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Effect of Inclined Load on Micropolar Theroelastic Medium Possessing Cubic Symmetry with Energy Dissipation

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The problem of the micropolar thermoelastic medium with (G-N) theory of types II (without energy dissipation) and type III (with energy dissipation) under the effect of the inclined load was investigated. The normal mode analysis is used to obtain the solution of the physical quantities. Comparisons are made between the results predicted by the (G-N) theory of type II and type III in different values of the angle of inclination.

Keywords: micropolar thermoelasticity, Green-Naghdi theory, normal mode method, inclined load.

1. Introduction

A micropolar elastic solid is an elastic solid whose deformation can be described by a 'macro' displacement together with a 'micro' rotation. Micropolar elastic materials are the elastic materials with an extra independent degree of freedom for local rotations. They include certain class of materials with fibrous and elongated grains. The theory of micropolar elasticity introduced and developed by Eringen [1-3] has aroused much interest in recent years, because of its possible utility in investigating deformation properties of solids for which the classical theory is inadequate. Furthermore, the micropolar elastic model is considered to be more realistic than the classical elastic model in studying earth science problems studied by Iesan [4]. The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effects by Nowacki [5] and Eringen [6]. Othman et al. [7] studied the effect of rotation and initial stress on generalized micropolar thermoelastic medium with three-phase-lag. Othman et al. [8] studied the influence of thermal loading due to laser pulse on generalized micropolar thermoelastic solid with a comparison of different theories. Othman and Singh [9] studied the effect of rotation on generalized micropolar thermoelasticity in a half-space under five theories. Othman et al. [10] studied the effect of rotation on micropolar generalized thermoelasticity with two temperatures using a dual-phase lag model. Abbas and Kumar [11] studied the deformation due to the thermal source in micropolar generalized thermo-elastic half-space by finite element method. Othman and Song [12] studied the effect of the thermal relaxation and magnetic field on generalized micropolar thermoelasticity.

Green and Naghdi [13–15] established a new generalized thermoelasticity theory (G-N) theory of three types based on the energy and entropy balances, in which the energy dissipation was not considered in the previous theories. The linearized form of type I was equivalent to the classical thermoelasticity (CT) theory. Type II describes the thermo-elastic system without energy dissipation, while type III permits the dissipation of the energy. Therefore, the (G-N) theory is an ideal thermoelasticity theory. Othman and Atwa [16] studied the two-dimensional problem of generalized thermo-microstretch elastic solid under Green-Naghdi theory. Othman et al. [17] studied the influence of the gravitational field and rotation on thermoelastic solid with voids under Green-Naghdi theory. Othman et al. [18] studied the effect of the gravitational field and temperature dependent properties on a two-temperature thermoelastic medium with voids under (G-N) Theory. Othman and Atwa [19] studied the effect of rotation on a fiber-reinforced thermoelastic under Green-Naghdi theory and the influence of gravity. Othman [20] studied generalized magneto-thermo-microstretch elastic solid under the gravitational effect with energy dissipation. Othman and Atwa [21] studied the 2-D problem of a mode-I crack for a generalized thermoelasticity under Green-Naghdi theory.

Kumar et al. [22, 23] investigated different problems in the micropolar elastic medium due to inclined load. Minagawa et al. [24] discussed the propagation of plane harmonic waves in a cubic micropolar medium. Kumar and Rani [25] studied time harmonic sources in a thermally conducting cubic crystal. The deformation due to other sources such as strip loads, continuous line loads, etc. can also be similarly obtained. The deformation at any point of the medium is useful to analyse the deformation field around mining tremors and drilling into the crust of the earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation fields in the entire volume surrounding the source region. Some works on (G-N) theory are discussed in Refs. [26-29].

In the present work, we are studying the effect of inclined load in the micropolar thermoelastic medium. The formulation is applied in the context of (G-N) theory of the both types II and III, the normal mode method used to obtain the exact expressions for all physical quantities, the distributions of the considered variables are represented graphically.

2. Formulation of the problem

We consider a homogeneous and micropolar thermoelastic medium under inclined load. All quantities considered are functions of the time t and the coordinates x, z and the dynamic displacement vector is $u = (u,\, 0,\, w)$. The system of governing equations of a linear micropolar thermo-elasticity under inclined load and without body forces based on (G-N) theory:

$$\sigma_{ji,j} = \rho u_{i,\,tt} \tag{1}$$

$$\varepsilon_{ijk}\sigma_{jr} + m_{ji,j} = j\rho\phi_{i,\,tt} \tag{2}$$

$$K^{*}\nabla^{2}T + K\nabla^{2}\frac{\partial T}{\partial t} = \rho\, C_{E}\frac{\partial^{2}T}{\partial t^{2}} + T_{0}\gamma_{1}\frac{\partial^{2}e}{\partial t^{2}} \tag{3}$$

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + k(u_{j,i} - \varepsilon_{ijk}\phi_{r}) - \gamma_{1}T\delta_{ij} \tag{4}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,\,i} \tag{5}$$

where, $i,\ j,\ r = 1,\ 2,\ 3$, T is the temperature above the reference temperature T_0 chosen so that $|(T - T_0)| << 1$,λ,μ are the Lame' constants, the components of displacement vector u are u_i,σ_{ij} are the components of the stress tensor, e is the dilatation, e_{ij} are the components of strain tensor, j the micro-inertia moment, m_{ij} is the couple stress tensor, $\delta_{ij}$is the Kronecker delta, α, β, γ and k are the micropolar constants, ε_{ijr} is the alternate tensor, the mass density is ρ, the specific heat at constant strain is C_E, the thermal conductivity is K^* and K is the material characteristic of the theory, $\gamma_1 = (3\lambda + 2\mu)\alpha_t$, α_t coefficient of linear thermal expansion.

The constitutive equations can be written as:

$$\sigma_{xx} = \lambda e + (2\mu + k)\frac{\partial u}{\partial x} - \gamma_{1}T \tag{6}$$

$$\sigma_{yy} = \lambda e - \gamma_{1}T \tag{7}$$

$$\sigma_{zz} = \lambda e + (2\mu + k)\frac{\partial w}{\partial z} - \gamma_{1}T \tag{8}$$

$$\sigma_{xz} = \mu\frac{\partial u}{\partial z} + (k + \mu)\frac{\partial w}{\partial x} + k\phi_{2} \tag{9}$$

$$\sigma_{zx} = \mu\frac{\partial w}{\partial x} + (k + \mu)\frac{\partial u}{\partial z} - k\phi_{2} \tag{10}$$

$$m_{xy} = \gamma\frac{\partial\phi_{2}}{\partial x} \tag{11}$$

$$m_{zy} = \gamma\,\frac{\partial\phi_{2}}{\partial z} \tag{12}$$

Using Eqs. (6–12) in Eqs. (1–3) we have:

$$\rho\frac{\partial^{2}u}{\partial t^{2}} = (\lambda + \mu)\,\frac{\partial e}{\partial x} + (\mu + k)\,\nabla^{2}u - k\frac{\partial\phi_{2}}{\partial z} - \gamma_{1}\frac{\partial T}{\partial x} \tag{13}$$

$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda+\mu)\,\frac{\partial e}{\partial z} + (\mu+k)\,\nabla^2 w + k\frac{\partial \phi_2}{\partial x} - \gamma_1 \frac{\partial T}{\partial z} \tag{14}$$

$$j\rho\,\frac{\partial^2 \phi_2}{\partial t^2} = k\,(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) + \gamma\,\nabla^2 \phi_2 - 2k\phi_2 \tag{15}$$

where: $e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$.
For simplifications we shall use the following non-dimensional variables:

$$(x',\, z') = \frac{\eta_0}{C_0}(x,\, z) \qquad (u',\, w') = \frac{\rho\eta_0 C_0}{\gamma_1 T_0}(u,\, w) \qquad t' = \eta_0\, t \qquad T' = \frac{T}{T_0}$$
$$\sigma'_{ij} = \frac{\sigma_{ij}}{\gamma_1 T_0} \qquad \phi'_2 = \frac{\rho C_0^2}{\gamma_1 T_0}\phi_2 \qquad m'_{ij} = \frac{\eta_0}{C_0 \gamma_1 T_0} m_{ij} \tag{16}$$

where, $\eta_0 = \frac{\rho C_E C_0^2}{K^*}$, $\gamma_1 = (3\lambda + 2\mu + k)\,\alpha_t$, α_t is the linear thermal expansion coefficient and $C_0^2 = (\lambda + 2\mu + k)\,/\rho$.
Eqs. (13–15) and (3) take the following form (dropping the dashed for convenience):

$$\frac{\partial^2 u}{\partial t^2} = \frac{\lambda+\mu}{\rho C_0^2}\frac{\partial e}{\partial x} + \frac{k+\mu}{\rho C_0^2}\nabla^2 u - \frac{k}{\rho C_0^2}\frac{\partial \phi_2}{\partial z} - \frac{\partial T}{\partial x} \tag{17}$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{\lambda+\mu}{\rho C_0^2}\frac{\partial e}{\partial z} + \frac{k+\mu}{\rho C_0^2}\nabla^2 w + \frac{k}{\rho C_0^2}\frac{\partial \phi_2}{\partial x} - \frac{\partial T}{\partial z} \tag{18}$$

$$j\rho\eta_0^2\frac{\partial^2 \phi_2}{\partial t^2} = k\,(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) + \frac{\gamma\eta_0^2}{C_0^2}\,\nabla^2\phi_2 - 2k\phi_2 \tag{19}$$

$$K^*\nabla^2 T + K\eta_0\nabla^2\frac{\partial T}{\partial t} = \rho\, C_E C_0^2\frac{\partial^2 T}{\partial t^2} + \frac{\gamma_1^2 T_0}{\rho}\frac{\partial^2 e}{\partial t^2} \tag{20}$$

We introduce the scalar potential $q(x,\, z,\, t)$and the vector potential $\psi(x,\, z,\, t)$ which are related to displacement components, we obtain:

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z} \qquad w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x} \qquad e = \nabla^2 q \qquad \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = \nabla^2\psi \tag{21}$$

Substituting from Eq. (21) in Eqs. (17–20) we obtain:

$$(a_1\nabla^2 - \frac{\partial^2}{\partial t^2})q - T = 0 \tag{22}$$

$$(a_2\nabla^2 - \frac{\partial^2}{\partial t^2})\psi - \frac{k}{\rho C_0^2}\,\phi_2 = 0 \tag{23}$$

$$k\nabla^2\psi + (a_3\nabla^2 - \rho j\eta_0^2\frac{\partial^2}{\partial t^2} - 2k)\phi_2 = 0 \tag{24}$$

$$K^*\nabla^2 T + K\eta_0\frac{\partial}{\partial t}(\nabla^2 T) = \rho C_E C_0^2\frac{\partial^2 T}{\partial t^2} + \frac{\gamma_1^2 T_0}{\rho}\,\frac{\partial^2}{\partial\, t^2}(\nabla^2 q) \tag{25}$$

where: $a_1 = \frac{\lambda+2\mu+k}{\rho C_0^2}$, $a_2 = \frac{\mu+k}{\rho C_0^2}$, $a_3 = \frac{\gamma\eta_0^2}{C_0^2}$.

3. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$[q, \psi, T, \phi_2, \sigma_{ij}, m_{ij}, F_1, F_2, f](x, z, t) \\ = [q^*, \psi^*, T^*, \phi_2^*, \sigma_{ij}^*, m_{ij}^*, F_1^*, F_2^*, f^*](z) \exp(\omega\, t + \mathrm{i} a x) \tag{26}$$

Where, ω is a frequency constant, $\mathrm{i} = \sqrt{-1}$, a is the wave number in the x direction. Using Eq. (26) in Eqs. (22–25), then we have:

$$[a_1\,(\mathrm{D}^2 - a^2) - \omega^2]\, q^* - T^* = 0 \tag{27}$$

$$[a_2(\mathrm{D}^2 - a^2)\; - \omega^2]\, \psi^* - \tfrac{k}{\rho C_0^2}\phi_2^* = 0 \tag{28}$$

$$k\,(\mathrm{D}^2 - a^2)\, \psi^* + [a_3(\mathrm{D}^2 - a^2) - \rho j \eta_0^2 \omega^2 - 2k]\, \phi_2^* = 0 \tag{29}$$

$$[b_1\, \mathrm{D}^2 - b_2]\, q^* + [b_3\, \mathrm{D}^2 - b_4\,]\, T^* = 0 \tag{30}$$

where:

$$b_1 = \frac{-\,\gamma_1^2 T_0 \omega^2}{\rho} \qquad b_2 = \frac{-\,\gamma_1^2 T_0 \omega^2 a^2}{\rho} \qquad b_3 = (K^* + \omega K \eta_0)$$

$$b_4 = a^2 (K^* + \omega K \eta_0) + \rho\, C_E C_0^2\, \omega^2$$

Eliminating q^*, T^* between Eqs. (28) and (30) we obtain:

$$(\mathrm{D}^4 - C_1 \mathrm{D}^2 + C_2)\{q^*, T^*\} = 0 \tag{31}$$

Eliminating ψ^*, ϕ_2^* between Eqs. (28) and (29) we obtain:

$$(\mathrm{D}^4 - C_3 \mathrm{D}^2 + C_4)\{\, \psi^*, \phi_2^*\} = 0 \tag{32}$$

where:

$$C_1 = \tfrac{b_4}{b_3} + a^2 + \tfrac{\omega^2}{a_1} - \tfrac{b_1}{a_1 b_3}$$
$$C_2 = \tfrac{a^2 b_4}{b_3} + \tfrac{\omega^2 b_4}{a_1 b_3} - \tfrac{b_2}{a_1 b_3}$$
$$C_3 = a^2 + \tfrac{\rho j \eta_0^2 \omega^2}{a_3} + \tfrac{2k}{a_3} + a^2 + \tfrac{\omega^2}{a_2} - \tfrac{k^2}{\rho C_0^2 a_2 a_3}$$
$$C_4 = a^4 + \tfrac{\rho j \eta_0^2 \omega^2}{a_3}(a^2 + \tfrac{\omega^2}{a_2}) + \tfrac{2k}{a_3}(a^2 + \tfrac{\omega^2}{a_2}) + \tfrac{a^2 \omega^2}{a_2} - \tfrac{k^2 a^2}{\rho C_0^2 a_2 a_3}$$

The solution of Eqs. (31) and (32), bound for $z \to \infty$, are given by:

$$q^* = \sum_{n=1}^{2} M_n e^{-k_n z} \tag{33}$$

$$T^* = \sum_{n=1}^{2} N_n M_n e^{-k_n z} \tag{34}$$

$$\psi^* = \sum_{m=3}^{4} M_m e^{-k_m z} \tag{35}$$

$$\phi_2^* = \sum_{m=3}^{4} N_m M_m e^{-k_m z} \tag{36}$$

where M_n and M_m are some constants, k_n^2, $(n = 1, 2)$ are the roots of the characteristic equation of Eq. (31), k_m^2, $(m = 3, 4)$ are the roots of the characteristic equation of Eq. (32) and N_n, N_m from Eqs. (27), (28) as follows:

$$N_n = a_1(k_n^2 - a^2) - \omega^2, \quad n = 1, 2, \quad N_m = \frac{\rho C_0^2}{k}(a_2(k_m^2 - a^2) - \omega^2), \quad m = 3, 4.$$

The roots $k_{1,2}^2$ and $k_{3,4}^2$ of Eqs. (31) and (32), respectively, are given by:

$$k_{1,2}^2 = \frac{1}{2}(C_1 \pm \sqrt{C_1^2 - 4C_2}\ ) \tag{37}$$

$$k_{3,4}^2 = \frac{1}{2}(C_3 \pm \sqrt{C_3^2 - 4C_4}\ ) \tag{38}$$

4. Boundary conditions

We consider an inclined load p acting in the direction who make an angle θ with the direction of the x axis:

$$\sigma_{zz} = F_1 = -p\cos\theta \qquad \sigma_{zx} = F_2 = -p\sin\theta \qquad m_{zy} = 0 \qquad T = f(x, t) \tag{39}$$

Using (16), (21), 926) on the non-dimensional boundary conditions and using (8), (10), (12), (34) we obtain the expressions of the displacement components, the stress components, the coupled stress distribution for micropolar thermoelastic medium:

$$u^* = ia(M_1 e^{-k_1 z} + M_2 e^{-k_2 z}) - k_3 M_3 e^{-k_3 z} - k_4 M_4 e^{-k_4 z} \tag{40}$$

$$w^* = -k_1 M_1 e^{-k_1 z} - k_2 M_2 e^{-k_2 z} - ia(M_3 e^{-k_3 z} + M_4 e^{-k_4 z}) \tag{41}$$

$$\sigma_{zz}^* = h_1 M_1 e^{-k_1 z} + h_2 M_2 e^{-k_2 z} + h_3 M_3 e^{-k_3 z} + h_4 M_4 e^{-k_4 z} \tag{42}$$

$$\sigma_{zx}^* = l_1 M_1 e^{-k_1 z} + l_2 M_2 e^{-k_2 z} + l_3 M_3 e^{-k_3 z} + l_4 M_4 e^{-k_4 z} \tag{43}$$

$$m_{zy}^* = r_3 M_3 e^{-k_3 z} + r_4 M_4 e^{-k_4 z} \tag{44}$$

where: $h_1 = -a^2 a_4 + a_1 k_1^2 - N_1$, $h_2 = -\, a^2 a_4 + a_1 k_2^2 - N_2$, $h_3 = -iaa_4 k_3 + iaa_1 k_3$, $h_4 = -\ iaa_4 k_4 + iaa_1 k_4$, $l_1 = -\ ia(a_5 k_1 + a_2 k_1)$, $l_2 = -\ ia(a_5 k_2 + a_2 k_2)$, $l_3 = a^2 a_5 + a_2 k_3^2 - a_6 N_3$, $l_4 = a^2 a_5 + a_2 k_4^2 - a_6 N_4$, $r_3 = -\ a_7 k_3 N_3$, $r_4 = -\ a_7 k_4 N_4$, $a_4 = \frac{\lambda}{\rho C_0^2}$, $a_5 = \frac{\mu}{\rho C_0^2}$, $a_6 = \frac{k}{\rho C_0^2}$, $a_7 = \frac{\gamma \eta_0^2}{\rho C_0^4}$.

Invoking the boundary conditions (39) at the surface $z = 0$ of the plate, we obtain a system of four equations. After applying the inverse of the matrix method, we have the values of the four constants M_n, $n = 1, 2$ and M_m, $m = 3, 4$. Hence, we obtain the expressions of the physical quantity distribution for the micropolar generalized thermo-elastic medium:

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} = \begin{pmatrix} h_1 & h_2 & h_3 & h_4 \\ l_1 & l_2 & l_3 & l_4 \\ 0 & 0 & r_3 & r_4 \\ N_1 & N_2 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} F_1^* \\ -F_2^* \\ 0 \\ f^* \end{pmatrix} \tag{45}$$

5. Numerical results

To study the effect of inclined load, we now present some numerical results. For this purpose, the copper is taken as the thermoelastic material for which we take the following values of the different physical constants:

$$\lambda = 7.7 \times 10^{10} kgm^{-1}s^{-2},\ \mu = 3.86 \times 10^{10} kgm^{-1}s^{-2},\ \alpha_t = 1.78 \times 10^{-5} K^{-1},$$

$$\rho = 8945\ kgm^{-3},\ c_e = 383.1\ \ Jkg^{-1}K^{-1},\ T_0 = 293K,\ K^* = 2.97 \times 10^2,$$

$$K = 170kgmK^{-1}s^{-3}, j = 0.2 \times 10^{-15}\, m,\ k = 1.0 \times 10^{11},\ a = 1.0,\ x = 0.4,$$

$$t = 0.1,\ p = 2.0,\ f = 1.0,\ \omega = \omega_0 + \mathrm{i}\xi,\ \omega_0 = 0.7,\ \xi = 0.3.$$

The above numerical technique, was used for the distribution of the real parts of the displacement components $u$and w, the temperature T, the stress components σ_{zz} and σ_{zx}, the couple stress component m_{zy} and the microrotation component ϕ_2 with the distance $z$in 2D for (G-N) theory of types II and III with inclined effect. All the physical quantities are shown graphically in figures 1-7 in the case of different values of angle ($\theta = 30°,\ 45°,\ 60°$).

Fig. 1 depicts that the distribution of u increases with the increase of θ, we note that the curves in the (G-N) of type III greater than the curves of type II. Fig. 2 depicts that the distribution of $w$decreases with the increase of θ, we note that the curves in the (G-N) of type III under the curves of type II. Fig. 3 shows that the angle has a low effect on the temperature distribution. Fig. 4 expresses that the distribution of σ_{zz} increases with the increase of θ, we note that the curves in the (G-N) of type III under the curves of type II. Fig. 5 explains that the distribution of σ_{zx}decreases with the increase of θ, we note that the curves in the (G-N) of type III under the curves of type II. Fig. 6 depicts that the distribution of the couple stress component m_{zy} increases with the increase of θ, we note that the curves in the (G-N) of type III above the curves of type II. Fig. 7 depicts that the distribution of the microrotation component ϕ_2 decreases with the increase of θ, we note that the curves in the (G-N) of type III under the curves of type II. It is noticed that all the curves converge to zero, and the angle has a significant role for the distributions of all physical quantities.

3D curves are representing the complete relations between physical quantities, and both distance components as shown in Figs. 8–12, with the angle $\theta = 15°$ in the context of (G-N III). These figures are very important to study the dependence of these physical quantities on the two displacement components. The curves obtained are highly depending on the distance from origin, all the physical quantities are moving in wave propagation.

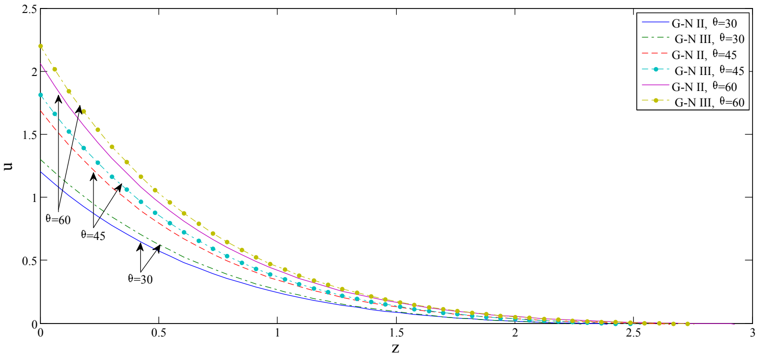

Figure 1 Variation of the displacement $u$with variation of the angle under (G-N) theory

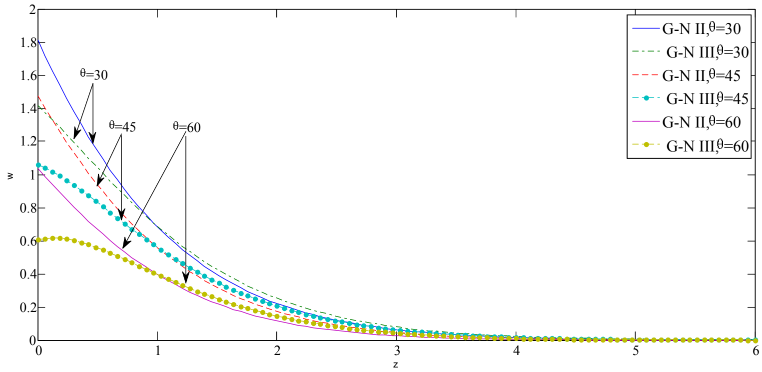

Figure 2 Variation of the displacement $w$with variation of the angle under (G-N) theory

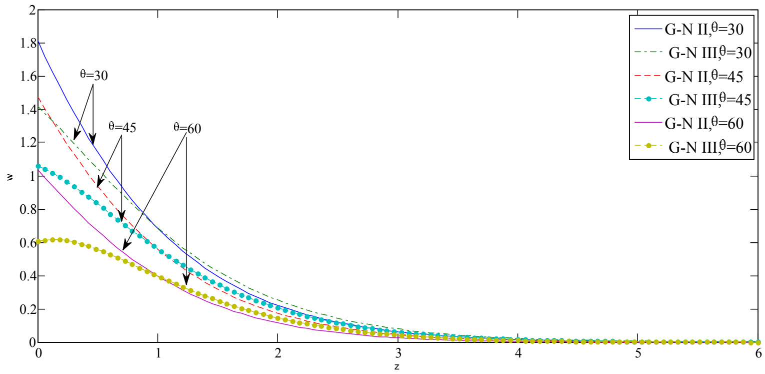

Figure 3 Variation of the temperature $T$with variation of the angle under (G-N) theory

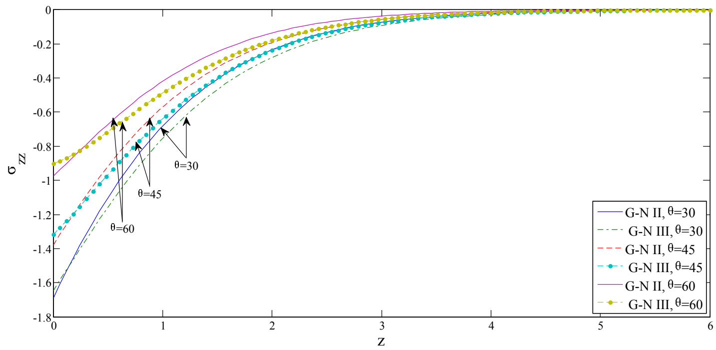

Figure 4 Variation of the stress σ_{zz} with variation of the angle under (G-N) theory

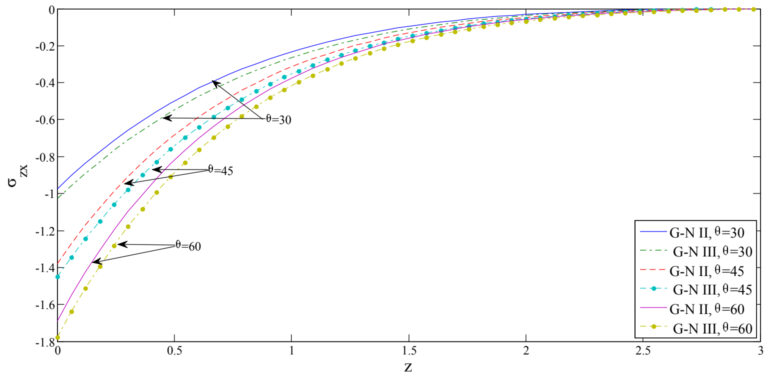

Figure 5 Variation of the stress $\sigma_{zx}$with variation of the angle under (G-N) theory

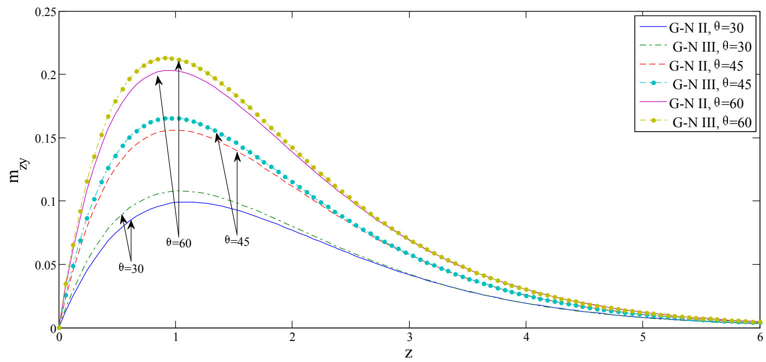

Figure 6 Variation of the tangential couple stress $m_{zy}$with variation of the angle under (G-N) theory

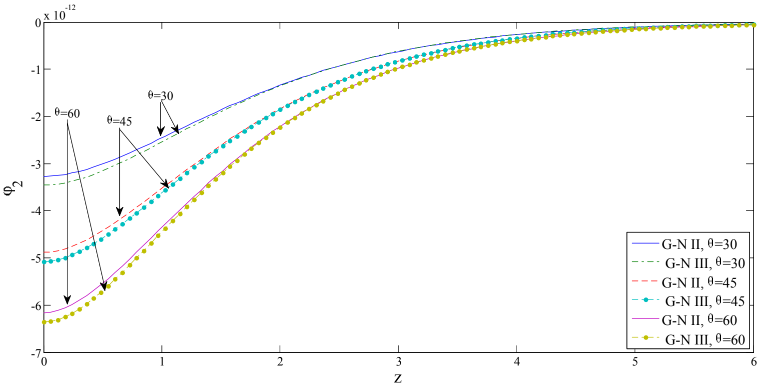

Figure 7 Variation of the microrotation component ϕ_2 with variation of the angle under (G-N) theory

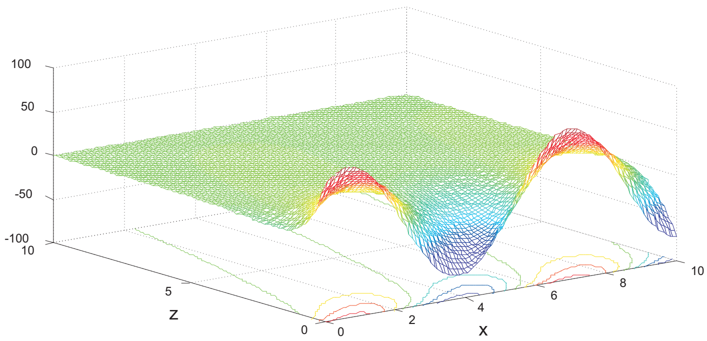

Figure 8 3D variation of the displacement u with the variation of x, z

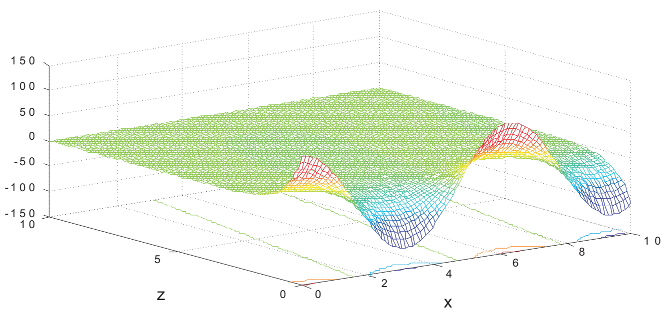

Figure 9 3D variation of the temperature T with the variation of x, z

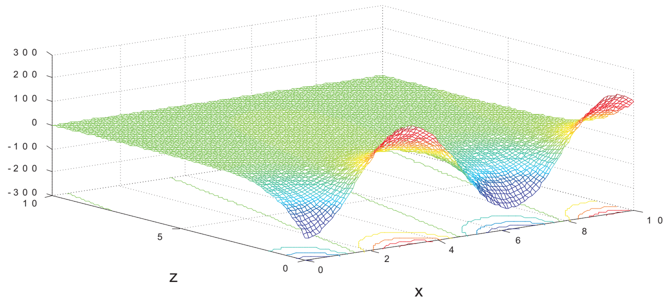

Figure 10 3D variation of the stress σ_{zz} with the variation of x, z

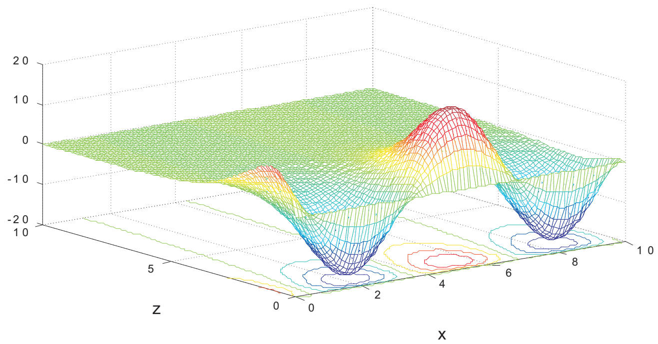

Figure 11 3D variation of the tangential couple stress m_{zy} with the variation of x, z

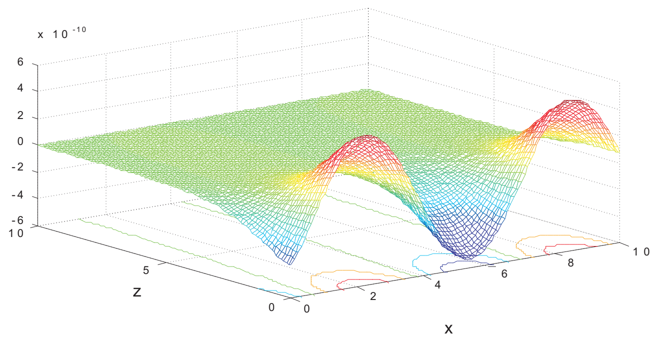

Figure 12 3D variation of the microrotation component ϕ_2 with the variation of x, z

6. Conclusion

According to the above results, we can conclude that:

1. The curves of the physical quantities with (G-N III) are most differing from the curves with (G-N II).

2. The used method in the present article is applicable to a wide range of problems in thermoelasticity.

3. The values of all the physical quantities converge to zero by increasing the distance z and all the functions are continuous.

4. The inclined load plays a significant role in the distribution of all the physical quantities.

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