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Modeling of Axisymmetric Waves in a Piezoelectric Circular Fiber Coated with Thin Film

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Mathematical modeling of axisymmetric waves in a piezoelectric fiber of circular cross section coated with thin film is studied using three-dimensional theory of piezoelectricity. Potential functions are introduced to uncouple the equations of motion, electric conduction equations. The surface area of the fiber is coated by a perfectly conducting gold material. The frequency equations are obtained for longitudinal and flexural modes of vibration and are studied numerically for PZT-4 ceramic fiber. The computed non-dimensional frequencies and attenuation for fiber with and without coating are presented in the form of dispersion curves.

Keywords: wave propagation in piezoelectric cylinder/fiber, forced vibration, Bessel function, actuators/sensors, thin film.

1. Introduction

Piezoelectric fiber with thin film coating plays vital role in many structural components, as a moisture barrier in the case of a packaging foil, a reflective layer for a car light reflector, an anti-reflection layer, a complex filter stack for optical components, highly reflective enhanced and protected layers for astronomical mirrors, or a heat insulation layer stack for architectural glazing applications. This type of Ceramic fiber obtained from the combination of lead zirconate/lead titanate reveals greater sensitivity and operating temperatures compare with other compositions and the materials PZT-4 are most widely used piezoelectric ceramics.

Mindlin [1, 2] developed the thermo-piezoelectric theory to derive the governing equations for a thermo-piezoelectric plate. The physical laws for the thermo-piezoelectric materials have been explored by Nowacki [3] (Foundations of linear piezoelectricity). Chandrasekharaiah [4, 5] has generalized Mindlin's theory of thermo-piezoelectricity for the finite speed of propagation of thermal disturbances.

Researchers (Pal [6], Paul and Ranganathan [7]) have respectively studied the surface waves in a thermo-piezo-electric medium of monoclinic symmetry and free vibrations of a pyroelectric layer of hexagonal (6 mm) class. Yang and Batra [8] analyzed the free vibrations of a thermo-piezoelectric body. Sharma and Kumar [9] disussed the plane harmonic waves in piezo-thermoelastic materials. The wave propagation in elastic solid has been discussed extensively in details by Graff [10] and Achenbach [11].

Sinha et al [12] have studied the axisymmetric wave propagation in circular cylindrical shell immersed in a fluid, in two parts. In Part I, the theoretical analysis of the propagation modes is discussed and in Part II, the axisymmetric modes excluding tensional modes are obtained theoretically and experimentally and are compared. Berliner and Solecki [13] have studied the wave propagation in a fluid loaded transversely isotropic cylinder. In that paper, Part I consists of the analytical formulation of the frequency equation of the coupled system consisting of the cylinder with inner and outer fluid and Part II gives the numerical results.

Ponnusamy and Selvamani [14, 15] have studied the wave propagation in magneto thermo elastic cylindrical panel and wave propagation in a transversely isotropic magneto-electro-elastic solid bar immersed in an inviscid fluid respectively, using Bessel function. Dayal [16] investigated the free vibrations of a fluid loaded transversely isotropic rod based on uncoupling the radial and axial wave equations by introducing scalar and vector potentials. Nagy [17] studied the propagation of longitudinal guided waves in fluid-loaded transversely isotropic rod based on the superposition of partial waves. Guided waves in a transversely isotropic cylinder immersed in a fluid was analyzed by Ahmad [18]. Selvamani [19, 20] has studied, the dispersion analysis in a fluid filled and immersed transversely isotropic thermo-electro-elastic hollow cylinder and Influence of thermo-piezoelectric field in a circular bar subjected to thermal loading due to laser pulse using the Bessel function in frequency equation.

2. Model of the problem

A homogeneous transversely isotropic piezoelectric circular fiber of infinite length coated by thin film is considered for this problem. The equations of motion and the piezoelectric, and dielectric matrices of the 6 mm crystal class is given as:

$$\sigma_{rr,r} + \sigma_{rz,z} + r^{-1}\sigma_{rr} = \rho u_{r,tt}$$

$$\sigma_{rz,r} + \sigma_{zz,z} + r^{-1}\sigma_{rz} = \rho u_{z,tt} \tag{1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rD_r) + \frac{\partial D_z}{\partial r} = 0 \tag{2}$$

$$\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - e_{31}$$

$$E_z\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - e_{31}E_z$$

$$\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - e_{33}E_z$$

$$\sigma_{rz} = 2c_{44}e_{rz} - e_{15}E_r \tag{3}$$

$$D_r = e_{15}e_{rz} + \varepsilon_{11}E_r$$

$$D_z = e_{31}\left(e_{rr} + e_{\theta\theta}\right) + e_{33}e_{zz} + \varepsilon_{33}E_z \tag{4}$$

where: σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, $\sigma_{r\theta}$, $\sigma_{\theta z}$, σ_{rz} are the stress components, e_{rr}, $e_{\theta\theta}$, e_{zz}, $e_{r\theta}$, $e_{\theta z}$, e_{rz} are the strain components, c_{11}, c_{12}, c_{13}, c_{33}, c_{44} and $c_{66} = \left(c_{11} - c_{12}\right)/2$ are the five elastic constants, e_{31}, e_{15}, e_{33} are the piezoelectric constants, ε_{11}, ε_{33} are the dielectric constants, ρ is the mass density. The comma in the subscripts denotes the partial differentiation with respect to the variables.
The strain e_{ij} are related to the displacements are given by:

$$\begin{aligned}
&e_{rr} = u_{r,r} \qquad e_{\theta\theta} = r^{-1}\left(u_r + u_{\theta,\theta}\right) \qquad e_{zz} = u_{z,z} \\
&e_{r\theta} = u_{\theta,r} + r^{-1}\left(u_{r,\theta} - u_\theta\right) \qquad e_{z\theta} = \left(u_{\theta,z} + r^{-1}u_{z,\theta}\right) \qquad e_{rz} = u_{z,r} + u_{r,z}
\end{aligned} \tag{5}$$

The comma in the subscripts denotes the partial differentiation with respect to the variables.
Substituting the Eqs. (3), (4) and (5) in the Eqs. (1) and (2), results in the following three-dimensional equations of motion, electric conductions are obtained as follows:

$$\begin{aligned}
&c_{11}\left(u_{rr,r} + r^{-1}u_{r,r} - r^{-2}u_r\right) + c_{44}u_{r,zz} + \left(c_{44} + c_{13}\right)u_{z,rz} + \left(e_{31} + e_{15}\right)V_{,rz} \\
&= \rho u_{r,tt} \\
&c_{44}\left(u_{z,rr} + r^{-1}u_{z,r}\right) + r^{-1}\left(c_{44} + c_{13}\right)\left(u_{r,z}\right) + \left(c_{44} + c_{13}\right)u_{r,rz} + c_{33}u_{z,zz} \\
&+e_{33}V_{,zz} + e_{15}\left(V_{,rr} + r^{-1}V_{,r}\right) = \rho u_{z,tt} \\
&e_{15}\left(u_{z,rr} + r^{-1}u_{z,r}\right) + \left(e_{31} + e_{15}\right)\left(u_{r,zr} + r^{-1}u_{r,z}\right) + e_{33}u_{z,zz} - \varepsilon_{33}V_{,zz} \\
&-\varepsilon_{11}\left(V_{,rr} + r^{-1}V_{,r}\right) = 0
\end{aligned} \tag{6}$$

3. Solution of the model

To obtain the propagation of harmonic waves in piezoelectric circular fiber, we assume the solutions of the displacement components to be expressed in terms of derivatives of potentials are taken from Paul [21]:

$$u_r\left(r, z, t\right) = \left(\phi_{,r}\right)e^{i(kz+\omega t)}$$

$$u_z\left(r, z, t\right) = \left(\frac{i}{a}\right)We^{i(kz+\omega t)}$$

$$V\left(r, z, t\right) = iVe^{i(kz+\omega t)}$$

$$E_r\left(r, z, t\right) = -E_{,r}e^{i(kz+\omega t)}$$

$$E_z\left(r, z, t\right) = E_{,z}e^{i(kz+\omega t)} \tag{7}$$

where$i = \sqrt{-1}$, k is the wave number, $\omega$is the angular frequency, $\phi\left(r\right)$, $W\left(r\right)$, $\psi\left(r\right)$ and $E\left(r\right)$are the displacement potentials and $V\left(r\right)$ is the electric potentials and a is the geometrical parameter of the bar. By introducing the dimensionless quantities such as $x = r/a$, $\zeta = ka$, $\Omega^2 = \rho\omega^2 a^2/c_{44}$, $\bar{c}_{11} = c_{11}/c_{44}$, $\bar{c}_{13} = c_{13}/c_{44}$, $\bar{c}_{33} = c_{33}/c_{44}$, $\bar{c}_{66} = c_{66}/c_{44}$ and substituting Eq. (7) in Eq. (6), we obtain:

$$\begin{gathered}
\left(\bar{c}_{11}\nabla^2 + \left(\Omega^2 - \zeta^2\right)\right)\phi - \zeta\left(1 + \bar{c}_{13}\right)W - \zeta\left(\bar{e}_{31} + \bar{e}_{15}\right)V = 0 \\
\zeta\left(1 + \bar{c}_{13}\right)\nabla^2\phi + \left(\nabla^2 + \left(\Omega^2 - \zeta^2\bar{c}_{33}\right)\right)W + \left(\bar{e}_{15}\nabla^2 - \zeta^2\right)V = 0 \\
\zeta\left(\bar{e}_{31} + \bar{e}_{15}\right)\nabla^2\phi + \left(\bar{e}_{15}\nabla^2 - \zeta^2\right)W + \left(\zeta^2\bar{\varepsilon}_{33} - \bar{\varepsilon}_{11}\nabla^2\right)V = 0
\end{gathered} \tag{8}$$

and

$$\left(\bar{c}_{66}\nabla^2 + \left(\Omega^2 - \zeta^2\right)\right)\psi = 0 \tag{9}$$

where: $\nabla^2 = \frac{\partial^2}{\partial x^2} + x^{-1}\frac{\partial}{\partial x} + x^{-2}\frac{\partial^2}{\partial \theta^2}$

The Eq. (8) can be written as:

$$\begin{vmatrix}
\left(\bar{c}_{11}\nabla^2 + \left(\Omega^2 - \zeta^2\right)\right) & -\zeta\left(1 + \bar{c}_{13}\right) & -\zeta\left(\bar{e}_{31} + \bar{e}_{15}\right) \\
\zeta\left(1 + \bar{c}_{13}\right)\nabla^2 & \left(\nabla^2 + \left(\Omega^2 - \zeta^2\bar{c}_{33}\right)\right) & \left(\bar{e}_{15}\nabla^2 - \zeta^2\right) \\
\zeta\left(\bar{e}_{31} + \bar{e}_{15}\right)\nabla^2 & \left(\bar{e}_{15}\nabla^2 - \zeta^2\right) & \left(\zeta^2\bar{\varepsilon}_{33} - \bar{\varepsilon}_{11}\nabla^2\right)
\end{vmatrix}(\phi, W, V) = 0 \tag{10}$$

Evaluating the determinant given in Eq. (10), we obtain a partial differential equation of the form:

$$\left(P\nabla^6 + Q\nabla^4 + R\nabla^2 + S\right)(\phi, W, V) = 0 \tag{11}$$

where:

$$P = c_{11}\left(\bar{e}_{15}^2 + \varepsilon_{11}\right)$$

$$Q = \left[\left(1 + \bar{c}_{11}\right)\bar{\varepsilon}_{11} + \bar{e}_{15}^2\right]\Omega^2 + \left\{\begin{array}{c} 2\left(\bar{e}_{31} + \bar{e}_{15}\right)\bar{c}_{13}\bar{e}_{15} - \left(1 + \bar{\varepsilon}_{11}\bar{c}_{33}\right)\bar{c}_{11} \\ +\bar{c}_{13}^2\bar{\varepsilon}_{11} + 2\bar{c}_{13}\bar{\varepsilon}_{11} - 2\bar{e}_{15}\bar{c}_{11} + 2\bar{e}_{13}^2 \end{array}\right\}\varsigma^2$$

$$\begin{aligned}
R = {} & \bar{\varepsilon}_{11}\Omega^4 - \left[\left(1 + \bar{c}_{13}\right)\bar{\varepsilon}_{11} + \left(1 + \bar{c}_{11}\right) + \left(\bar{e}_{31} + \bar{e}_{15}\right) + 2\bar{e}_{15}\right]\varsigma^2\Omega^2 + \left\{\bar{c}_{11}\left(1 + \bar{c}_{33}\bar{\varepsilon}_{33}\right)\right. \\
& \left. - \left[\left(\bar{e}_{31} + \bar{e}_{15}\right)^2 + \bar{\varepsilon}_{11}\right] - 2\bar{e}_{31}\left(1 + \bar{c}_{13}\right) - \bar{c}_{13}\bar{\varepsilon}_{33}\left(\bar{c}_{33} + \bar{c}_{13}\right) + 2\bar{e}_{15}\right\}\varsigma^4
\end{aligned}$$

$$S = -\left\{\left(1 + \bar{c}_{33}\right)\varsigma^6 - \left[2\left(1 + \bar{c}_{33}\right)\bar{\varepsilon}_{33} + 1\right]\varsigma^4\Omega^2 + \bar{\varepsilon}_{33}\varsigma^2\Omega^4\right\}$$

Solving the Eq. (11), we get solutions for a circular fiber as:

$$\begin{gathered}
\phi = \sum_{i=1}^{3} A_i J_n\left(\alpha_i a x\right)\cos n\theta \\
W = \sum_{i=1}^{3} a_i A_i J_n\left(\alpha_i a x\right)\cos n\theta \\
V = \sum_{i=1}^{3} b_i A_i J_n\left(\alpha_i a x\right)\cos n\theta
\end{gathered} \tag{12}$$

Here $\left(\alpha_i a\right)^2 > 0, (i = 1, 2, 3)$are the roots of the algebraic equation:

$$A\left(\alpha a\right)^6 - B\left(\alpha a\right)^4 + C\left(\alpha a\right)^2 + D = 0 \tag{13}$$

The Bessel function J_n is used when the roots $(\alpha_i a)^2$, $(i = 1, 2, 3)$are real or complex and the modified Bessel function $I_n$is used when the roots $(\alpha_i a)^2$, $(i = 1, 2, 3)$are imaginary.
The constants a_i, b_i defined in the Eq. (12) can be calculated from the equations:

$$(1 + \bar{c}_{13})\, \varsigma a_i + (\bar{e}_{31} + \bar{e}_{15})\, \varsigma b_i = -\left(\bar{c}_{11}\,(\alpha_i a)^2 - \Omega^2 + \varsigma^2\right)$$

$$\left((\alpha_i a)^2 - \Omega^2 + \varsigma^2 \bar{c}_{33}\right) a_i + \left(\bar{e}_{15}\,(\alpha_i a)^2 + \varsigma^2\right) b_i = -\left(\bar{c}_{13} + 1\right) \varsigma\, (\alpha_i a)^2 \qquad (14)$$

Solving the Eq. (9), we obtain:

$$\psi = A_{4n} J_n\,(\alpha_4 a x) \sin n\theta \qquad (15)$$

where$(\alpha_4 a)^2 = \Omega^2 - \zeta^2$. If$(\alpha_4 a)^2 < 0$, the Bessel function J_n is replaced by the modified Bessel functionI_n.

4. Boundary conditions and Frequency equations

In this problem, the free axisymmetric vibration of transversely isotropic piezoelectric fiber of circular cross-section coated with thin film is considered. For the solid-fluid problems, the continuity conditions require that the displacement components, the surface stress components and electric potential must be equal. The boundary conditions can be written as

$$\sigma_{rj} = -\delta_{j\,b}\, 2\, \mu^{'}\, h^{'} \left[\left(\frac{3\lambda^{'} + 2\mu^{'}}{\lambda^{'} + 2\mu^{'}}\right) U_{a,ab} + U_{b,aa}\right] + 2\; h^{'}\; \rho^{'}\; \ddot{U}_j \qquad (16)$$

$$V = 0$$

where: $\lambda^{'}$, $\mu^{'}$, $\rho^{'}$ and $h^{'}$ are Lame's constants, density, thickness of the material coating, respectively, $\delta_{j\,b}$ is the Kronecker delta function with a, b takes the value of θ, z and j takes r, θ and z. In order to get the axisymmetric waves a, b can ta es only z. Then the transformed boundary conditions is as follows:

$$\begin{aligned} &\sigma_{rr} = 2h^{'}\rho^{'}\ddot{U} \\ &\sigma_{rz} = -2h^{'}\mu^{'}G^2 W_{,zz} + 2h^{'}\rho^{'}\ddot{W} \\ &V = 0 \qquad \text{at} \qquad r = a \end{aligned} \qquad (17)$$

where: $G^2 = \frac{1 + C_{12}^{'}}{C_{11}^{'}}$
Substituting the solutions given in the Eqs. (12), (15) in the boundary condition Eq. (17), we obtain a system of linear algebraic equations as follows:

$$[B]\,\{X\} = \{0\} \qquad (18)$$

where $[B]$ is a 5×5 matrix of unknown wave amplitudes, and $\{X\}$ is an 5×1 column vector of the unknown amplitude coefficients B_1, B_2, B_3, B_4, B_5. The solution of Eq. (18) is nontrivial when the determinant of the coefficient of the wave amplitudes $\{X\}$ vanishes, that is:

$$|B| = 0 \qquad (19)$$

The components of $[B]$ are obtained as:

$$
\begin{aligned}
&B_{1i} = 2\overline{c}_{66}\left\{n\,(n-1) - \overline{c}_{11}\,(\alpha_i a)^2 - \varsigma\,(\overline{c}_{13}a_i + \overline{e}_{31}b_i)\right\} J_n\,(\alpha_i a) \\
&+2\overline{c}_{66}\,(\alpha_i a)\,J_{n+1}\,(\alpha_i a) \qquad i = 1,2 \\
&B_{13} = 2\overline{c}_{66}n\,\{(n-1)\,J_n\,(\alpha_4 a) - (\alpha_4 a)\,J_{n+1}\,(\alpha_4 a)\} \\
&B_{14} = 2\,(\alpha_1 a)\left[\left(\rho' h' \big/ a\rho\,(Ca)^2 - \overline{C_{66}}\right)\right] J_n\,(\alpha_5 a) \\
&B_{2i} = 2n\,\{(n-1)\,J_n\,(\alpha_i a) + (\alpha_i a)\,J_{n+1}\,(\alpha_i a)\} \qquad i = 1,2 \\
&B_{23} = \left\{\left[(\alpha_4 a)^2 - 2n\,(n-1)\right] J_n\,(\alpha_4 a) - 2\,(\alpha_4 a)\,J_{n+1}\,(\alpha_4 a)\right\} \\
&B_{24} = 2\,(\alpha_1 a)\left[\left(\rho' h' \big/ a\rho\,(Ca)^2 - \overline{C_{66}}\right)\right] J_n\,(\alpha_5 a) \\
&B_{3i} = ((\varsigma + a_i) + \overline{e}_{15}b_i)\,\{nJ_n\,(\alpha_i a) - (\alpha_i a)\,J_{n+1}\,(\alpha_i a)\}\,, i = 1,2 \\
&B_{33} = n\varsigma J_n\,(\alpha_4 a) \qquad B_{34} = 0
\end{aligned}
$$

5. Numerical results and discussion

The frequency equation given in Eq. (19) is transcendental in nature with unknown frequency and wave number. The solutions of the frequency equation are obtained numerically by fixing the wave number. The material chosen for the numerical calculation is PZT-4 ceramics coated with gold material. The material properties of PZT-4 and Gold is taken from Berlincourt et al [22]:

$$c_{11} = 13.9 \times 10^{10}Nm^{-2},\ c_{12} = 7.78 \times 10^{10}Nm^{-2},\ c_{13} = 7.43 \times 10^{10}Nm^{-2}$$

$$c_{33} = 11.5 \times 10^{10}Nm^{-2},\ c_{44} = 2.56 \times 10^{10}Nm^{-2},\ c_{66} = 3.06 \times 10^{10}Nm^{-2}$$

$$e_{31} = -5.2Cm^{-2},\ e_{33} = 15.1Cm^{-2},\ e_{15} = 12.7Cm^{-2}$$

$$\varepsilon_{11} = 6.46 \times 10^{-9}C^2N^{-1}m^{-2},\ \varepsilon_{33} = 5.62 \times 10^{-9}C^2N^{-1}m^{-2},\ \rho = 7500Kgm^{-2}$$

In this problem, by choosing n = 0 and n = 1, we can obtain the non-dimensional frequencies of two kinds of basic independent modes, namely, longitudinal and flexural modes of vibrations.

5.1. *Dispersion curves*

The results of non dimensional frequency and attenuation for longitudinal and flexural modes are plotted in the form of dispersion curves. The notation used in the figures, namely Lm, Fsm, and FAsm respectively denote the longitudinal mode, flexural symmetric mode and flexural anti symmetric mode. The 1 refers the first mode, 2 refers the second mode and so on.

The dispersion curves are drawn for non-dimensional frequency Ω versus the dimensionless wave number $|\varsigma|$ for longitudinal modes of piezoelectric circular fiber with and without thin film coating, respectively shown in Figs. 1 and 2. From the Figs. 1 and 2, it is observed that the non-dimensional frequencies are increased with respect to its wave number. A comparison is made between the non-dimensional frequency Ω versus the dimensionless wave number $|\varsigma|$ for flexural modes of vibration is respectively shown in the Figs. 3 and 4, for the fiber with and without coating. From the Figs. 3 and 4, it is observed that, the third and fourth modes of frequency are merges for a particular period after that, it starts increases. The cross-over points between the flexural modes of frequency show that, there is energy transfer between the modes of vibrations due to the impact of coating. A dispersion curve is drawn to compare the frequency responses of flexural anti symmetric modes of vibration for a piezoelectrical cylindrical fiber with coating and without coating is shown respectively in the Figs. 5 and 6. From the Figs. 5 and 6, it is noticed that the dimensionless frequencies are increases with respect to its non-dimensional wave numbers. In this analysis it is observed that the frequency of the fiber with coating is deviated from that of the uncoated fiber.

Figs. 7-12 represents the variation of attenuation for the real and imaginary part of longitudinal, flexural symmetric and flexural anti symmetric modes with respect to thickness of the fiber. Whenever the thickness of the fiber increases the attenuation is oscillating both in real and imaginary part of all the three fundamental modes of vibration.

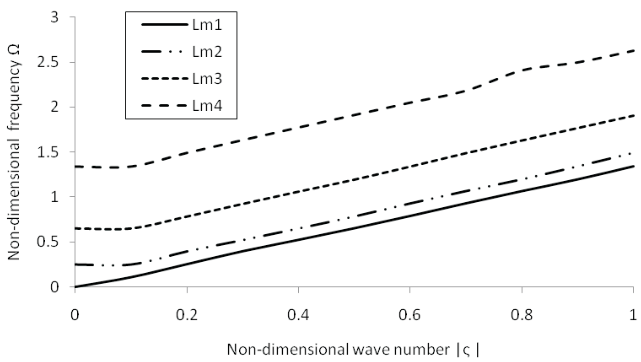

Figure 1 Non-dimensional wave number $|\varsigma|$ versus Non-dimensional frequency Ω of longitudinal modes of vibration for a piezoelectric cylindrical fiber with coating

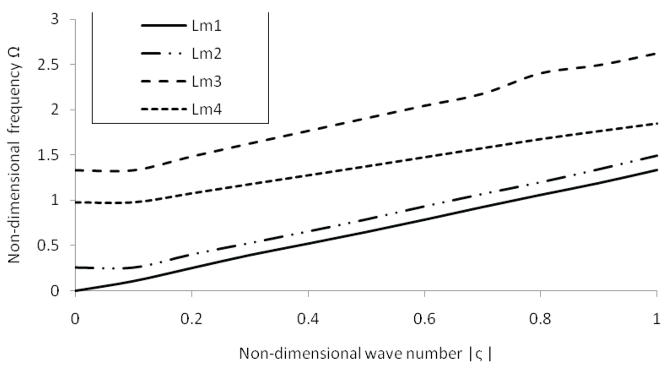

Figure 2 Non-dimensional wave number $|\varsigma|$ versus Non-dimensional frequency Ω of longitudinal modes of vibration for a piezoelectric cylindrical fiber without coating

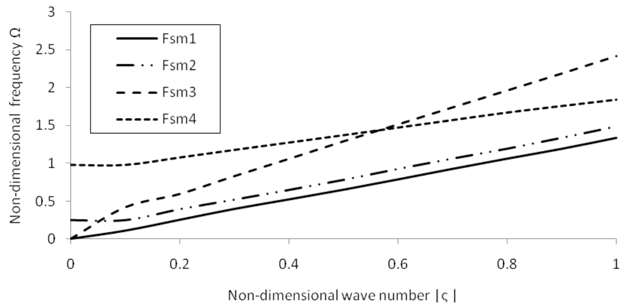

Figure 3 Non-dimensional wave number $|\varsigma\ |$ versus Non-dimensional frequency Ω of flexural symmetric modes of vibration for a piezoelectric cylindrical fiber with coating

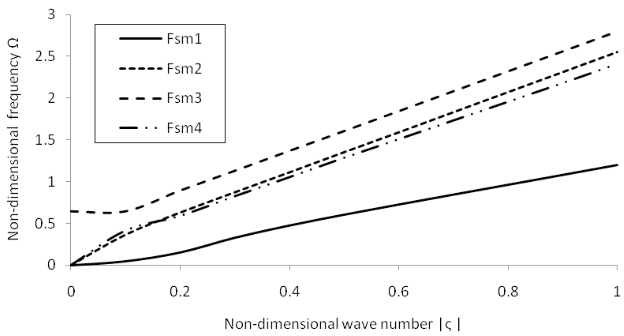

Figure 4 Non-dimensional wave number $|\varsigma\ |$ versus Non-dimensional frequency Ω of flexural symmetric modes of vibration for a piezoelectric cylindrical fiber without coating

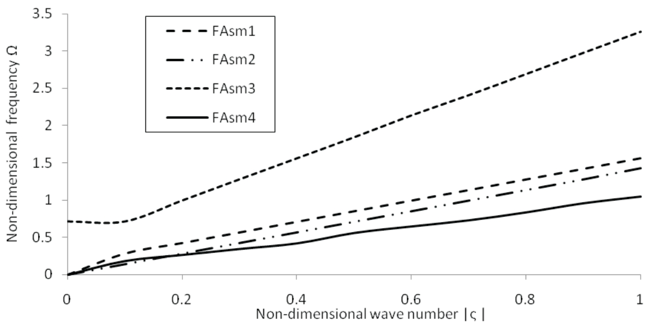

Figure 5 Non-dimensional wave number $|\varsigma\ |$ versus Non-dimensional frequency Ω of flexural antisymmetric modes of vibration for a piezoelectric cylindrical fiber with coating

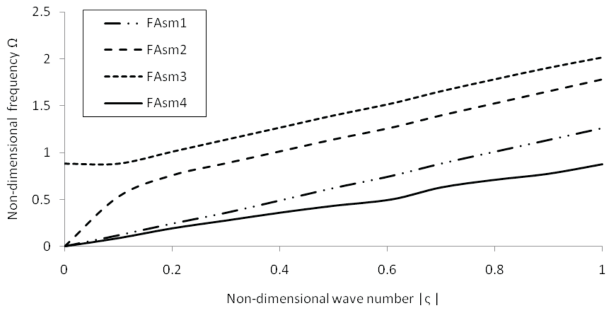

Figure 6 Non-dimensional wave number $|\varsigma\ |$ versus Non-dimensional frequency Ω of flexural antisymmetric modes of vibration for a piezoelectric cylindrical fiber without coating

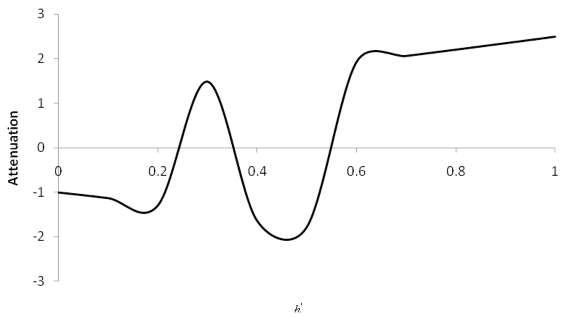

Figure 7 Variation of Attenuation versus thickness of the coating material $h^{'}$ for real part longitudinal mode

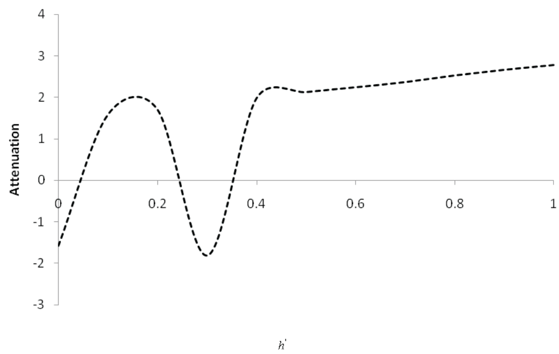

Figure 8 Variation of Attenuation versus thickness of the coating material $h^{'}$ for imaginary part of longitudinal mode

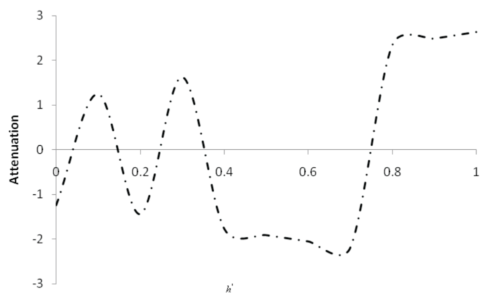

Figure 9 Variation of Attenuation versus thickness of the coating material h' for real part of flexural anti symmetric mode

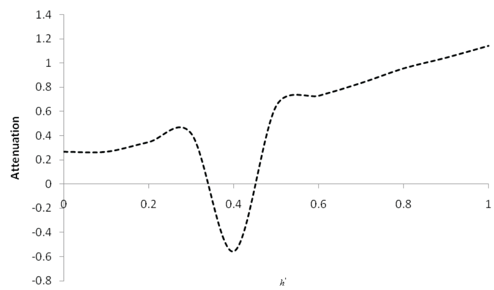

Figure 10 Variation of Attenuation versus thickness of the coating material h' for imaginary part of flexural anti symmetric mode

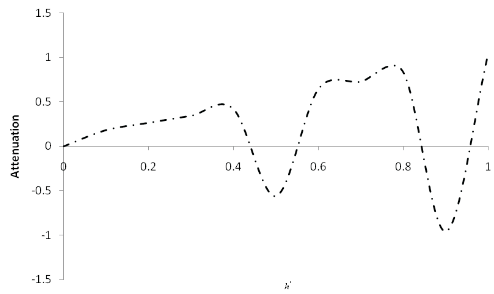

Figure 11 Variation of Attenuation versus thickness of the coating material h' for real part of flexural symmetric mode

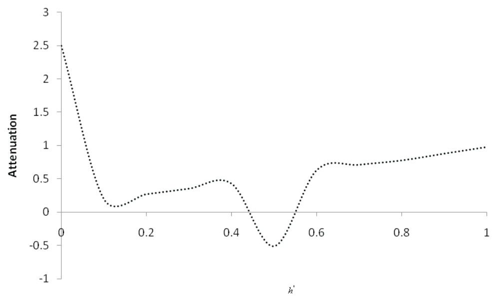

Figure 12 Variation of Attenuation versus thickness of the coating material $h^{'}$ for imaginary part of flexural symmetric mode

6. Conclusions

The axisymmetric wave propagation in a piezoelectric circular fiber coated with thin film is discussed using three-dimensional theory of piezoelectricity. Three displacement potential functions are introduced to uncouple the equations of motion, electric conduction. The frequency equations are obtained for longitudinal and flexural modes of vibration and are studied numerically for PZT-4 material fiber with gold coating. The computed non-dimensional frequency and attenuation are presented in the form of dispersion curves. From the graphical pattern, it is observed that the frequency of the fiber with coating is deviated from that of the uncoated fiber and also the attenuation is oscillating with increasing thickness of the coated material.

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