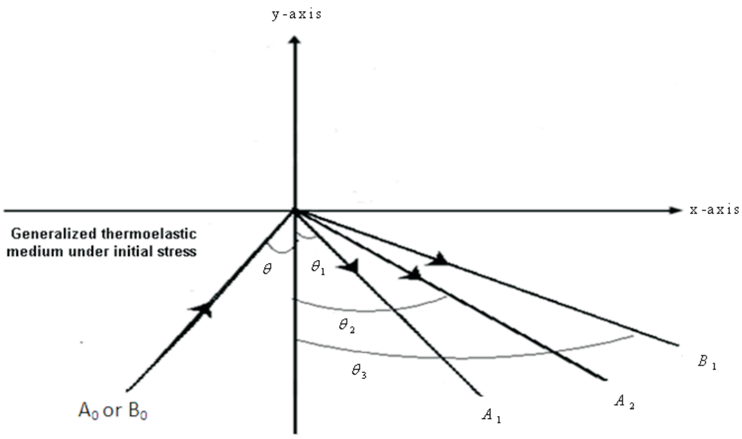

Figure 1 Geometry of the problem

Substituting from Eqs. (38), (41), (42) into the boundary conditions in Eq. (44), we obtain a system of three algebraic equations takes the form:

$$\sum A_{ij}X_j=B_i \qquad i,j=1,2,3 \tag{46}$$

$$\begin{aligned}
&A_{1j}=D_j\xi_j\cos\theta_j\\
&A_{2j}=(a_3+a_4)\frac{\xi_j^2}{2}\sin 2\theta_j+(-\,a_3\cos^2\theta_j+a_4\sin^2\theta_j)\xi_j^2\eta_j\\
&A_{3j}=-\mu_e H_0^2\xi_j^2-\sin^2\theta_j\xi_j^2+(1-a_1)\tfrac{\eta_j}{2}\sin 2\theta_j\xi_j^2+a_1\cos^2\theta_j\xi_j^2\\
&+(1-i\omega\,\upsilon_0+\xi_j^2a-i\omega\,\upsilon_0a\xi_j^2)D_j
\end{aligned}$$

and:

$$X_1=\frac{A_1}{A_0} \qquad X_2=\frac{A_2}{A_0} \qquad X_3=\frac{A_3}{A_0} \qquad B_1=D_0\xi_0\cos\theta_0$$

$$B_2 = (a_3 + a_4)\frac{\xi_0^2}{2}\sin 2\theta_0 + (a_3\cos^2\theta_0 - a_4\sin^2\theta_0)\xi_0^2\eta_0$$

$$B_3 = \mu_e H_0^2\xi_0^2 + \sin^2\theta_0\xi_0^2 + (1-a_1)\tfrac{\eta_j}{2}\sin 2\theta_0\xi_0^2 + a_1\cos^2\theta_0\xi_0^2$$
$$-(1 - i\omega\, \upsilon_0 + \xi_0^2 a - i\omega\, \upsilon_0 a\xi_0^2)D_0$$

where A_0, A_1, A_2, A_3 are the amplitudes of the incident and reflected waves respectively.

4. Numerical results and discussion

In the view to illustrate the computational work, the following material constants at $T_0 = 293\ ^o$C are considered a copper material for an elastic solid with generalized thermoelastic solid as follow:

$$\rho = 8954\, kgm^{-3} \quad \mu = 3.86\times 10^{10}\, kgm^{-1}s^{-2} \quad \lambda = 7.76\times 10^{10}\, kgm^{-1}s^{-2}$$
$$\mu_e = 4\pi(10)^{-7}Hm^{-1} \quad \mu_0 = 4.0\times 10^{11}\, dyne/cm^2 \quad k = 386\, m^{-1}k^{-1}$$
$$\alpha_t = 1.78\times 10^{-5}k^{-1} \quad C_E = 383.1\ J/kgK \quad \varepsilon_0 = 0.1\times 10^{-4}$$
$$a^* = 0.1\times 10^{-3} \quad \omega = \omega_0 + i\zeta \quad \omega_0 = 2 \quad \zeta = 1$$

From Fig. 2–13, we can see that for the reflection coefficient $X_1 = 1$ while, the reflection coefficients X_2 and X_3 equal zero when the angle of incidence $\theta = 90^o$.

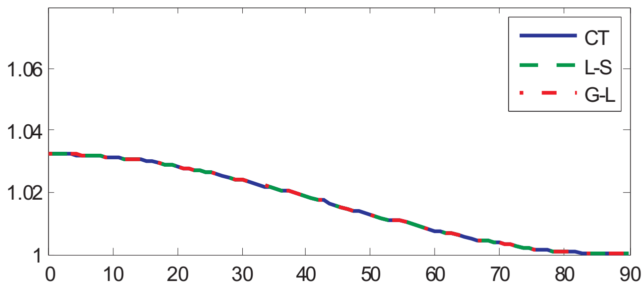

Figure 2 Variation of the reflection coefficient X_1 with varies values of the angle of incidence under three theories

Figs. 2–4 display a comparison between the three thermoelastic theories (i.e. CT, L–S and G–L). It is shown from Fig. 4 that the values of the reflection coefficient X_3 considering (CT) theory is less than the corresponding value considering (L–S) theory less than it takes into account (G–L) theory attending to zero at $\theta = 90^o$, while in Figs. 2 and 3 it is shown that X_1 and X_2 do not affect by the variation of three theories.

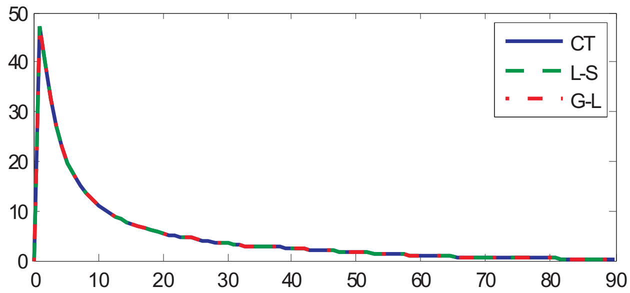

Figure 3 Variation of the reflection coefficient X_2 with varies values of the angle of incidence under three theories

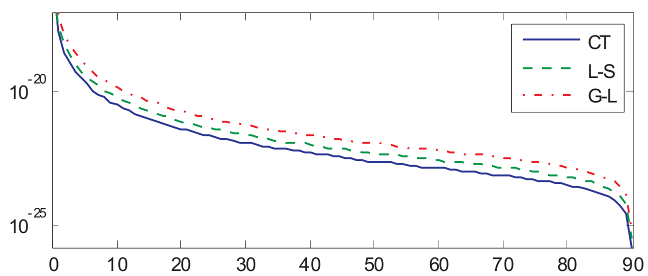

Figure 4 Variation of the reflection coefficient X_3 with varies values of the angle of incidence under three theories

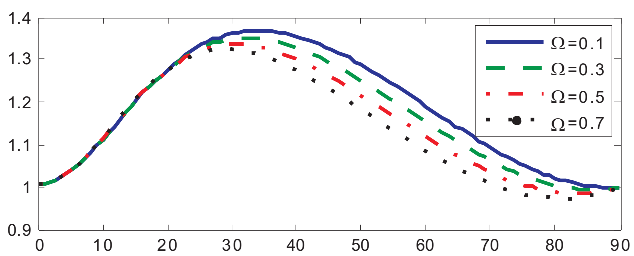

Figure 5 Variation of the reflection coefficient X_1 with varies values of the angle of incidence under the rotation

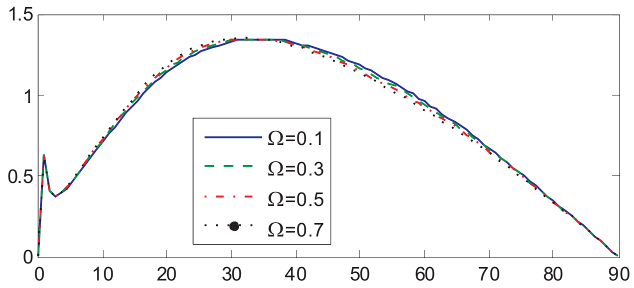

Figure 6 Variation of the reflection coefficient X_2 with varies values of the angle of incidence under the rotation

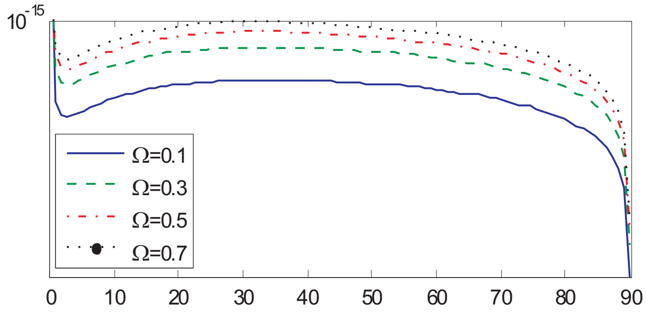

Figure 7 Variation of the reflection coefficient X_3 with varies values of the angle of incidence under the rotation

Figs. 5–7 display the influence of rotation parameter $\Omega = 0.1, 0.3, 0.5, 0.7$ on the amplitudes of the reflected waves p–, T–, and SV–waves due to the incident p–wave with respect to the angle of incidence θ. We note that the amplitude of reflected p–wave with smaller values of the angle of incidence (i.e. $\theta = 0^o$, 25^o) increases with the increase of the rotation, but it increases with a decreasing of it with the increasing of angle of incidence (i.e. $\theta = 25^o$, 90^o) attends to unity at $\theta = 90^o$. Concern the amplitude of X_2 for the reflected T–wave, increases with an increasing of the rotation with smaller values of the angle of incidence (i.e. $\theta = 0^o, 40^o$), while decreases with the increase of the angle of incidence (i.e. $\theta = 40^o$, 90^o). Concern the amplitude of X_3 for the reflected SV–wave, increases with an increasing of the rotation with smaller values of the angle of incidence (i.e. $\theta = 0^o$, 90^o) attend to zero at $\theta = 90^o$.

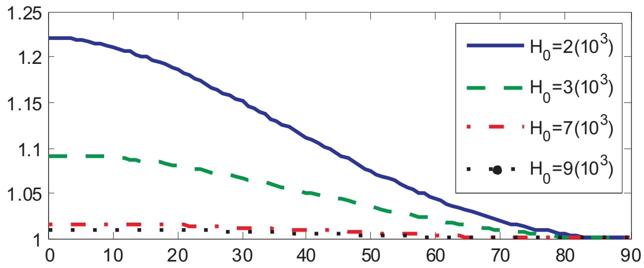

Figure 8 Variation of the reflection coefficient X_1 with varies values of the angle of incidence under the magnetic field

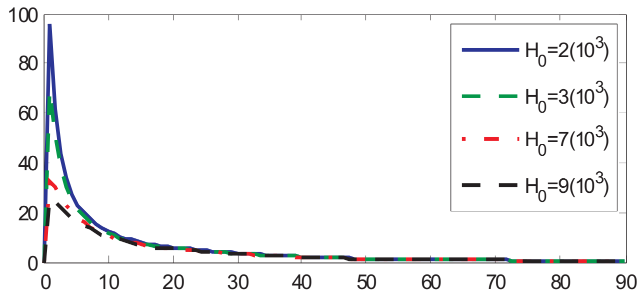

Figure 9 Variation of the reflection coefficient X_2 with varies values of the angle of incidence under the magnetic field

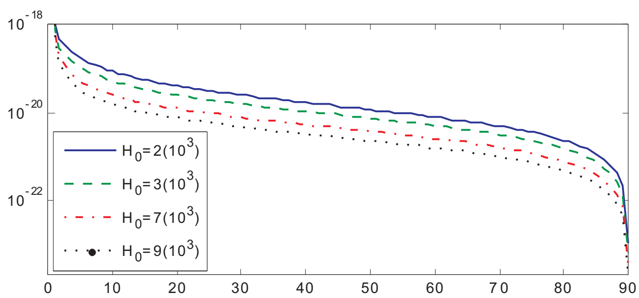

Figure 10 Variation of the reflection coefficient X_3 with varies values of the angle of incidence under the magnetic field

Figs. 8–10, display the influence of magnetic field parameter $H_0 = (2, 3, 7, 9) \times 10^3$on the amplitudes of the reflected waves p–, T–, and SV–waves due to the incident p–wave with respect to the angle of incidence θ. We note that the amplitude of reflected p–wave with smaller values of the angle of incidence (i.e. $\theta = 0^o$, 90^o) decrease with an increasing of the magnetic field. Concern the amplitude X_2 for reflected T–wave, increases with a decreasing of the rotation with smaller values of the angle of incidence (i.e. $\theta = 0^o$, 15^o), while decreasing with an increasing of it with the increase of the angle of incidence (i.e., $\theta = 15^o$, 90^o) attend to zero at $\theta = 90^o$. Concern the amplitude of X_3 of reflected SV–wave, decreases with an increasing of the rotation with smaller values of the angle of incidence (i.e. $\theta = 0^o$, 90^o) attend to zero at $\theta = 90^o$.

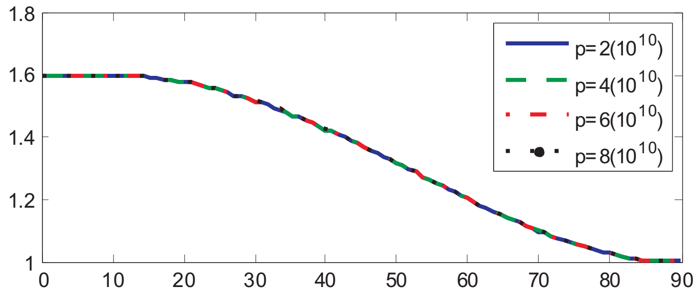

Figure 11 Variation of the reflection coefficient X_1 with varies values of the angle of incidence under the initial stress

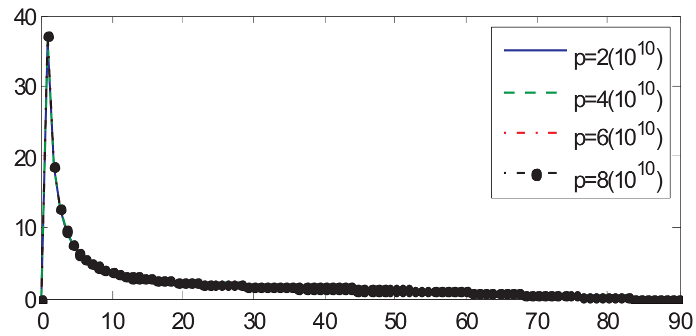

Figure 12 Variation of the reflection coefficient X_2 with varies values of the angle of incidence under the initial stress

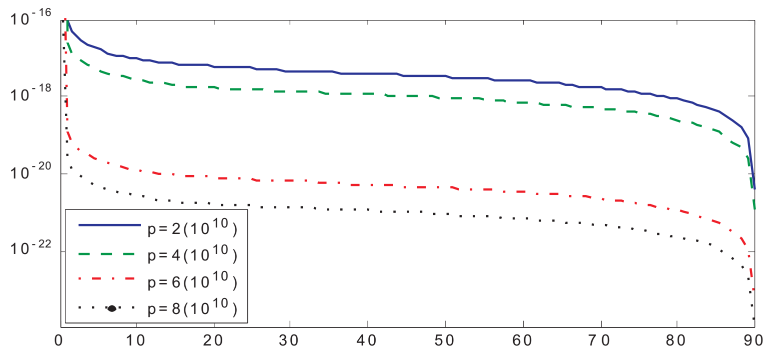

Figure 13 Variation of the reflection coefficient X_3 with varies values of the angle of incidence under the initial stress

Figs. 11–13, display the influence of initial stress parameter $P = (2, 4, 6, 8) \times 10^{10}$ on the amplitudes of the reflected waves p–, T–, and SV–waves due to the incident p-wave with respect to the angle of incidence θ. We note that the amplitude of reflected p- wave with smaller values of the angle of incidence increases with an increasing of the initial stress attends to unity at $\theta = 90^o$. Concern the amplitude of X_2 of the reflected T- wave, increases with the decrease of the initial stress with small values of the angle of incidence (i.e. $\theta = 0^o$, 7^o), while decreases with the increase of it with the increasing of the angle of incidence (i.e. $\theta = 7^o$, 90^o) attend to zero at $\theta = 90^o$. Concern the amplitude of X_3 of the reflected SV-wave, decreases with an increasing of the initial stress with smaller values of the angle of incidence (i.e. $\theta = 0^o$, 90^o) attend to zero at $\theta = 90^o$.

5. Conclusion

The reflection of p–wave at the free surface under initial stress, two temperature, magnetic field and rotation is discussed in the context of three theories. The derived expressions of reflection coefficient are obtained from incident p–wave for a copper material. The reflection coefficients ratios are computed and presented graphically with the angle of incidence θ for different values of rotation and magnetic and also angle of incidence. From the graphical results representation, it is obvious that the presence of rotation and magnetic field parameters affect significantly to the reflection coefficients.
We concluded the following remarks:

1. The rotation and magnetic field influence strongly on the amplitudes that has a significant role and a lot of applications in diverse fields, especially, geophysics, industries, engineering, astrophysics, . . . , etc.

2. The angle of incidence θ has a significant effect on the reflection coefficient, so tends to unity for p-waves, but tend to zero for T- and SV-waves when the incident wave in normalized on the free surface.

3. Also, it is obvious that initial stress affects strongly on X_3 but do not affect on the other amplitudes.

4. Finally, it is obvious that there is a variation between the three models of thermo–elasticity on X_3 but X_1 and X_2 do not affect.

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