Mechanics and Mechanical Engineering
Vol. 21, No. 4 (2017) 813–821

Cam–clay Models in Mechanics of Granular Materials

Ałła V. Ilyashenko
Moscow State University of Civil Engineering
26 Yaroslavscoe sh., Moscow, 117526, Russia

Sergey V. Kuznetsov
Institute for Problems in Mechanics
101 Prosp. Verndskogo, Moscow, 129526, Russia

Received (27 June 2017)
Revised (19 July 2017)
Accepted (23 August 2017)

The mathematical models for granular materials utilizing concept of the critical state, is reviewed. Several extensions of the critical state Modified Cam-Clay (MCC) models are reviewed, including kinematic hardening with bounding surface (BS), the general plasticity (GP) model, extension of the MCC model to include finite strain, and different variants of the pressure hardening rule, including bi-modulus extension, hypoplastic, and the hyperelastic potential extension. The associated flow rules coupled with different hardening equations are considered. In the review the main attention is paid to the case of the infinitesimal strains.

Keywords: critical state, cam-clay, hardening, isotropy, plasticity.

1. Introduction

The original Cam-Clay critical state model is mainly due to (Roscoe, Schofield, and Wroth, 1958; Roscoe, Schofield, 1963). Later on (Roscoe, Burland, 1968), this model was modified by substituting the logarithmic yield surface by the elliptic one, shown in Fig.1. Such a modification is known as the Modified Cam-Clay (MCC) model.

There is a large number of publications on applying both CC and MCC models to simulating static monotonic loading of granular materials with negligible cohesion (Borja, Lee, 1990; Bigoni, Hueckel, 1991; Alawaji et al., 1992; Borja, Tamagnini, 1998; Armero, Pérez-Foguet, 2002; Borja, Sama, Sanz, 2003; Dal Maso, De Simone, 2009; Dal Maso, Solombrino, 2010; Dal Maso, De Simone, Solombrino, 2011; Buscarnera, Dattola, di Prisco, 2011; Conti, Tamagnini, De Simone, 2013), along with simulations of cyclic loadings (Mroz, 1967; Sangrey, 1978; Takahashi, Hight, 1980; Selig, 1981; Carter, Booker, Wroth, 1982; Uzan, 1985; Al Tabbaa, Wood, 1989; Wood, 1990; Puppala, Mohammad, Allen, 1999; Zhou, Gong, 2001; Andersen, 2009; Liu, Xiao, 2010).

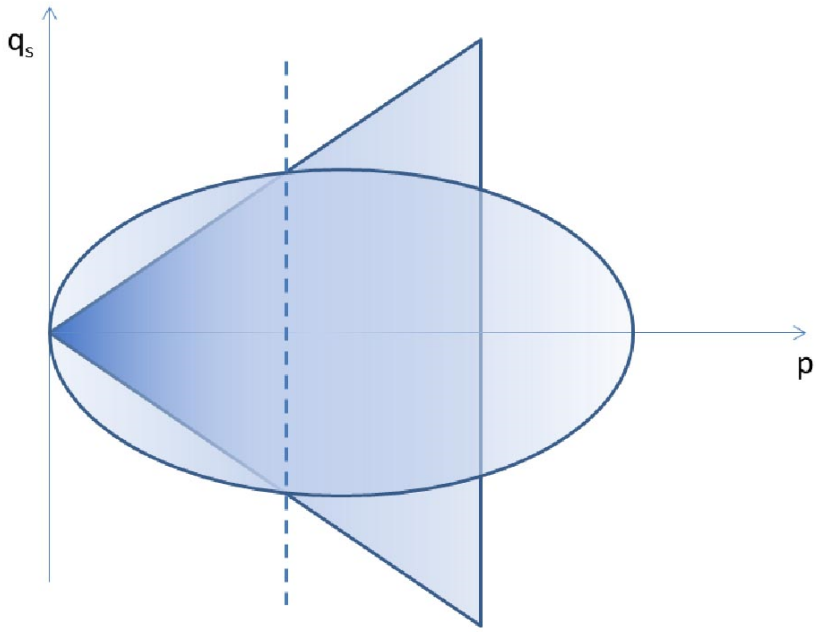

Figure 1 Yield and critical state surfaces for the MCC model: dashed line corresponds to intersection of the ellipsoidal yield surface with the critical state cone

Most of the latter works are concerned with either uniaxial (Sangrey, 1978; Takahashi, Hight, 1980) or triaxial loading conditions (Mroz, 1967; Carter, Booker, Wroth, 1982; Uzan, 1985; Al Tabbaa, Wood, 1989; Wood, 1990; Puppala, Mohammad, Allen, 1999; Zhou, Gong, 2001; Andersen, 2009; Liu, Xiao, 2010). Quite often (Carter, Booker, Wroth, 1982; Uzan, 1985; Al Tabbaa, Wood, 1989; Wood, 1990; Puppala, Mohammad, Allen, 1999), the triaxial loading is analyzed in the stress-state space only.

In the papers (Mroz, 1967; Al Tabbaa, Wood, 1989; Wood, 1990) the kinematic hardening is also considered to overcome difficulties related to degeneracy of the hysteresis loops at cyclic loadings with large volumetric component, and to include ability to model the Bauschinger effect. Some of the MCC models (Carter, Booker, Wroth, 1982; Zhou, Gong, 2001; Shahin, Loh, Nikraz, 2011; Ni et al., 2014) contain additional parameters to account cyclic loading frequency, damage degradation, etc.

The present work gives a review of the mathematical methods used in formulating the MCC model at infinitesimal deformations.

2. Principle equations

2.1. *Basic notations*

The following decomposition in volumetric and deviatoric parts of an arbitrary symmetric second-order tensor $\mathbf{A} \in sym(R^3 \otimes R^3)$ is needed:

$$\mathbf{A}_{vol} = \tfrac{1}{3}\mathbf{I} \otimes (\mathbf{I} \cdot \cdot \mathbf{A}) \qquad \mathbf{A}_{dev} = \mathbf{A} - \mathbf{A}_{vol} \tag{1}$$

where I is the unite tensor. The following tensorial invariants are needed for further analysis:

$$I_{\mathbf{A}} \equiv \mathbf{I} \cdot \cdot \mathbf{A} \qquad II_A \equiv \mathbf{A} \cdot \cdot \mathbf{A}, \quad III_{\mathbf{A}} \equiv \det(\mathbf{A}). \tag{2}$$

Combining (1), (2) yields:

$$II_{\mathbf{A}_{dev}} \equiv \mathbf{A} \cdot \cdot \mathbf{A} - \tfrac{1}{3} I_{\mathbf{A}}^2. \tag{3}$$

The decomposition (1) for stress $\boldsymbol{\sigma}$ and infinitesimal strain $\boldsymbol{\epsilon}$ tensors is usually written in a slightly modified form:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \mathbf{s}, \quad \boldsymbol{\epsilon} = -\frac{1}{3}\theta\mathbf{I} + \mathbf{e} \tag{4}$$

where:

$$p = -\tfrac{1}{3} I_{\boldsymbol{\sigma}} \qquad \mathbf{s} = \boldsymbol{\sigma}_{dev} \qquad \theta = -I_{\boldsymbol{\epsilon}}, \qquad \mathbf{e} = \boldsymbol{\epsilon}_{dev} \tag{5}$$

Remark 1. In theories of elasticity and plasticity the sign of volumetric strain θ usually coincides with the sign of $I_{\boldsymbol{\epsilon}}$. However, in the critical state theories like the considered MCC it is more convenient to use opposite sign, that is taken into account in (5).

According to (2), (3) and definitions (4), (5) the following notation for Schur's norms of the corresponding deviators is adopted:

$$q_{\mathbf{s}} = \sqrt{II_{\mathbf{s}}} \qquad q_{\mathbf{e}} = \sqrt{II_{\mathbf{e}}} \tag{6}$$

Remark 2. Along with deviatoric norms (6) the corresponding signed parameters can be introduced (Papuga, 2011):

$$q_{\mathbf{s}}^{\pm} = \mathrm{sign}(f(\boldsymbol{\sigma}))\sqrt{II_{\mathbf{s}}} \qquad q_{\mathbf{e}}^{\pm} = \mathrm{sign}(f(\boldsymbol{\epsilon}))\sqrt{II_{\mathbf{e}}} \tag{7}$$

where $f(g)$ is a function of the corresponding tensors. In many applications, including fatigue analyses $f(g)$ is chosen as the first invariant (Papuga, 2011) of the corresponding tensor.

Instead of norms (6), in theories of plasticity the following deviatoric norms, known as Tresca stress or strain, are also used:

$$t_{\mathbf{s}} = \sigma_1 - \sigma_3 \qquad t_{\boldsymbol{\epsilon}_{dev}} = \epsilon_1 - \epsilon_3, \tag{8}$$

where σ_k, ϵ_k are the principle components of the corresponding tensors. By analogy with (7) the signed Tresca parameters $t_{\mathbf{s}}^{\pm}$ and $t_{\mathbf{e}}^{\pm}$ can also be introduced.

2.2. *Elastic state*

Assuming that the strain tensor can be decomposed in elastic and plastic parts, yields:

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^p + \boldsymbol{\epsilon}^e \tag{9}$$

At the elastic stress-strain state the material behavior can be modeled by either:

(A) Linear elastic relation:

$$\boldsymbol{\sigma} = K_e \theta^e \mathbf{I} + 2\mu_e \mathbf{e}^e \tag{10}$$

where K_e and μ_e are elastic volumetric and shear moduli respectively.

(B) Hypoelastic relations associated with linear elastic response in semi-logarithmic coordinates (Roscoe, Burland, 1968; Bigoni, Hueckel, 1991) for volumetric strain - pressure:

$$\begin{cases} d\ln\frac{p-p_t}{p_0-p_t} = K_e d\theta^e \\ d\mathbf{s} = 2\mu_e d\mathbf{e}^e \end{cases} \tag{11}$$

where $p_t \leq 0$ is the elastic tensile pressure limit; p_0 is the initial pressure value at the reference configuration at which $\theta^e = 0$; it is assumed that $p_0 > p_t$, ensuring absence of the singularity in the left-hand side of Eq. (11).

(C) Hyperelastic potential (Borja, Tamagnini, 1998; Borja, Sama, Sanz, 2003):

$$W = p_0 k \exp\left(\frac{\theta^e}{k}\right)\left(1 + \frac{\alpha}{k} I I_{\mathbf{e}^e}\right) \tag{12}$$

where k and α are dimensionless constants associated with the volumetric and shear moduli respectively. Hyperelastic potential (12) leads to the following representation for the nonlinear stress-strain relations

$$\begin{cases} p = p_0 \exp\left(\frac{\theta^e}{k}\right)\left(1 + \frac{\alpha}{k} I I_{\mathbf{e}^e}\right) \\ \mathbf{s} = 2\alpha p_0 \exp\left(\frac{\theta^e}{k}\right)\mathbf{e}^e \end{cases} \tag{13}$$

The second equation in (13) reveal exponential dependence of the shear modulus $2\alpha p_0 \exp\left(\frac{\theta^e}{k}\right)$ upon θ^e.

Remark 3. a) The volumetric hypoelastic equation (11) can be rewritten in terms of increments $dp,\ d\theta^e$:

$$dp = K_e^*(p) d\theta^e \tag{14}$$

where $K_e^*(p) = (p - p_t)K_e$.

b) In hypoelastic relations (11) the shear modulus is usually taken as constant (Carter, Booker, Wroth, 1982) or derived from assumption of the constant Poisson's ratio (Borja, Lee, 1990). In the latter case:

$$\mu_e = \frac{3(1-2\nu)}{2(1+\nu)} K_e^*(p) \tag{15}$$

The case of constant Poisson's ration appears more realistic than the case of constant shear modulus, since the latter case can lead to negative Poisson's ratio values (Borja, Lee, 1990).

2.3. *Yield surface*

In the MCC theory the yield surface is defined by the following equation (Roscoe, Burland, 1968, Bigoni, Hueckel, 1991):

$$f(p, q_s, p_c) \equiv \frac{1}{\beta}\left(\frac{p}{a} - 1\right)^2 + \left(\frac{q_s}{Ma}\right)^2 - 1 = 0 \tag{16}$$

where β is a dimensionless parameter specifying the ellipsoid shape: in a subcritical zone $\beta = 1$ (left side), in a supercritical zone $\beta \leq 1$ (right side); the dimensionless parameter M, known as the critical cone tangent, specifies ellipsoid dimension along q_s-axis; a is the "central" point of the ellipsoid, this parameter defines ellipsoid dimension along p-axis:

$$a = \frac{p_c}{1 + \beta} \tag{17}$$

where p_c is the current yield pressure value, note, that at $\beta = 1$ parameter a takes value $p_c/2$. Actually, parameter p_c specifies evolution of the ellipsoidal surface (16).

2.4. *Volumetric hardening*

According to (Bigoni, Hueckel, 1991; Borja, Tamagnini, 1998) volumetric hardening law can be described in terms of either (D) continuation of linear elastic response (10), or (E) hypoelastic equations (11), or (F) hyperelastic potential (12). It should be noted that in the MCC model only volumetric hardening should be specified.

(D) Volumetric behavior expressed at active loading admits following representation in terms of incremental linear equation of state (10):

$$dp = K(p)d\theta \qquad K(p) = \begin{cases} K_e & 0 < p < p_{c0} \\ K_p & p > p_{c0} \end{cases} \tag{18}$$

where K_p is plastic hardening modulus ($K_p < K_e$) and p_{c0} is the initial yield pressure. The initial yield pressure p_{c0} corresponds to the yield pressure at the beginning of the active loading cycle; when active loading finishes, p_{c0} is set to either max(p), if max(p)>p_{c0}, or leaved unaltered in the opposite case. Integrating equation (18) yields:

$$p = \begin{cases} K_e\theta & 0 < p < p_{c0} \\ p_{c0} + K_p\theta_p & p > p_{c0} \end{cases} \tag{19}$$

where θ_p=$\theta - \theta_{c0}$ and $\theta_{c0} = p_{c0}/K_e$ is the initial (at the beginning of the active loading) ultimate elastic volumetric strain. At unloading $K(p)$ is taken as K_e.

(E) Continuation of the hypoelastic equation (11) to include plastic hardening is straightforward: the first equation in (11) can be rewritten in terms of two moduli similar to equation (18) (Schofield, 1968; Hashiguchi, 1995):

$$d\ln\frac{p - p_t}{p_0 - p_t} = \begin{cases} K_e d\theta_e, & 0 < p < p_{c0} \\ K_p d\theta_p, & p > p_{c0} \end{cases} \tag{20}$$

Remark 4. Similarly to equation (14), the second equation in (20) can be rewritten in terms of variable tangent plastic modulus:

$$dp = K_p^*(p) d\theta_p \tag{21}$$

where $K_p^*(p) = (p - p_t) K_p$.

(F) Continuation of the hyperelastic potential (12) to plastic hardening zone includes dependencies of parameters α and k upon pressure level $p > p_{c0}$ in plastic state:

$$W = p_0 k(p) \exp\left(\frac{\theta}{k}\right)\left(1 + \frac{\alpha(p)}{k(p)} II_{\mathbf{e}^e}\right) \tag{22}$$

The simplest form of dependencies for $\alpha(p)$, $k(p)$ relies on bilinear functions:

$$k(p) = \begin{cases} k_e & 0 < p < p_{c0} \\ k_p, & p > p_{c0} \end{cases} \qquad \alpha(p) = \begin{cases} \alpha_e & 0 < p < p_{c0} \\ \alpha_p, & p > p_{c0} \end{cases} \tag{23}$$

2.5. ***Flow rule***

Equation (16) should be supplemented by the equation of flow rule. The associated flow rule is adopted in the most of the MCC approaches (Roscoe, Schofield, and Wroth, 1958; Roscoe, Schofield, 1963; Roscoe, Burland, 1968; Borja, Lee, 1990). That means coincidence of the flow potential and an equation for the yield surface, defined by (16), at every points belonging to $f(p, q_{\mathbf{s}}, p_c)$:

$$d\boldsymbol{\epsilon}_p = d\gamma \nabla_{\boldsymbol{\sigma}} f \tag{24}$$

where $d\gamma$ is the plastic flow intensity parameter. That results in the following expressions for the increment of plastic deformations:

$$d\mathbf{e}_p \equiv d\gamma\, \nabla_{\mathbf{s}} f(p, q_{\mathbf{s}}, p_c) \qquad d\theta_p = 3 d\gamma\, \partial_p f(p, q_{\mathbf{s}}, p_c) \tag{25}$$

Performing differentiation in (25) yields:

$$d\mathbf{e}_p = d\gamma \frac{2\mathbf{s}}{(Ma)^2} \qquad d\theta_p = d\gamma\, \frac{6}{\beta a}\left(\frac{p}{a} - 1\right) \tag{26}$$

Taking into account relation (17), second equation in (26) takes the form:

$$d\theta_p = d\gamma\, \frac{6(1+\beta)}{\beta p_c}\left(\frac{p(1+\beta)}{p_c} - 1\right) \tag{27}$$

Assuming $p = p_c$ and $\mathbf{e} = 0$, equation (27) yields:

$$d\theta_p = d\gamma\, \frac{6(1+\beta)}{p_c} \tag{28}$$

At a point (p_c, 0) one of hardening equations (18) - (23) is adopted, allowing to rewrite equation (28) in terms of plastic increment dp_c:

$$dp_c = d\gamma\, \frac{6(1+\beta)}{p_c} g(p_c) \tag{29}$$

where function $g(p_c)$ is derived from the corresponding hardening equations. For example, for hardening equation $(19)_2$:

$$g(p_c) = K_p \tag{30}$$

Similarly, for hardening equation (21):

$$g(p_c) \equiv K_p^*(p_c) = (p_c - p_t)K_p. \tag{31}$$

Now, in view of (29) the increment $d\gamma$ can be found from the Prager consistency condition:

$$\nabla_\sigma f \cdot \cdot d\boldsymbol{\sigma} + \partial_{p_c} f \, dp_c = 0. \tag{32}$$

Substituting into equation (32) representation for dp_c from equation (29) yields the desired equation for increment: $d\gamma$:

$$d\gamma = -\frac{\nabla_{\boldsymbol{\sigma}} f \cdot \cdot d\boldsymbol{\sigma}}{\partial_{p_c} f} \times \frac{p_c}{6(1+\beta)g(p_c)}. \tag{33}$$

Equation (33) completes yield equations (26).

3. Concluding remarks

Several extensions of the MCC model are known. The kinematic hardening introduced within the MCC model in (Uzan, 1985; Al Tabbaa, Wood, 1989) is extended in (van Eekelen, van den Berg, 1994) by changing the ellipsoidal shape of both yield and bounding surfaces to the egg shape.

In (Aboim, Roth, 1982; Hirai, 1987) the extended plasticity model by Zienkiewicz and Mroz (1984) is applied to include plastic strain generated by an increment stress vector directed inside the elastic region that is confined by the yield surface.

Another extension of the MCC model relies on the concept of the Bounding Surface (BS) (Dafalias, Herrmann, 1980); the BS comprises all the admissible elastic-plastic states, and the hardening volumetric modulus is a function of a distance from the current state and BS. Further extensions of the BS concept known as the General Plasticity (GP) model, are proposed in (Auricchio, Taylor, Lubliner, 1992; Auricchio, Taylor, 1999). Both BS and GP models are introduced to ensure smooth transition from elastic to plastic states.

The case of finite deformation for the MCC model is considered in (Simo, Meschke, 1993; Borja, Tamagnini, 1998; Callari, Auricchio, Sacco, 1998). In (Simo, Meschke, 1993) the linear response $v - \ln p$ and assumption of the constant shear modulus is considered, while in (Borja, Tamagnini, 1998; Callari, Auricchio, Sacco, 1998) an assumption of the constant Poisson's ratio coupled with potential (22), and hence, variable shear modulus, is adopted.

References

[1] **Aboim, C., Roth, W.**: Bounding surface plasticity theory applied to cyclic loading of sand, in: *Int. Symp.Num. Models Geomech.*, 65–72, **1982**.

[2] **Al Tabbaa, A., Wood, D. M.**: An experimentally based bubble model for clay. in: *Proc. 3rd Int. Symp. Num. Models Geomech. (NUMOG III)*, 90–99, **1989**.

[3] **Alawaji, H., Runesson, K., Sture, S., Axelsson, K.**: Implicit integration in soil plasticity under mixed control for drained and undrained response, *Int. J. Numer. Anal. Methods Geomech.,* 13, 737–756, **1992**.

[4] **Andersen, K. H.**: Bearing capacity under cyclic loading - offshore, along the coast, and on land, *Can. Geotech J.,* 46, 513–535, **2009**.

[5] **Armero, F., Pérez-Foguet, A.**: On the formulation of closest-point projection algorithms in elastoplasticity – Part I: The variational structure, *Int. J. Numer. Methods Eng.,* 53, 297–329, **2002**.

[6] **Auricchio, F., Taylor R.**: A return-map algorithm for general associative isotropic elasto-plastic materials in large deformation regimes, *Int. J. Plasticity,* 15, 1359–1378, **1999**.

[7] **Auricchio, F., Taylor, R.L., Lubliner J.**: Application of a return map algorithm to plasticity models, in: Owen D.R.J. et al. (Eds.), *Computational Plasticity*, CIMNE, Barcelona, 2229–2248, **1992**.

[8] **Bigoni, D., Hueckel T.**: Uniqueness and localization associative and non-associative elasto–plasticity, *Int. J. Solids Struct.,* 28, 197–213, **1991**.

[9] **Borja, R.I., Lee S.R.**: Cam-Clay plasticity. Part I: Implicit integration of elasto-plastic constitutive relations, *Comput. Methods Appl. Mech. Eng.,* 78, 49–72, **1990**.

[10] **Borja, R., Sama, K., Sanz, P.**: On the numerical integration of three-invariant elastoplastic constitutive models, *Comput. Methods Appl. Mech. Eng.,* 192, 1227–1258, **2003**.

[11] **Borja, R., Tamagnini, C.**: Cam-Clay plasticity, Part III: Extension of the infinitesimal model to include finite strains, *Comput. Methods Appl. Mech. Eng.,* 155, 73–95, **1998**.

[12] **Buscarnera, G., Dattola, G., di Prisco, C.**: Controllability, uniqueness and existence of the incremental response: A mathematical criterion for elastoplastic constitutive laws, *Int. J. Solids Struct.,* 48, 1867–1878, **2011**.

[13] **Callari, C., Auricchio, F., Sacco, E.**: A finite-strain cam-clay model in the framework of multiplicative elasto-plasticity, *Int. J. Plasticity,* 14, 1155–1187, **1998**.

[14] **Carter, J.P., Booker, J.R., Wroth, C.P.**: *A critical state soil model for cyclic loading, in: Soil mechanics—Transient and cyclic loading*, *G. N. Pande, O. C. Zienkiewicz, eds.*, Wiley, Chichester, 219–252, **1982**.

[15] **Conti, R., Tamagnini, C., De Simone, A.**: Critical softening in Cam-Clay plasticity: Adaptive viscous regularization, dilated time and numerical integration across stress–strain jump discontinuities, *Comput. Methods Appl. Mech. Eng.,* 258, 118–133, **2013**.

[16] **Dafalias, Y. F., Herrmann, L. R.**: A bounding surface soil plasticity model, in: *Proc. Int. Symp. Soils Cyclic Trans. Load.*, 335–345, **1980**.

[17] **Dal Maso, G., De Simone, A.**: Quasistatic evolution for cam-clay plasticity: Examples of spatially homogeneous solutions, *Math. Models Methods Appl. Sci.,* 19, 1643–1711, **2009**.

[18] **Dal Maso, G., De Simone, A., Solombrino, F.**: Quasistatic evolution for cam-clay plasticity: A weak formulation via viscoplastic regularization and time rescaling, *Calc. Var.,* 40, 125–181, **2011**.

[19] **Dal Maso, G., Solombrino, F.**: Quasistatic evolution for Cam-Clay plasticity: The spatially homogeneous case, *Netw. Heter. Media.,* 5, 97–132, **2010**.

[20] **Hashiguchi, K.**: On the linear relations of V–ln p and ln v–ln p for isotropic consolidation of soils, *Int. J. Num. Anal. Methods Geomech.,* 19, 367-376, **1995**.

[21] **Hirai, H.**: An elastoplastic constitutive model for cyclic behavior of sands, *Int. J. Num. Anal. Methods Geomech.*, 11, 5, 503–520, **1987**.

[22] **Liu, J., Xiao, J.**: Experimental study on the stability of railroad silt subgrade with increasing train speed, *J. Geotech. Geoenviron. Eng., (ASCE)* GT.1943-5606.0000282, 10.1061, 833–841, **2010**.

[23] **Mroz, Z.**: On the description of anisotropic work hardening, *J. Mech. Phys. Solids.,* 15, 3, 163–175, **1967**.

[24] **Ni, J., Indraratna, B., Geng, X., Carter, J., Chen Y.**: Model of soft soils under cyclic loading. *Int. J. Geomech. Eng. (ASCE)* GM.1943-5622.0000411, 2014, 10.1061, 1–10, **2014**.

[25] **Papuga, J.**: A survey on evaluating the fatigue limit under multiaxial loading, *Int. J. Fatigue.* 33, 153–165, **2011**.

[26] **Puppala, A. J., Mohammad, L. N. Allen, A.**: Permanent deformation characterization of subgrade soils from RLT test, *J. Materials Civil Engng.,* 11, 274-282, **1999**.

[27] **Roscoe, K.H., Burland, J.B.**: On the generalized stress?strain behavior of wet clay, *Engineering Plasticity, eds. J. Heyman and F. A. Leckie. Cambridge Univ. Press*, 535-609, **1968**.

[28] **Roscoe, K. H., Schofield, A. N., Wroth C. P.**: On the yielding of soils, *Geotechnique*, 8, 22–53, **1958**.

[29] **Roscoe, K. H., Schofield, A. N.**: Mechanical behavior of an idealized \wet clay, In: *Proc. 2nd European Conf. Soil Mechanics and Foundation Engineering*, vol. I, 47–54, **1963**.

[30] **Sangrey, D. A.**: Cyclic loading of sands, silts and clays. Earthquake engineering and soil dynamics, *Proc. ASCE Geot. Engineering Div. Conf.*, 836-851, **1978**.

[31] **Schofield, A. N., Wroth, C.P.**: *Critical State Soil Mechanics*, McGraw-Hill, **1968.**

[32] **Selig, E. T.**: Soil failure modes in undrained cyclic loading, *J. Geot. Engineering Div.,* 107, 539–551, **1981**.

[33] **Shahin, M. A. Loh, R. B. H., Nikraz, H. R.**: Some observations on the behaviour of soft clay under undrained cyclic loading, *J. Geo Engineering, TGS,* 6, 109–112, **2011**.

[34] **Simo, J. C., Meschke, G.**: A new class of algorithms for classical plasticity extended to finite strains. Application to geomaterials, *Comput. Mech.,* 11, 253–278, **1993**.

[35] **Takahashi, M., Hight, D. W., Vaughan, P. R.**: Effective stress changes observed during undrained cyclic triaxial tests on clay, in: *Proc. Int. Symp. on Soils under Cyclic and Transient Loading*, Balkema, Rotterdam, 201–209, **1980**.

[36] **Uzan, J.**: Characterization of granular materials, *Transportation Research Record 1022, TRB. Washington DC: National Research Council*, 52–59, **1985**.

[37] **Van Eekelen, S.J., Van den Berg, P.**: The Delft egg model, a constitutive model for clay, *DIANA Computational Mechanics '94*, Springer, N.Y., 103–116, **1994**.

[38] **Wood, D.M.**: Soil behavior and critical state soil mechanics, *Cambridge University Press*, **1990**.

[39] **Zhou, J., Gong, X.**: Strain degradation of saturated clay under cyclic loading, *Can. Geotech. J.,* 38, 208–212, **2001**.

[40] **Zienkiewicz, O., Mroz, Z.**: Generalized plasticity formulation and applications to geomechanics, *Mech. Eng. Materials*, Wiley, Chichester, 655–679, **1984**.