Mechanics and Mechanical Engineering
Vol. 21, No. 3 (2017) 657–677


# The Effect of Gravity and Diffusion on Micropolar Thermoelasticity with Temperature–Dependent Elastic Medium under G–N Theory

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Received (25 April 2017)
Revised (26 May 2017)
Accepted (11 June 2017)

This article studies the effect of the gravity field and the diffusion on micropolar thermoelastic medium with dependence on the temperature properties. The analytic method used to obtain the exact formula of the physical quantities was the normal mode analysis. The comparisons established graphically in the presence and the absence of the gravity, the temperature-dependent properties, the diffusion and the micropolar in the context of two types of Green-Naghdi (G-N) theory II and III.

*Keywords*: diffusion, gravity, Green-Naghdi, micropolar thermoelastic, temperature–dependent.

## 1. Introduction

The generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory of thermoelasticity in which the parabolic type heat conduction equation is based on Fourier's law of heat conduction which introduced by Biot [1]. There are two important generalized theories of thermoelasticity. The first is due to Lord and Shulman [2] who developed the theory with one relaxation time. The second generalization to the coupled theory of thermoelasticity is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-rate-dependent thermoelasticity introduced by Green and Lindsay (G-L) who's obtained another version of the constitutive equations [3]. The classical Fourier law violated if the medium under consideration has a center of symmetry. Green and Naghdi [4-6] proposed three new thermoelastic theories based on entropy equality than the usual entropy inequality. The constitutive assumption for the heat flux vector is different in each theory. Thus, they obtained three theories which are called thermoelasticity of type I, thermoelasticity of type II and thermoelasticity of type III. When type I theory is linearized we obtain the classical system of thermoelasticity. The type II theory (is a limiting case of type III) does not admit energy dissipation.

The response of the material to the external stimuli depends heavily on the motions of its inner structures. Classical elasticity does not contain this effect, where only translational degrees of freedom of the material point of the body are considered. Eringen [7] developed the linear micropolar theory of elasticity, which included the intrinsic rotations of the microstructure. It provides a model that can support the body and the surface couples and display high frequency optical branch of the wave spectrum. For the engineering applications, this theory establishes the composites with rigid chopped fibers, elastic solid with rigid granular inclusions, and other industrial materials such as liquid crystals. Smith [8] studied the wave propagation in micropolar elastic solids. Parfitt and Eringen [9] investigated the reflection of the plane waves from a flat boundary of a micropolar elastic half-space. Ariman [10] also studied the wave propagation in micro-polar elastic half-space solid. Eringen [11] presented the microcontinuum field theory. Othman *et al.* [12-14] investigated some problems for micropolar thermoelasticity.

The diffusion can be defined as the random walk, of an ensemble of particles, from regions of high concentration to regions of lower concentration. There is now a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industrial applications. In integrated circuit fabrication, diffusion is used to introduce the pants in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors. In most of these applications, the concentration is calculated using what is known as Flick's law. This is a simple law that does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of the temperature on this interaction. Nowacki [15-18] established the theory of thermo-elastic diffusion. In this theory, the coupled thermoelastic model is used.

The effect of the gravity on the wave propagation in an elastic medium was first considered by Bromwich [19], which treating the force of the gravity as a type of a body force. Love [20] extended the work of Bromwich investigated the influence of the gravity on superficial waves and showed that the Rayleigh wave velocity is affected by the gravitational field. Sezawa [21] studied the dispersion of elastic waves propagated on curved surfaces.

Othman *et al.* [22, 23] discussed two problems on the effect of the gravitational field on thermoelastic solids.

Most of the studies in applied sciences of mechanics of materials were done under the assumption of the temperature-independent material properties, which limit the applicability of the obtained solutions to certain ranges of temperature. At high temperature, the material characteristics such as the modulus of elasticity, Poisson's ratio, the coefficient of thermal expansion and the thermal conductivity are no longer constants. In recent years due to the progress in various fields in science and technology the necessity of taking into consideration the real behavior of the material characteristics became actual. Othman et al. [24] discussed the effect of the gravitational field and temperature-dependent properties on two-temperature thermoelastic medium with voids under G–N theory. Recently, Othman and Hilal [25] established the rotation effect on two-temperature porous thermoelastic material and temperature-dependent properties of type III. Elmaklizi and Othman [26] studied the effect of rotation on thermoelastic diffusion with temperature-dependent elastic moduli comparison of different theories. Othman and Elmaklizi [27] discussed the 2-D problem of generalized magneto-thermo-elastic diffusion with temperature-dependent elastic moduli.

This paper studies the effect of diffusion on a linear, isotropic, homogeneous micropolar thermoelastic solid influenced by the gravitational field with temperature-dependent properties based on (G-N) theory. The analytic methodology used to get the exact solutions of the considered physical quantities was the normal mode analysis. The obtained physical quantities represented graphically in the absence and presence of the physical effects.

## 2. Basic equations

Consider an isotropic, homogeneous, linear thermoelastic diffusive micropolar medium Following Sherief et al. [13], Green and Naghdi [20] and Aouadi [27], the field equations and the constitutive relations without body forces, body couples and heat sources can be considered in the form:

$$\sigma_{ij,\,i} = \rho\, u_{i,\,tt} \tag{1}$$

$$m_{ij,\,i} + \varepsilon_{ijr}\, \sigma_{ir} = j\rho\, \phi_{i,\,tt} \tag{2}$$

$$d\,\beta_2\, e_{,\,ii} + d\, a^*\, T_{,\,ii} + d\, b\, C_{,\,ii} + C_{,t} = 0 \tag{3}$$

$$K\, T_{,\,ii} + K^*\, T_{,\,iit} - a^*\, T_0\, C_{,tt} = \rho\, C_e\, T_{,\,tt} + \beta_1\, T_0\, u_{i,\,i\,tt} \tag{4}$$

$$\sigma_{ij} = \lambda\, u_{r,r}\, \delta_{ij} + \mu\,(u_{i,j} + u_{j,i}) + k^*\,(u_{j,i} - \varepsilon_{ijr}\, \phi_r) - \beta_1\, T\, \delta_{ij} - \beta_2\, C\, \delta_{ij} \tag{5}$$

$$m_{ij} = \alpha\, \phi_{r,r}\, \delta_{ij} + \beta\, \phi_{i,j} + \gamma\, \phi_{j,i} \tag{6}$$

$$\rho\, T_0\, S = \rho\, C_e\, T + \beta_1\, T_0\, e_{kk} + a^*\, T_0\, C \tag{7}$$

$$P = -\beta_2\, e_{kk} + b\, C - a^*\, T \tag{8}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{9}$$

where: $\lambda$, $\mu$ are the Lame' constants, $\alpha$, $\beta$, $\gamma$ and $k^*$ are the micropolar constants, $\beta_1 = (3\,\lambda + 2\,\mu + k^*)\,\alpha_t$,while $\alpha_t$ is the linear thermal expansion coefficient,$\rho$ is the density, $\beta_2 = (3\,\lambda+2\,\mu+k^*)\,\alpha_c$, while $\alpha_c$ is the linear diffusion expansion coefficient, $C_e$ is the specific heat at constant strain, $K$ is the thermal conductivity, $K^*$ is the material constant characteristic for this theory, $u_i$ is the displacement vector, $\theta$ is the absolute temperature, where, $T = \theta - T_0$, $T_0$ is the reference temperature chosen so that $|(T - T_0)/T_0| << 1$, $\phi_i$ is the microrotation vector, $\sigma_{ij}$ are the components of the stresses, $e_{ij}$ are the components of strains, $\delta_{ij}$ is the Kronecker delta, $\varepsilon_{ijr}$is the permutation symbol, $e_{kk}$ is the cubic dilation, $p$ is the chemical potential, $m_{ij}$is the couple stresses, $j$ is the microinertia, $S$ is the entropy per unit mass, $a^*$ is a measure of the thermo-diffusive effect, $d$ is the diffusion coefficient, $b$ is the measure of diffusion effect, $C$ is the concentration of the diffusive material in the elastic body.

## 3. Formulation of the problem and solution

Consider an isotropic, linear, homogeneous, micropolar thermoelastic medium with a half-space ($y \geq 0$), under the effect of a constant gravitational field $g$ with temperature- dependent properties. The rectangular Cartesian coordinate system $(x, y, z)$ having originated on the surface $z = 0$. For two dimensional problem assume the dynamic displacement vector as $\mathbf{u} = (\mathrm{u},\ \mathrm{v},\ 0)$. The microrotation vector $\boldsymbol{\phi}$ will be $\boldsymbol{\phi} = (0, 0, \phi_3)$. All quantities considered will be a function of the time variable $t$ and of the coordinates $x$ and $y$, in the used equations a dot denotes differentiation with respect to time, while a comma denotes the coordinate system derivatives. Eqs. (1)-(4) under the effect of the gravitational field will be on the form:

$$(\mu + k^*)\,\nabla^2 u + (\lambda + \mu)\,\frac{\partial e}{\partial x} + k^*\,\frac{\partial \phi_3}{\partial y} - \beta_1\,\frac{\partial T}{\partial x} - \beta_2\,\frac{\partial C}{\partial x} + \rho\, g\,\frac{\partial v}{\partial x} = \rho\,\frac{\partial^2 u}{\partial t^2} \quad (10)$$

$$(\mu + k^*)\,\nabla^2 v + (\lambda + \mu)\,\frac{\partial e}{\partial y} - k^*\,\frac{\partial \phi_3}{\partial x} - \beta_1\,\frac{\partial T}{\partial y} - \beta_2\,\frac{\partial C}{\partial y} - \rho\, g\,\frac{\partial u}{\partial x} = \rho\,\frac{\partial^2 v}{\partial t^2} \quad (11)$$

$$\gamma\,\nabla^2\,\phi_3 - 2\,k^*\phi_3 + k^*(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) = j\rho\,\frac{\partial^2\,\phi_3}{\partial t^2} \quad (12)$$

$$d\,\beta_2\,\nabla^2\,e + d\,a^*\,\nabla^2\,T + d\,b\,\nabla^2 C + \frac{\partial C}{\partial t} = 0 \quad (13)$$

$$K\,\nabla^2\,T + K^*\,\frac{\partial}{\partial t}\nabla^2\,T - a^*\,T_0\,\frac{\partial^2\,C}{\partial t^2} = \rho\,C_e\,\frac{\partial^2\,T}{\partial t^2} + \beta_1\,T_0\,\frac{\partial^2\,e}{\partial t^2} \quad (14)$$

To investigate the effect of the temperature dependence properties on the micropolar thermoelastic medium assume that:

$$\begin{aligned} &\lambda = \lambda_0\,f(T) \qquad \mu = \mu_0\,f(T) \qquad \beta_1 = \beta_{10}\,f(T) \qquad \beta_2 = \beta_{20}\,f(T) \\ &\alpha = \alpha_0\,f(T) \qquad \beta = \beta_0\,f(T) \qquad \gamma = \gamma_0\,f(T) \qquad k^* = k_0^*\,f(T) \end{aligned} \quad (15)$$

where: $\lambda_0$, $\mu_0$, $\beta_{10}$, $\beta_{20}$, $\alpha_0, \beta_0$, $\gamma_0$, $k_0^*$ are constants, $f(T)$is a given non-dimensional function of temperature. In the case of a temperature independent modulus of elasticity, $f(T) = 1$, such that $f(T) = (1 - \alpha^* T_0)$, where $\alpha^*$ is called the empirical

material constant, in the case of the reference temperature independent of modulus of elasticity and thermal conductivity $\alpha^* = 0$. In Eq. (14) $K^* = 0$; this system converted into (G–N II) theory without energy dissipation.
Define the non-dimensional variables by expressions:

$$x_i^{'} = \frac{\omega_1^*}{c_1} x_i \qquad u_i^{'} = \frac{\rho\, c_1\, \omega_1^*}{\beta_{10}\, T_0} u_i \qquad \phi_3^{'} = \frac{\rho\, c_1^2}{\beta_{10}\, T_0} \phi_3 \qquad m_{ij}^{'} = \frac{\omega_1^*}{\beta_{10}\, c_1\, T_0} m_{ij}$$
$$t^{'} = \omega_1^*\, t \qquad g^{'} = \frac{g}{c_1\, \omega_1^*} \qquad C' = \frac{\beta_{20}}{\beta_{10}\, T_0} C$$
$$(T^{'} p_2^{'}) = \frac{1}{T_0} (T, p_2) \qquad (\sigma_{ij}^{'} p_1^{'}) = \frac{1}{\beta_{10}\, T_0} (\sigma_{ij}, p_1) \tag{16}$$
$$P' = \frac{1}{\beta_{20}} P \qquad \omega_1^* = \frac{\rho\, C_e\, c_1^2}{K} \qquad c_1^2 = \frac{\lambda_0 + 2\,\mu_0 + k_0^*}{\rho}$$

Assuming the potential functions $\psi_1(x, y, t)$, $\psi_2(x, y, t)$, on the dimensionless form:

$$u = \frac{\partial\, \psi_1}{\partial\, x} + \frac{\partial\, \psi_2}{\partial\, y} \quad \text{and} \quad v = \frac{\partial\, \psi_1}{\partial\, y} - \frac{\partial\, \psi_2}{\partial\, x} \tag{17}$$

To get the solution for the physical quantities, consider the solution in the form of the normal mode:

$$[\psi_1, \psi_2, \phi_3, C, T]\, (x, y, t) = [\psi_1^*, \psi_2^*, \phi_3^*, C^*, T^*]\, (y)\, e^{\mathrm{i}\,(a\, x - \xi\, t)} \tag{18}$$

Where, $[\psi_1^*, \psi_2^*, \phi_3^*, C^*, T^*]\, (y)$ are the amplitude of the physical quantities, $\xi$ is the angular frequency, $\mathrm{i} = \sqrt{-1}$ and $a$ is the wave number in the $x-$ direction.
Apply Eqs. (15–18) into Eqs. (10–14) and drop the prime to obtain

$$[\mathrm{D}^2 - m_2]\ \psi_1^* - m_3\ \psi_2^* - m_4\ C^* - m_4\ T^* = 0 \tag{19}$$

$$m_5\ \psi_1^* + [\mathrm{D}^2 - m_6]\ \psi_2^* + l_2\ \phi_3^* = 0 \tag{20}$$

$$-\ l_6\, [\mathrm{D}^2 - a^2]\ \psi_2^* + [\mathrm{D}^2 - m_7]\ \phi_3^* = 0 \tag{21}$$

$$l_8\ [(\mathrm{D}^2 - a^2)\ (\mathrm{D}^2 - a^2)]\ \psi_1^* + [\mathrm{D}^2 - m_8]\ C^* + l_{10}\ [\mathrm{D}^2 - m_8]\ T^* = 0 \tag{22}$$

$$m_{10}\, [\mathrm{D}^2 - a^2]\ \psi_1^* - m_{11}\ C^* + [\mathrm{D}^2 - m_{12}]\ T^* = 0 \tag{23}$$

All the constants are in Appendix B.
Eliminate $\psi_1^*$, $\psi_2^*$, $\phi_3^*$, $C^*$ and $T^*$ between Eqs. (19–23), to obtain the following differential equation:

$$[\mathrm{D}^{10} - \delta_1\, \mathrm{D}^8 + \delta_2\, \mathrm{D}^6 - \delta_3\, \mathrm{D}^4 + \delta_4\, \mathrm{D}^2 - \delta_5]\{\psi_1^*(y), \psi_2^*(y), \phi_3^*(y), C^*(y), T^*(y)\} = 0 \tag{24}$$

where: $\delta_{\mathrm{n}}\, (\mathrm{n} = 1, 2, ..., 5)$ can be obtained from elimination the functions among Eqs. (19–23).
Equation (24) can be factored as:

$$\begin{gathered}[(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)(\mathrm{D}^2 - k_3^2)\, (\mathrm{D}^2 - k_4^2)(\mathrm{D}^2 - k_5^2)] \\ \{\psi_1^*(y), \psi_2^*(y), \phi_3^*(y), C^*(y), T^*(y)\} = 0\end{gathered} \tag{25}$$

where: $k_{\mathrm{n}}^2\,(\mathrm{n}=1,2,...,5)$ are the roots of the characteristic equation of the Eq. (25). The general solution of the equation (25), bound at $y \to \infty$, given by:

$$\psi_1\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{26}$$

$$\psi_2\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} B_{1\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{27}$$

$$u\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} A_{1\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{28}$$

$$v\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} A_{2\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{29}$$

$$\phi_3\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} B_{2\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{30}$$

$$C\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} B_{3\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{31}$$

$$T\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} B_{4\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{32}$$

$$m_{yz}\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} A_{6\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{33}$$

$$m_{xz}\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} A_{7\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{34}$$

$$\sigma_{xx}\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} A_{3\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{35}$$

$$\sigma_{yy}\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} A_{4\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{36}$$

$$\sigma_{xy}\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} A_{5\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{37}$$

$$\sigma_{xz}\,(x,y,t)=\sigma_{yz}\,(x,y,t)=0 \tag{38}$$

$$P\,(x,y,t)=\sum_{\mathrm{n}=1}^{5} A_{8\mathrm{n}}\, R_{\mathrm{n}}\, e^{-k_{\mathrm{n}}\, y+\mathrm{i}\,(a\,x-\,\xi\,t)} \tag{39}$$

Since, $R_{\mathrm{n}}(\mathrm{n}=1,2,...,5)$ some coefficients. All constants defined in the Appendix.

## 4. Applications

Consider the following non-dimensional boundary conditions to determine the coefficients $R_{\mathrm{n}}(\mathrm{n} = 1, 2, ..., 5)$ and neglect the positive exponential to avoid the unbounded solutions at infinity. Then, the surface of the medium at $y = 0$, assumes these conditions:

1. The mechanical boundary conditions are:

   - The normal stress condition (mechanically stressed by constant force $p_1$), so that:
   $$\sigma_{yy} = -p_1\, e^{\mathrm{i}\,(a\,x - \xi\, t)} \tag{40}$$
   - The tangential stress condition (stress free), then:
   $$\sigma_{xy} = 0 \tag{41}$$

2. The condition of the couple stress (the couple stress is constant in y-direction). This implies that:
$$m_{yz} = 0 \tag{42}$$

3. The concentration condition (no variation on the mass concentration on the surface $y$, that means:
$$\frac{\partial C}{\partial y} = 0 \tag{43}$$

4. The thermal condition (the half-space was taken to be insulated thermal space) is:
$$\frac{\partial T}{\partial y} = 0 \tag{44}$$

Substitute the expressions of the considered quantities in these boundary conditions (40–44), to obtain the equations satisfied by the parameters. Then, one can obtain a system of five equations. After applying the inverse of matrix method, we have the values of the constants $R_{\mathrm{n}}(\mathrm{n} = 1, 2, ..., 5)$:

$$\begin{pmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{pmatrix} = \begin{pmatrix} A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} \\ A_{91} & A_{92} & A_{93} & A_{94} & A_{95} \\ -k_1 B_{41} & -k_2 B_{42} & -k_3 B_{43} & -k_4 B_{44} & -k_5 B_{45} \end{pmatrix}^{-1} \begin{pmatrix} -p_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{45}$$

Thus, we obtain the expressions for the physical quantities of the plate surface.

## 5. Numerical results and discussion

In order to illustrate the obtained theoretical results in the preceding section, following Eringen [28], the magnesium crystal-like micropolar thermoelastic material was chosen for purposes of numerical evaluations. All the units of the used parameters are given in SI units. The constants were taken as:
$\lambda = 9.4 \times 10^{10}$ N/m$^2$, $\mu = 4 \times 10^{10}$ N/m$^2$, $K = 1.7 \times 10^2$ N/s K, $\rho = 1.74 \times 10^3$ kg/m$^3$, $\alpha_t = 7.4033 \times 10^{-7}$ 1/K, $C_e = 1.04 \times 10^3$ J/kg K, $k^* = 85$ W/m K, $\gamma = 7.779 \times 10^{-8}$ N, $j = 2 \times 10^{-20}$ m$^2$, $T_0 = 298$ K, $b = 32 \times 10^5$ kg$^{-1}$ m$^5$ s$^{-2}$, $d = 0.85 \times 10^{-8}$ kg m$^{-3}$ s, $p_1 = 1$ N/m$^2$, $\alpha_c = 2.65 \times 10^{-4}$ kg$^{-1}$ m$^3$ , $a = 1.5$ m, $t = 0.5$ s, $\xi = \eta + \mathrm{i}\eta_1$, $\eta = 0.6$ rad/s, $\eta_1 = 2.8$ rad/s, $x = 0.45$ m, $0 \leq y \leq 0.35$ m.

These numerical values used to obtain the distribution of the real parts of the displacement component $u$, the temperature $T$, the stress components $\sigma_{yy}$, $\sigma_{xy}$, the couple stresses $m_{tyz}$, the microrotation $\phi_3$, the chemical potential $P$, and the concentration $C$ with the distance $y$ in the case of two types II and III of (G–N) theory.

Figs. 1-8 represent the change in the behavior of these physical quantities against distance $y$ in 2-D when $\alpha^* = 8$ K$^{-1}$, in the case of $g = 9.8$ m/s$^2$, 0. Figs. 9–16 show the behavior of these physical quantities against distance $y$ in 2-D during $g = 9.8$ $m$/s$^2$, in the case of $\alpha^* = 8$, 0 K$^{-1}$.

Fig. 1 shows that the distribution of the displacement component $u$decreases in the case of (G-N II) for $y > 0$, but in the case of (G-N III) it decreases in the range $0 \leq y \leq 0.03$, then it increases for $y > 0.03$ with the increase of the gravity. Fig. 2 clarifies that the distribution of the temperature $T$increases in the case of (G-N II) in the range $0 \leq y \leq 0.07$, then it decreases in the range $0.07 \leq y \leq 0.35$, while it increases in the case of (G-N III) for $y > 0$ with the increase of the gravity. Fig. 3 explains that the distribution of the stress component $\sigma_{yy}$increases in the case of (G-N II) for $y > 0$; while it increases in the range $0 \leq y \leq 0.035$, then it decreases in the range $0.035 \leq y \leq 0.35$ in the case of (G-N III) with the increase of the gravity. Fig. 4 shows that the distribution of the stress component $\sigma_{xy}$increases in the range $0 \leq y \leq 0.04$, then it decreased in the range $y > 0.04$, in the case of (G-N II), while it increases in the case of (G-N III) for $y > 0$ with the increase of the gravity.

Figs. 5 and 6 clarify that the distribution of the couple stress $m_{yz}$and the microrotation vector $\phi_3$ respectively. The distribution of $m_{yz}$ increases in both two types II and III of (G-N) theory for $y > 0$ with the increase of the gravity, while the distribution of $\phi_3$ decreases in the case of (G-N II) and increases in the case of (G-N III) for $y > 0$ with the increase of the gravity. Figs. 7 and 8 depict that the chemical potential $P$and the concentration $C$respectively. The distribution of $P$ decreases for both two types II and III of (G-N) theory for $y > 0$ with the increase of the gravity, while the distribution of $C$ increases for both two types II and III of (G-N) theory for $y > 0$ with the increase of the gravity. It can deduce that all the functions are continuous and all the curves converge to zero. The gravity has an effective role in the distribution of all the physical quantities of the problem. Fig. 9 shows that the distribution of the displacement component $u$ increases in the ranges $0 \leq y \leq 0.01$, and $0.06 \leq y \leq 0.35$, while it decreases in the range $0.01 \leq y \leq 0.06$, in the case of (G-N II), while it decreases in the range $0 \leq y \leq 0.05$, then it increases in the range $0.05 \leq y \leq 0.35$, in the case of (G-N III) with the increase of $\alpha^*$.

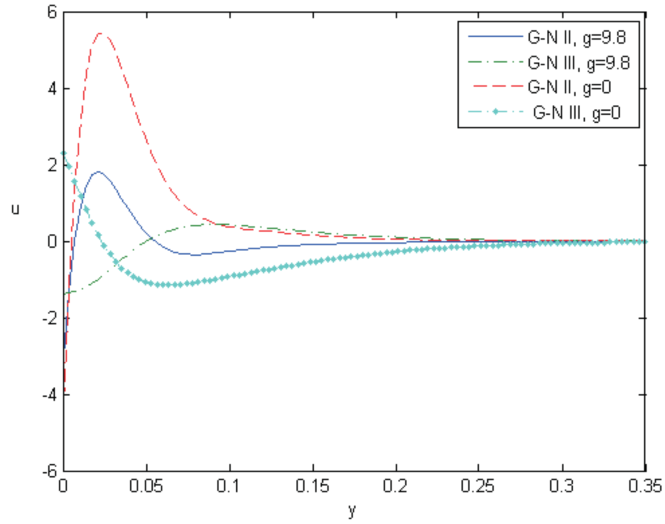


**Figure 1** Distribution of the displacement $u$ against $y$ while $g = 9.8\ \mathrm{m/s^2}$, 0

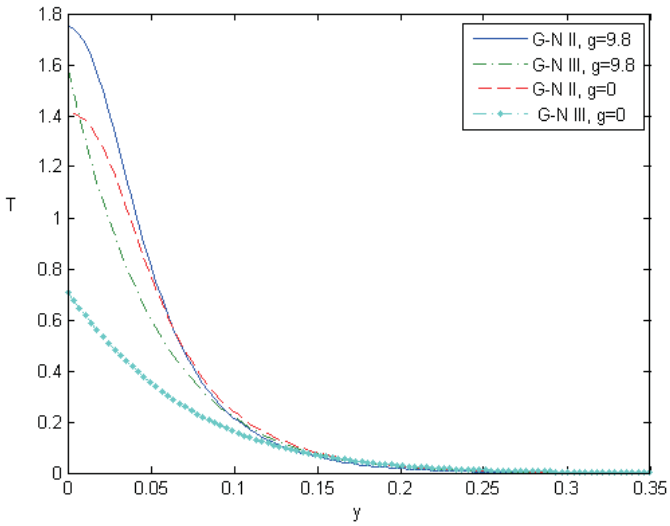


**Figure 2** Distribution of the displacement $T$ against $y$ while $g = 9.8\ \mathrm{m/s^2}$, 0

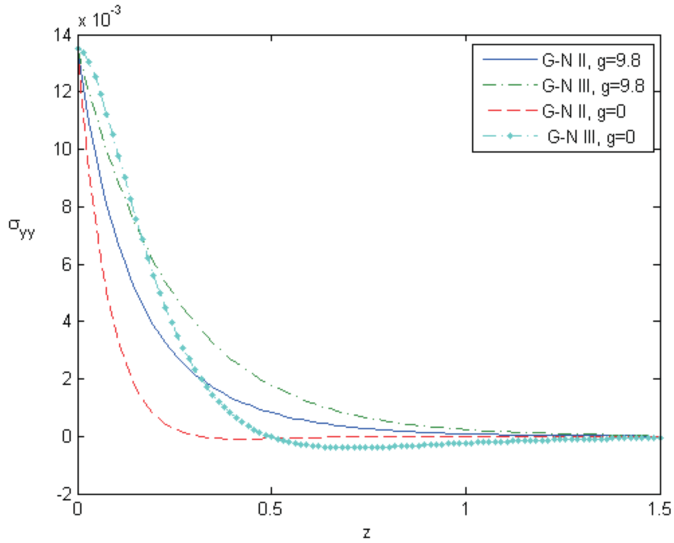


**Figure 3** Distribution of the displacement $\sigma_{yy}$ against $y$ while $g = 9.8$ m/s$^2$, 0

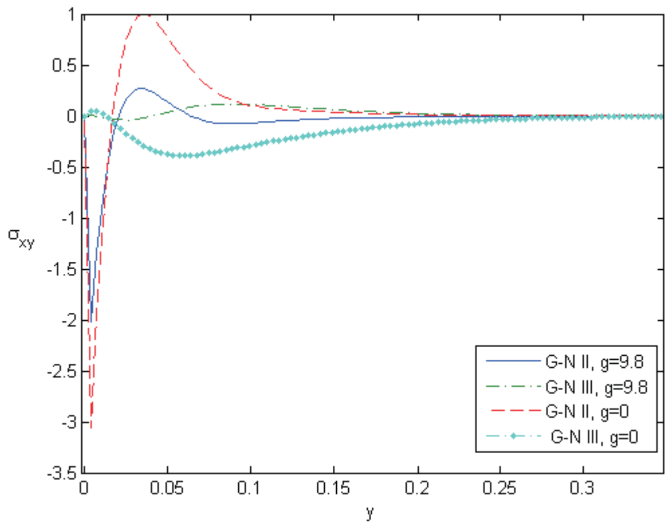


**Figure 4** Distribution of the displacement $\sigma_{xy}$ against $y$ while $g = 9.8$ m/s$^2$, 0

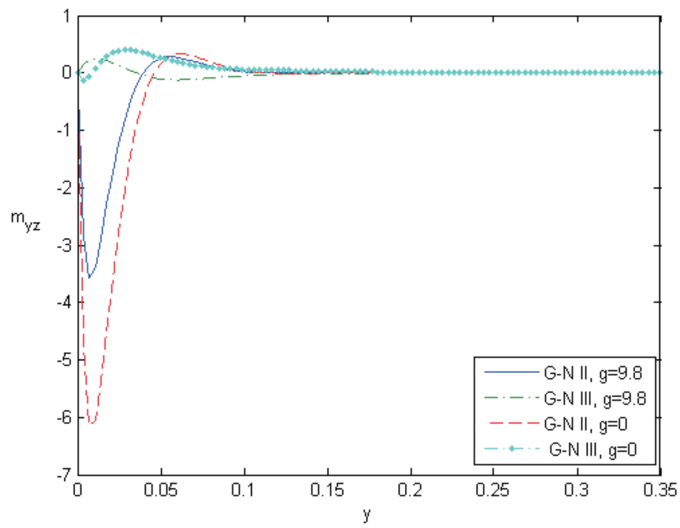


**Figure 5** Distribution of the displacement $m_{yz}$ against $y$ while $g = 9.8$ m/s$^2$, 0

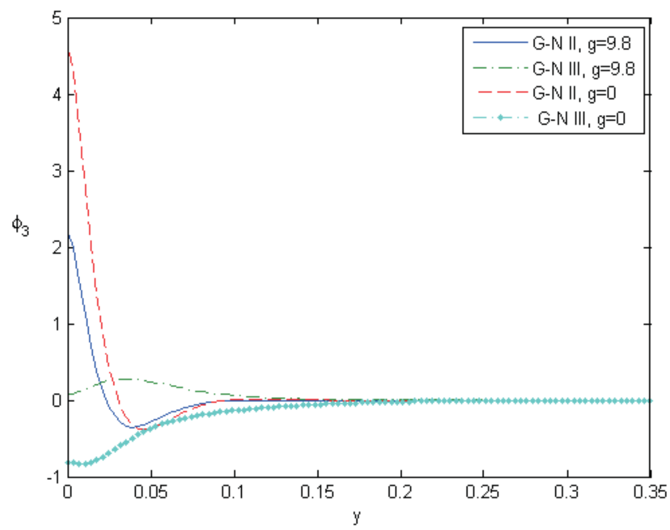


**Figure 6** Distribution of the displacement $\Phi_3$ against $y$ while $g = 9.8$ m/s$^2$, 0

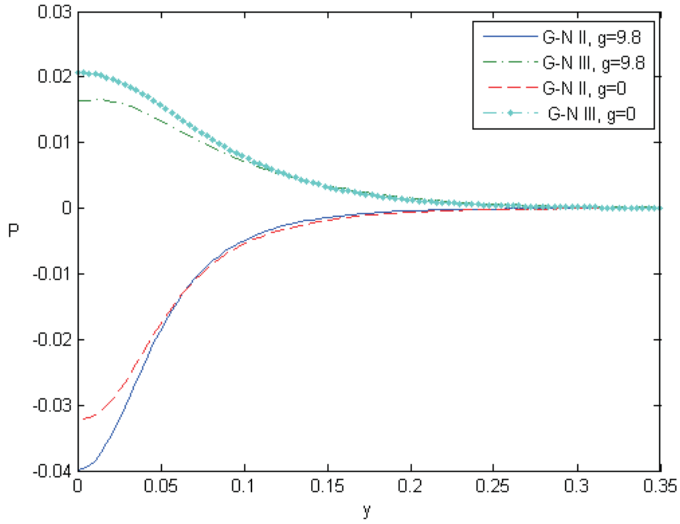


**Figure 7** Distribution of the displacement $P$ against $y$ while $g = 9.8$ m/s$^2$, 0

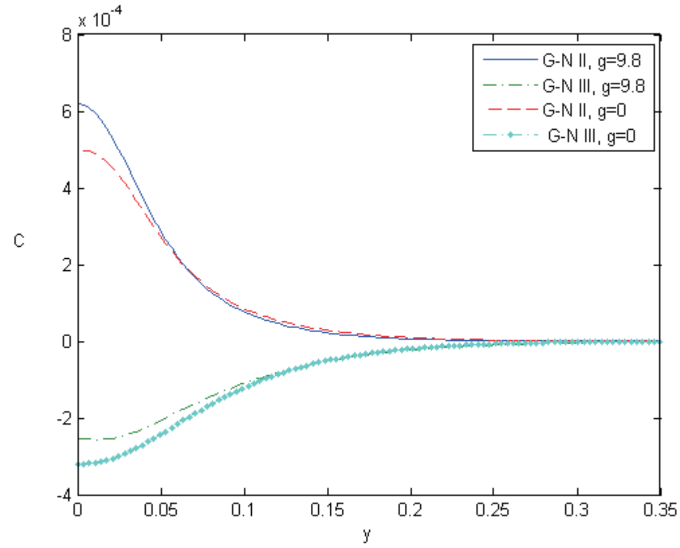


**Figure 8** Distribution of the displacement $C$ against $y$ while $g = 9.8$ m/s$^2$, 0

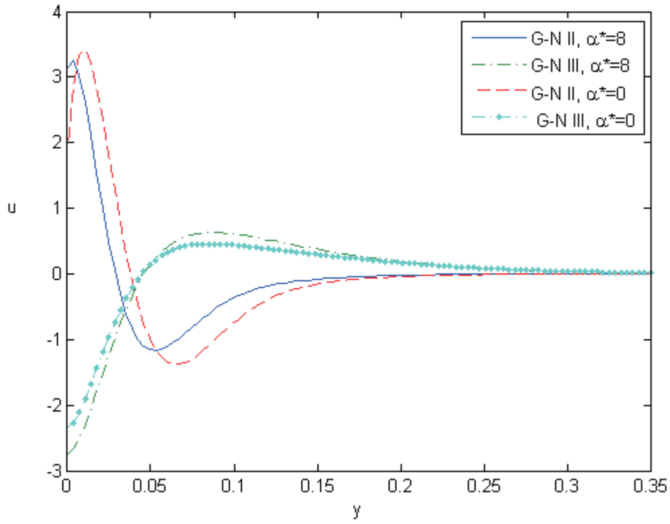


**Figure 9** Distribution of the displacement $u$ against $y$ while $\alpha^* = 8,\, 0$

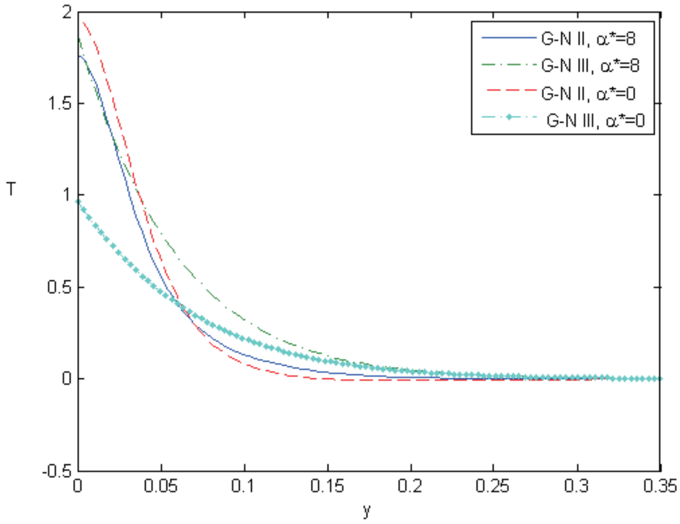


**Figure 10** Distribution of the displacement $T$ against $y$ while $\alpha^* = 8,\, 0$

Fig. 10 clarifies that the distribution of the temperature $T$decreases in the case of (G-N II) in the range $0 \leq y \leq 0.06$, then it increases in the range $0.06 \leq y \leq 0.35$, while it increases in the case of (G-N III) for $y > 0$ with the increase of $\alpha^*$. Fig. 11 explains that the distribution of the stress component $\sigma_{yy}$ decreases in the case of (G-N II) for $y > 0$; while it decreases in the range $0 \leq y \leq 0.06$, then it increases in the range $0.06 \leq y \leq 0.35$ in the case of (G-N III) with the increase of $\alpha^*$. Fig. 12 shows that the distribution of the stress component $\sigma_{xy}$ increases in the ranges $0 \leq y \leq 0.05$, and $0.1 \leq y \leq 0.35$, then it decreased in the range $0.05 \leq y \leq 0.1$, in the case of (G-N II), while it increases in the range $0 \leq y \leq 0.06$, then decreases in the range $0.06 \leq y \leq 0.35$,in the case of (G-N III) with the increase of $\alpha^*$. Figs. 13 and 14 clarify that the distribution of the couple stress $m_{yz}$and the microrotation vector $\phi_3$ respectively. The distribution of $m_{yz}$in the context of (G-N II) increases in the range $0 \leq y \leq 0.05$, then decreases in the range $0.05 \leq y \leq 0.35$, while in the context of (G-N III) theory it increases in the range $0 \leq y \leq 0.04$, moreover it seems to be identically with the increase of $\alpha^*$. The distribution of $\phi_3$ decreases in the case of (G-N II) in the range $0 \leq y \leq 0.03$, then increases in the range $0.03 \leq y \leq 0.35$, and increases in the case of (G-N III) for $y > 0$ with the increase of $\alpha^*$. Figs. 15 and 16 depict that the chemical potential $P$ and the concentration $C$ respectively. The distribution of $P$in the context of (G-N II) increases in the range $0 \leq y \leq 0.07$, then decreases in the range $0.07 \leq y \leq 0.35$, while in the context of (G-N III) it increases in the range $y > 0$ with the increase of $\alpha^*$. The distribution of $C$in the context of (G-N II) decreases in the range $0 \leq y \leq 0.1$, then increases in the range $0.1 \leq y \leq 0.35$, while in the context of (G-N III) it decreases in the range $y > 0$ with the increase of $\alpha^*$.

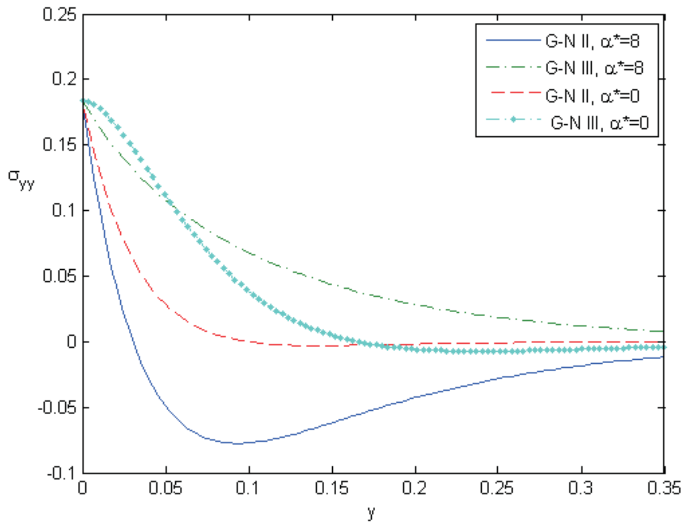


**Figure 11** Distribution of the displacement $\sigma_{yy}$ against $y$ while $\alpha^* = 8, 0$

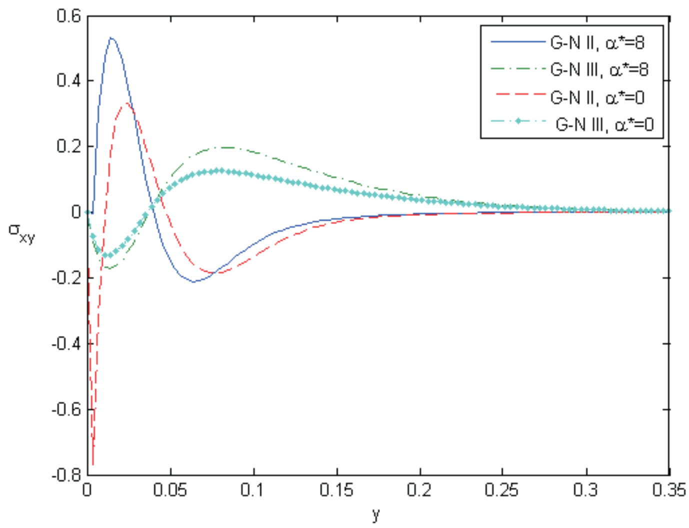


**Figure 12** Distribution of the displacement $\sigma_{xy}$ against $y$ while $\alpha^* = 8$, 0

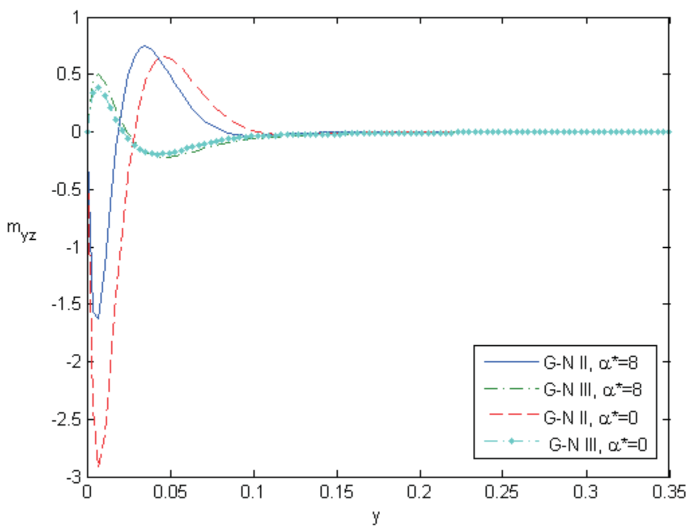


**Figure 13** Distribution of the displacement $m_{yz}$against $y$while $\alpha^* = 8$, 0

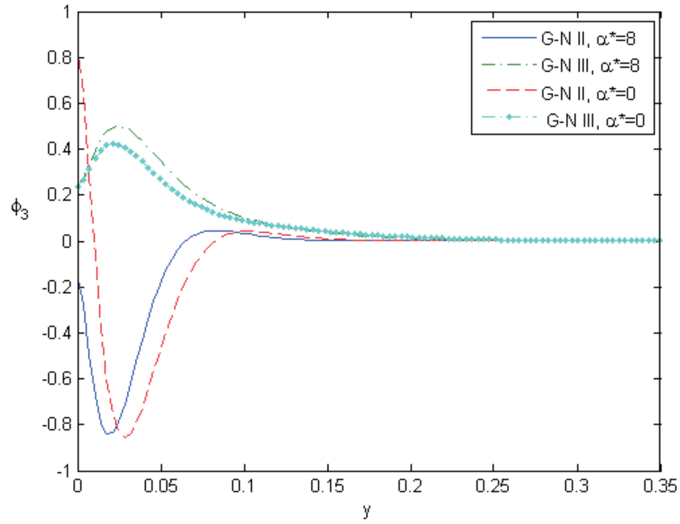


**Figure 14** Distribution of the displacement $\Phi_3$ against $y$ while $\alpha^* = 8,\ 0$

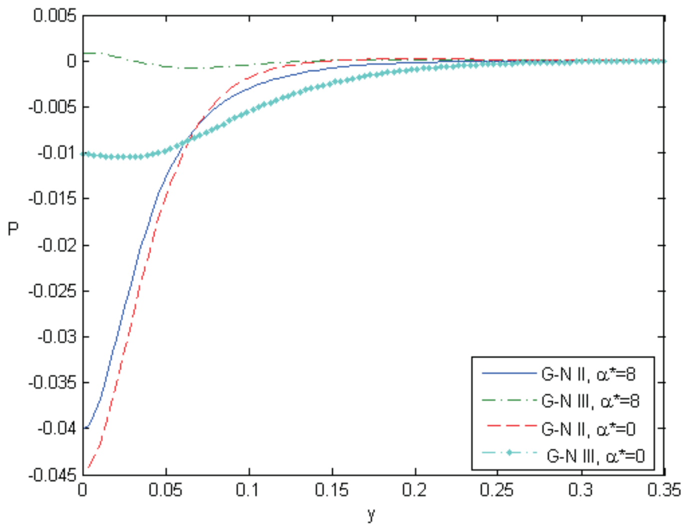


**Figure 15** Distribution of the displacement $P$ against $y$ while $\alpha^* = 8,\ 0$

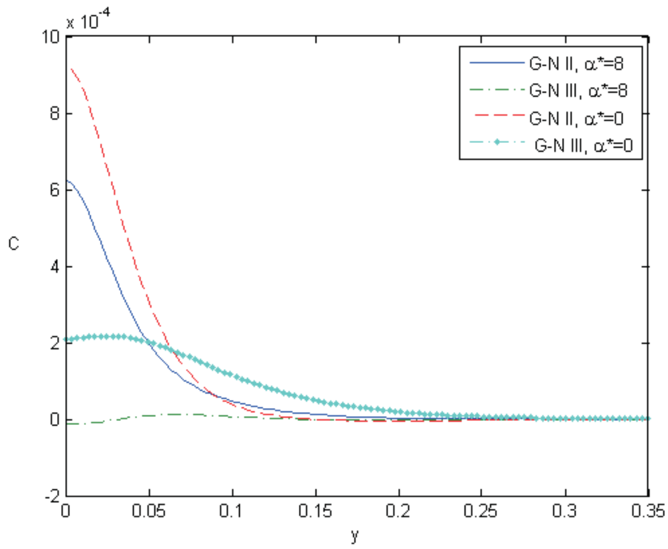


**Figure 16** Distribution of the displacement $C$ against $y$ while $\alpha^* = 8$, 0

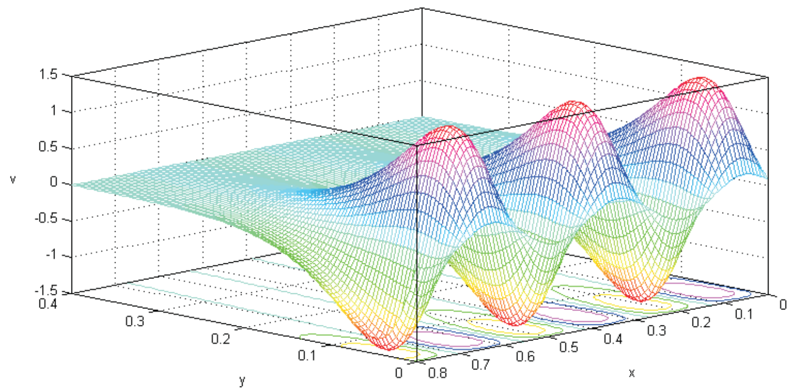


**Figure 17** Distribution of the displacement $v$

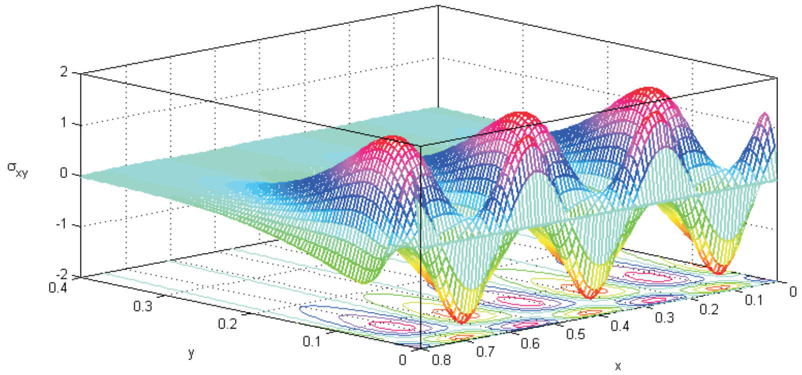


**Figure 18** Distribution of the stress $\sigma_{xy}$

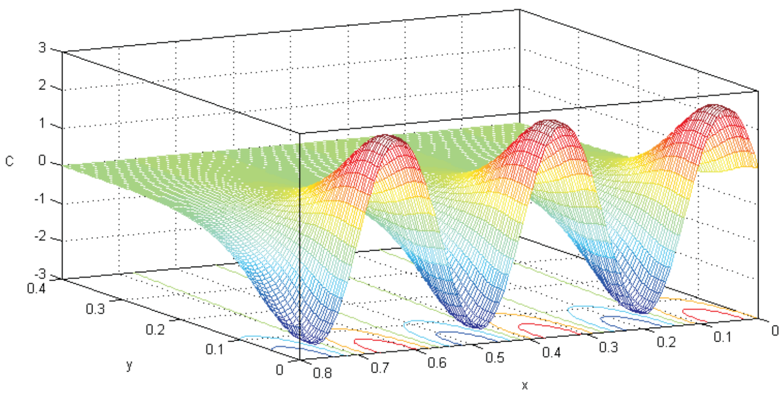


**Figure 19** Distribution of the concentreation $C$

It can deduce that all the functions are continuous and all the curves converge to zero. The empirical material constant $\alpha^*$ is an effective physical operator in the distribution of all the physical quantities of the problem.

The 3D curves are representing the complete relation between $v$, $\sigma_{xy}$, and $C$ against both components of the distance as shown in Figs. 17-19, where, $g = 9.8$ m/s$^2$ and $\alpha^* = 8$ K$^{-1}$ with the presence of the micropolar effect at $t = 0.5$ s. These figures are very important to study the dependence on the distances $x$ and $y$ while they are moving in wave propagation.

## 6. Concluding remarks

The results concluded from the above analysis can be summarized as:
1. The gravity and the temperature dependent properties as effective physical operators having an effective role on the distribution of the physical quantities; since the behavior of them varying with the increase of the value of these physical operators.
2. The micropolar is an important property; the presence and the absence of this property is observable effect on the variation of the considered physical quantities.
3. The diffusion is a significant characteristic property of the used medium in the study.
4. The value of all physical quantities converges to zero with an increase in the distance $y$ and all functions are continuous.

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## Appendix

$$m_1 = l_1 + 1 \qquad m_2 = a^2 - \frac{l_5\,\xi^2}{m_1} \qquad m_3 = \frac{\mathrm{i}\,a\,l_4}{m_1} \qquad m_4 = \frac{l_3}{m_1} \qquad m_5 = \mathrm{i}\,a\,l_4$$

$$m_6 = a^2 - l_5\,\xi^2 \qquad m_7 = a^2 - 2\,l_6 - l_7\,\xi^2 \qquad m_8 = a^2 - \mathrm{i}\,\xi\,l_9 \qquad m_9 = \varepsilon_3 - \mathrm{i}\,\varepsilon_2$$

$$m_{10} = \frac{\varepsilon_1\,\xi^2}{m_9} \qquad m_{11} = \frac{-\,\xi^2\,l_{11}}{m_9} \qquad m_{12} = a^2 - \frac{\xi^2}{m_9}$$

$$B_{1\mathrm{n}} = \frac{k_{\mathrm{n}}^2 - m_4(B_{3\mathrm{n}} + B_{4\mathrm{n}}) - m_2}{m_3} \qquad B_{2\mathrm{n}} = \frac{l_6\,B_{1\mathrm{n}}(k_{\mathrm{n}}^2 - a^2)}{(k_{\mathrm{n}}^2 - m_7)}$$

$$B_{3\mathrm{n}} = \frac{k_{\mathrm{n}}^2\,(B_{4\mathrm{n}} + m_{10}) - (m_{10}\,a^2 + m_{12}\,B_{4\mathrm{n}})}{m_{11}}$$

$$B_{4\mathrm{n}} = \frac{k_{\mathrm{n}}^4\,(l_8\,m_{11} + m_{10}) - k_{\mathrm{n}}^2\,(m_8\,m_{10} + m_{10}\,a^2 + 2\,l_8\,m_{11}\,a^2) + (l_8\,m_{11}\,a^4 + m_8\,m_{10}\,a^2)}{k_{\mathrm{n}}^4 - k_{\mathrm{n}}^2\,(m_8 + m_{12} - l_{10}\,m_{11}) + (m_8\,m_{12} - l_{10}\,m_{11}\,a^2)}$$

$$A_{1\mathrm{n}} = \mathrm{i}\,a - k_{\mathrm{n}}\,B_{1\mathrm{n}} \qquad A_{2\mathrm{n}} = -\,(k_{\mathrm{n}} + \mathrm{i}\,a\,B_{1\mathrm{n}})$$

$$A_{3\mathrm{n}} = l_{12}\,(\mathrm{i}\,a\,A_{1\mathrm{n}} - k_{\mathrm{n}}\,A_{2\mathrm{n}}) + \mathrm{i}\,a\;l_{13}\,A_{1\mathrm{n}} - l_{14}\,B_{3\mathrm{n}} - l_{14}\,B_{4\mathrm{n}}$$

$$A_{4\mathrm{n}} = l_{12}\,(\mathrm{i}\,a\,A_{1\mathrm{n}} - k_{\mathrm{n}}\,A_{2\mathrm{n}}) - k_{\mathrm{n}}\;l_{13}\,A_{2\mathrm{n}} - l_{14}\,B_{3\mathrm{n}} - l_{14}\,B_{4\mathrm{n}}$$

$$A_{5\mathrm{n}} = l_{15}\,(-k_{\mathrm{n}}A_{1\mathrm{n}} + \mathrm{i}\,a\,A_{2\mathrm{n}}) + l_{16}\,(\mathrm{i}\,a\,k_{\mathrm{n}}A_{2\mathrm{n}} - B_{2\mathrm{n}})$$

$$A_{6\mathrm{n}} = -k_{\mathrm{n}}\,B_{2\mathrm{n}}\,l_{17} \qquad A_{7\mathrm{n}} = \mathrm{i}\,a\,B_{2\mathrm{n}}\,l_{17}$$

$$A_{8\mathrm{n}} = -l_{18}\,(\mathrm{i}\,a\,A_{1\mathrm{n}} - k_{\mathrm{n}}\,A_{2\mathrm{n}}) + l_{19}\,B_{3\mathrm{n}} + l_{20}\,B_{4\mathrm{n}}$$

$$A_{9\mathrm{n}} = -k_{\mathrm{n}}\,B_{3\mathrm{n}} \qquad \mathrm{n} = 1, 2, 3, 4, 5.$$

$$l_1 = \frac{\lambda_0 + \mu_0}{\mu_0 + k_0^*} \qquad l_2 = \frac{k_0^*}{\mu_0 + k_0^*} \qquad l_3 = \frac{\rho\,c_1^2}{\mu_0 + k_0^*} \qquad l_4 = \frac{\rho\,g\,c_1^2}{(\mu_0 + k_0^*)f(T)}$$

$$l_5 = \frac{\rho\,c_1^2}{(\mu_0 + k_0^*)f(T)} \qquad l_6 = \frac{c_1^2\,k_0^*}{\gamma_0\,\omega_1^{*2}} \qquad l_7 = \frac{j\,\rho\,c_1^2}{\gamma_0\,f(T)} \qquad l_8 = \frac{\omega_1^*\,\beta_{20}^2\,f(T)}{\rho\,c_1^2\,b}$$

$$l_9 = \frac{c_1^2}{d\,b} \qquad l_{10} = \frac{a^*\,\omega_1^*\,\beta_{20}}{b\,\beta_{10}} \qquad l_{11} = \frac{a^*\,T_0\,\beta_{10}}{\rho\,C_e\,\beta_{20}\omega_1^*} \qquad l_{12} = \frac{\lambda_0\,f(T)}{\rho\,c_1^2}$$

$$l_{13} = \frac{(2\,\mu_0 + k_0^*)\,f(T)}{\rho\,c_1^2} \qquad l_{14} = f(T) \qquad l_{15} = \frac{\mu_0 f(T)}{\rho c_1^2} \qquad l_{16} = \frac{k_0^*\,f(T)}{\rho\,c_1^2}$$

$$l_{17} = \frac{\gamma_0\,f(T)\,\omega_1^*}{\rho\,c_1^3} \qquad l_{18} = \frac{\beta_{10}^2\,T_0}{\beta_{20}^2\,\rho\,c_1^2} \qquad l_{19} = \frac{b\,\beta_{10}\,T_0}{\beta_{20}^2} \qquad l_{20} = \frac{a^*\,T_0}{\beta_{20}}$$

$$\varepsilon_1 = \frac{\beta_{10}^2\,T_0\,f(T)}{\rho^2\,c_1^2\,C_e} \qquad \varepsilon_2 = \frac{K^*\,\omega_1^*}{\rho\,c_1^2\,C_e} \qquad \varepsilon_3 = \frac{K}{\rho\,c_1^2\,C_e}$$