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Comparative Effectiveness of the Mechanical Behaviour of Sandwich Beams under Uncoupled Bending and Torsion Loadings

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Sandwich geometries, mainly panels and beams, are widely used in several transportation industries, namely aerospace, aeronautic and automotive. Sandwich geometries are known for their advantages in structural applications: high specific stiffness, low weight, and possibility of design optimization prior to manufacturing. This study aims to know the influence of the number of reinforcements (ribs), and of the thickness on the mechanical behaviour of sandwich beams subjected to bending and torsion uncoupled loadings. In this study, four geometries are compared: simple web-core beam, corrugated core, honeycomb core, and joined honeycomb core. The last three are asymmetric, due to the use of odd number of ribs. The influence of the geometry on the results is discussed by means of a parameter that establishes a relation between the stiffness behaviour and the mass of the object. It is shown that all relations are non-linear, despite the elastic nature of the analysis in both the FEM software and in the practical application.

Keywords: sandwich beams, mechanical behavior, FEM – Finite Element Method, solid mechanics.

1. Introduction

The engineering structures used in this work are known as all-metal sandwich structures. All-metal sandwich panels have weight efficiency [1–3], multifunctional characteristics, and high impact resistance [2], [4–7]. Sandwich structures have been in use since the early 1940's, when their main applications were aeronautics and astronautics [8]. In the late 1960's, Allen presented the laws of mechanics of sandwich structures [9]. In the mid-80's, the sandwich structures began to be widely used in engineering components. For the sandwich beams mechanical behaviour characterization, three methods were used by Dai and Zhang [10] – the FE discretization method, the homogenization method, and the classical beam theory. The use of sandwich panels in the transportation industry leads to economical savings, because there is lower fuel consumption with less weight [11]. Valdevit el al. presented relevant design variables for optimization processes for steel sandwich panels with corrugated cores subjected to bending loads [12]. Silva and Meireles studied several sandwich geometries in order to determine its effective mechanical behaviour [13]. Silva and Meireles also studied the feasibility of incorporating sandwich panels in reinforced beams for industrial applications [14]. This study does not involve changing the material, as it was already demonstrated that the geometry has a higher potential than the material for the improvement of the mechanical behaviour [15]. There is the need of using light and stiff structures in some engineering applications, namely such involving mobile parts, such as laser cutting machines and plotters [16]. This need balances the need of studying of structural solutions which are adequate for these requirements, such as sandwich geometries. The present study aims to increase the knowledge about influence of the number of ribs and of the thickness on the effective mechanical behaviour of engineered beams, and to determine its best values for practical structural applications.

2. Numerical procedure

2.1. *FEM procedure*

FEM models having different number of ribs or different thickness were built on the commercial software *ANSYS Mechanical APDL*. The deflections were measured on two points, far enough from the point of load application, in order for the effect of the concentrated load to be not relevant. The considered geometries have equally distributed ribs along the beams width, as this is the best way to distribute deflections as evenly as possible on every zone of the object [17]. The same number of reinforcing segments and the same thickness value was studied in the four geometries, as shown in Tab. 1 together with values of parameters.

Table 1 Values of the studied parameters

Parameter 1	Parameter 2
Number of ribs	Thickness
0	0.0005
1	0.0010
3	0.0015
5	0.0020
7	0.0025
9	0.0030

The type of support is double cantilevered, by applying boundary conditions to the lines of the FEM models located at the ends. In Fig. 1 one of the FEM models with the DOF constraints is shown.

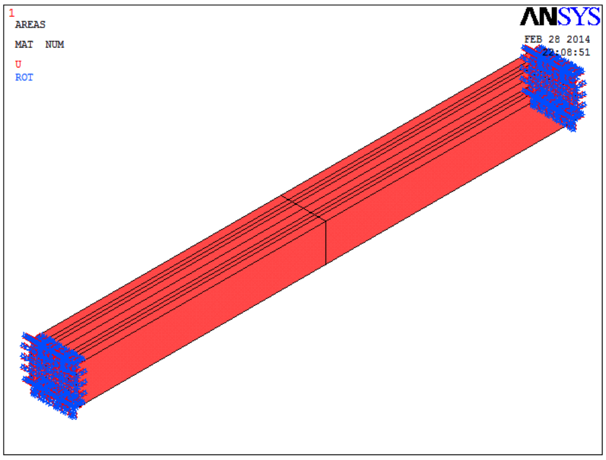


Figure 1 Boundary conditions applied to the FEM model

Figures 2 and 3 show the four geometries, all having 9 ribs. They are based on assumptions similar to those discussed by authors earlier in [13,18]. The deflections were measured on two points, far enough from the point of load application (Figs. 4 and 5) in order for the effect of the concentrated load to be insignificant. The thickness of the entire part was also studied. Figs. 2 and 3 show the section of the four studied beams: Fig. 2 shows the geometries numbered 1 and 2 in Table. 1, and the Fig. 3 shows the geometries 3 and 4.

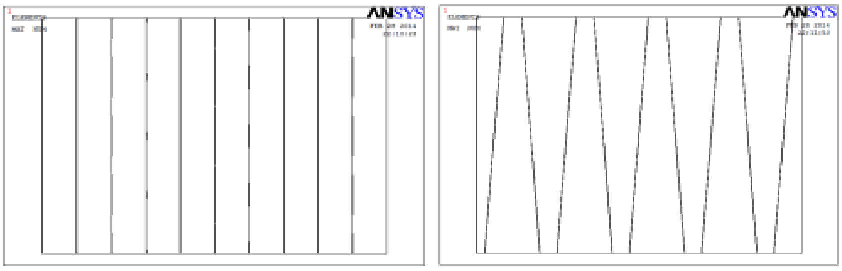


Figure 2 Section view of the sandwich beams with 9 ribs: geometry 1(left), geometry 2 (right)

The element type taken to modelling is *SHELL63*. The results were analysed on the nodes attached to two keypoints, located on the top face of the beam, on the midspan, and 0.0225 m from the edge, as shown in Fig. 5. The keypoints are therefore located at $\frac{1}{4}$ and $\frac{3}{4}$ of the total width. Two types of loadings were analysed separately: bending and torsion (Fig. 4). On bending, a load of 1500 N was applied and on torsion, two loads of 2000 N were applied. The general dimensions

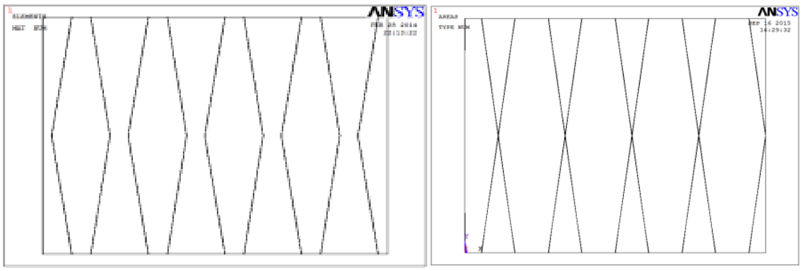


Figure 3 Section view of the sandwich beams with 9 ribs: geometry 3 (left), geometry 4 (right)

of the beams are: section width $b = 0.09$ m, section height $h = 0.075$ m, length $L = 1.0$ m and thickness $t = 0.002$ m, [13,18]. The material properties used in *ANSYS* were assumed as, Young modulus $2.1 \cdot 10^{11}$ Pa, Poisson coefficient 0.29 and density 7890 kg/m^3. As a simplification, the material is considered to be isotropic.

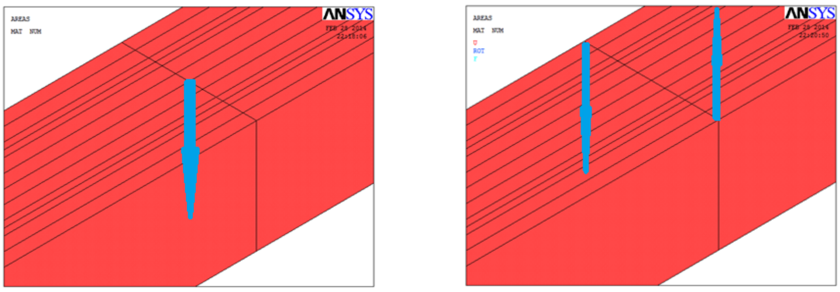


Figure 4 Bending (left) and torsion (right) loading applied to the FEM models

In Fig. 5 the points P1 and P2 were the results were queried are shown.

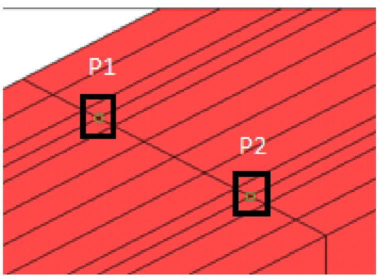


Figure 5 Keypoints where the results were collected

2.2. *Effective mechanical behaviour of sandwich beams*

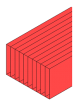
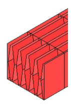
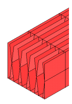
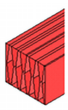
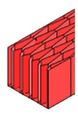
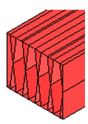
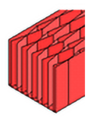
For the calculation of the effective deflection, the average absolute value of the displacement was multiplied by the mass:

$$\delta_{eff} = \frac{|\delta_1| + |\delta_2|}{2} m \tag{1}$$

where: δ_{eff} is the effective deflection, $\delta_{1,2}$ are displacements on the point P1 and P2 (left and right in Fig. 5, respectively), and m is the mass. Such a calculation allows to establish a parameter which is an indication of a better stiffness behaviour when its value is close to zero. This happens because the task is to obtain the lowest possible deflection and also the lowest possible mass. So, the multiplication of both quantities allows to obtain the relation between the increase in mass and the decrease in deflection when comparing the effectiveness between the several studied FEM models [13, 18].

2.3. *About the FEM results*

Fig. 6 shows all the different FEM models, all possessing 9 ribs. On the geometries marked as b, c and d, the top area is unselected to allow inner view. Two orientations were studied, both with and without transversal reinforcements. In the first orientation, the reinforcement section is oriented in the same direction as the section while in the second the reinforcements are rotated 90° with relation to the axis along the width (perpendicular with relation to the section). The b and d models have 13 transversal reinforcements in relation to the longitudinal axis, with 0.075 m distance between each other. The ribs located closest to the support on each side are located with 0.05 m of distance between them and the supports.

Designation	FEM model	Designation	FEM model
a1		b3	
a2		b4	
a3		c3	
a4		c4	

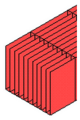
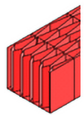
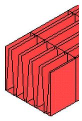
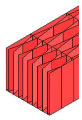
Designation	FEM model	Designation	FEM model
b1		d3	
b2		d4	

Figure 6 The FEM models discussed

Relations between the geometry designations and geometry modelled in the Finite Element software ANSYS are presented in Tabs. 2 and 3. They follow assumption taken by Authors in their earlier works [13, 18].

Table 2 Relation between the modelled geometries and their designations for the first orientation

Geometries	First orientation	
Transversal ribs	No	Yes
Geometry 1	a1	b1
Geometry 2	a2	b2
Geometry 3	a3	b3
Geometry 4	a4	b4

Table 3 Relation between the modelled geometries and their designations for the second orientation

Geometries	Second orientation	
Transversal ribs	No	Yes
Geometry 1	–	–
Geometry 2	–	–
Geometry 3	c3	d3
Geometry 4	c4	d4

3. Results

3.1. *Mesh sensitivity analysis*

In order to ensure accuracy of the results, a mesh sensitivity analysis was performed for all geometries separately under bending and torsion uncoupled loadings. It is obvious, the more complex a structure is, the more refined the mesh must be in order to obtain accurate results. Therefore, the geometries used were those with 9 ribs, which are the more complex. Thus, the mesh is ensured to be refined enough to produce accurate results on the other geometries. The results are considered accurate enough if the error for both nodal points is lower than 0.2%, with a mesh having average element length of 0.0025 m. Quadrilateral free mesh was used on all geometries. The refinement level is related to the mean element size according to values in Table 5:

Table 4 Relation between the refinement levels with the mean element size, from [13, 18]

Refinement level	Element size	
Re	first value	second value
1	0.0200	0.01000
2	0.0100	0.00500
3	0.0050	0.00250
4	0.0025	0.00125

The error was calculated using the second values in relation to the first ones, as:

$$\delta_{error} = \frac{|\delta_{Re+1}| - |\delta_{Re}|}{|\delta_{Re+1}|} * 100\% \,, \tag{2}$$

where Re is the refinement level.

Figure 7 present errors of the mesh sensitivity analysis for bending load, at points P1 and P2, respectively.

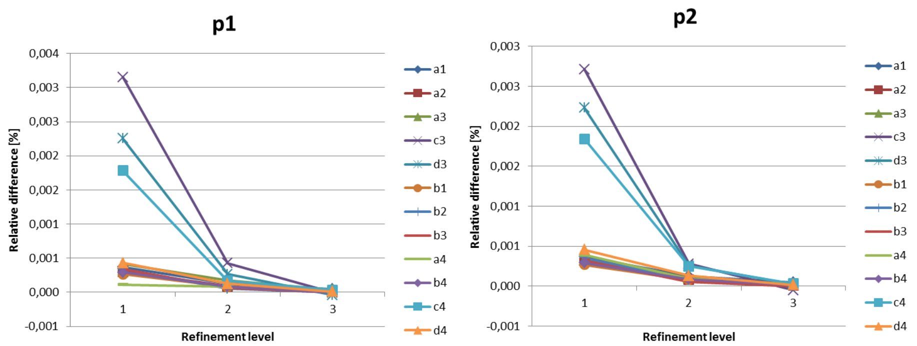


Figure 7 Mesh sensitivity analysis at point P1 (left) and P2 (right) for bending load

Figure 8 demonstrate errors of the mesh sensitivity analysis for torsion load, on points 1 and 2, respectively.

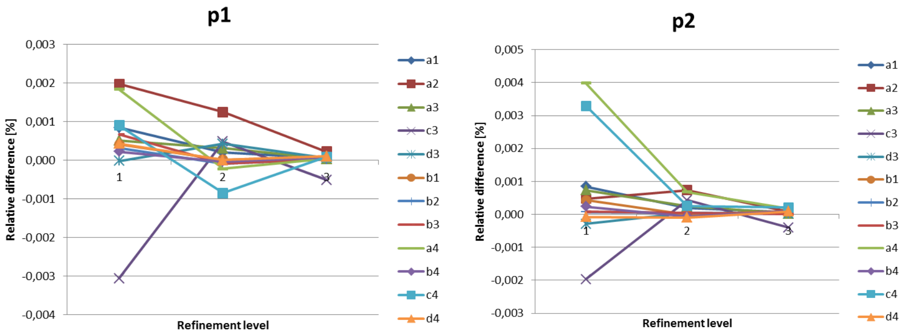


Figure 8 Mesh sensitivity analysis at point P1 (left) and P2 (right) for torsion load

Primary conclusion states that the geometries originate results with enough accuracy for mesh sizes of 0.0025 m. Therefore, this was the value for selected mesh size for all geometries.

3.2. *Global FEM results*

Figures 9 and 10 show the FEM calculation results for the model marked as a2 with 9 ribs, under uncoupled bending and torsion loads, respectively. An effect of the concentrated load is visible and happens locally in a small area of the top face. Due to the effect of the concentrated loads, both in the case of bending and of torsion, two points located far enough from the applied loads were chosen, as shown in Fig. 5. This allows to obtain more realistic results.

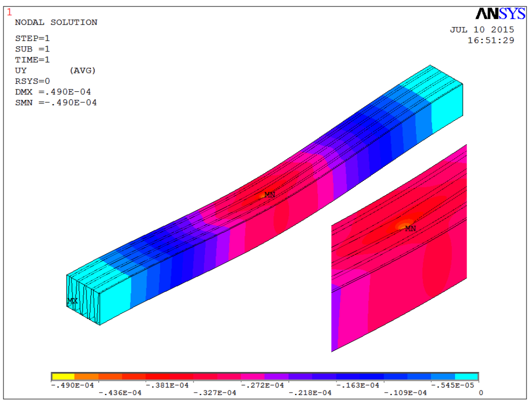


Figure 9 FEM results of the model a2 with 9 ribs under bending load

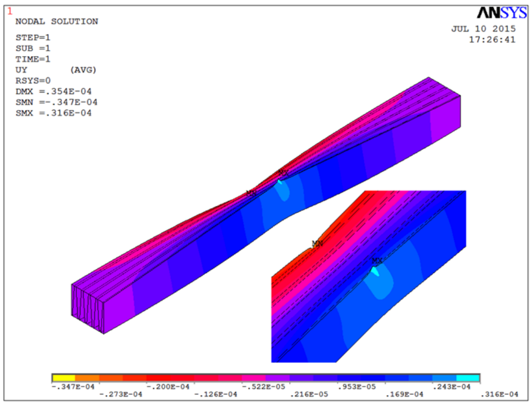


Figure 10 FEM results of the model a2 with 9 ribs under torsion load

3.3. *Local FEM results*

Results of performed calculations can be summarized in form of graphs in Figs. 11–18. Influence of the number of ribs and of the thickness on the deflections (Fig. 11 and 12 for bending and Fig. 13 and 14 for torsion) and on the deflections multiplied by the mass (Fig. 15 and 16 for bending and Fig. 17 and 18 for torsion).

3.3.1. *Bending*

Figure 11 shows influence of the number of ribs on the stiffness behaviour of the beams under bending loading. Figure 12 shows influence of thickness on the stiffness behaviour of the beams under bending loading.

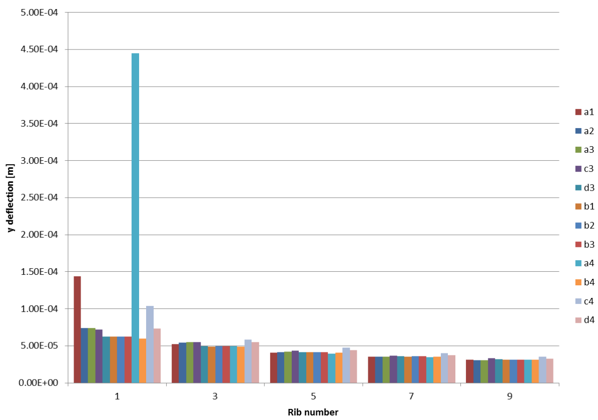


Figure 11 Influence of the number of ribs on the stiffness behaviour of the beams under bending loading: deflection vs. rib number

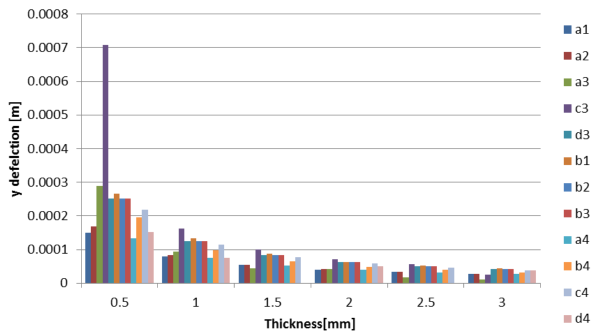


Figure 12 Influence of the thickness on the stiffness behaviour of the beams under bending loading: deflection vs. thickness [mm]

Figure 13 presents influence of the number of ribs on the effective mechanical behaviour of the beams under bending loading using expression (1). Figure 14 shows influence of thickness on the effective mechanical behaviour of the beams under bending loading.

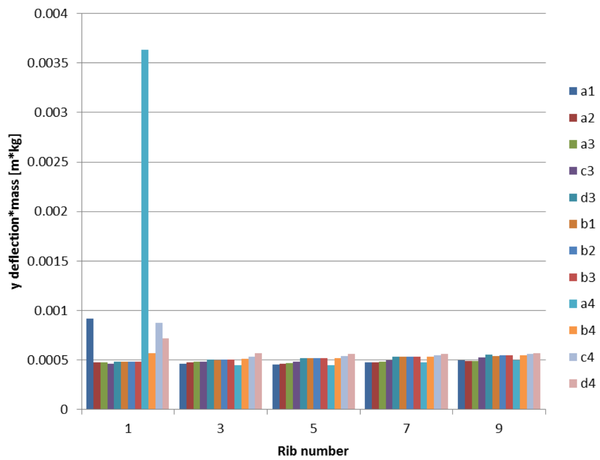


Figure 13 Influence of the number of ribs on the effective mechanical behaviour of the beams under bending load: deflection*mass vs. rib number

Conclusions taken from these results lead to the following statements. Under bending load, the geometry which behaves best is the a4 for rib number from 3 to 9; However, it is worst for the rib number 1. The results are close for all geometries from rib numbers 3–9, with sample d4 being the worst. When studying the thickness, the best model is a4 for thickness values of 0.5, 1 and 2 mm. In other cases,

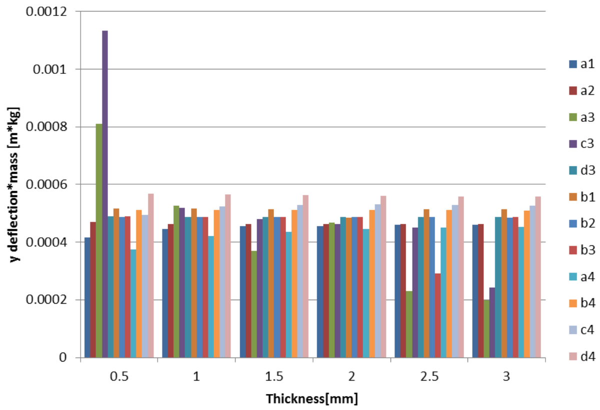


Figure 14 Influence of the number of ribs on the effective mechanical behaviour of the beams under bending load: deflection*mass vs. thickness [mm]

the geometry a3 is the best. The worst sample is also d4, except for a thickness of 0.5 mm.

3.3.2. *Torsion*

In case of torsion loading the obtained results are summarized in Figures 15–18. Fig. 15 shows the influence of the number of ribs on the stiffness behaviour of the beams under torsion loading. Fig. 16 shows the influence of the thickness on the stiffness behaviour of the beams under torsion loading.

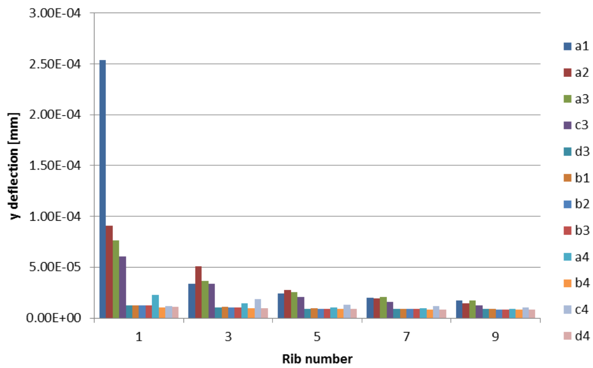


Figure 15 Influence of the number of ribs on the stiffness behaviour of the beams under torsion loading: deflection vs. rib number

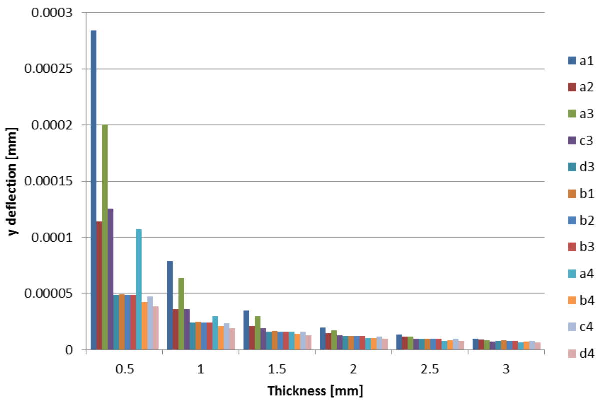


Figure 16 Influence of the thickness on the stiffness behaviour of the beams under torsion loading: deflection vs. thickness [mm]

Figures 17 and 18 show the influence of the number of ribs and thickness on the effective mechanical behaviour of the beams under torsion loads using (1), respectively. The effective mechanical behaviour worsens for all geometries with increasing rib number, except for the geometries a1, a2, a3 and c3, under torsion loading. For rib number equal to 1, the worst geometry is a1.

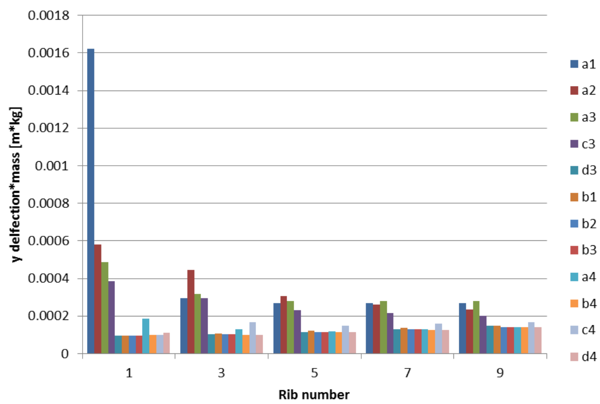


Figure 17 Influence of the number of ribs on the effective mechanical behaviour of the beams under torsion load: deflection*mass [m*kg] vs. rib number

When varying the thickness, the effective mechanical behaviour does not vary significantly, except for the sample a4, for which the y deflection multiplied by the mass has an inverse proportionality relation with the thickness.

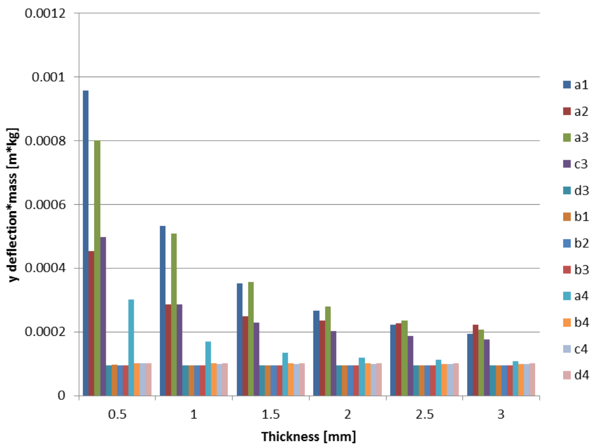


Figure 18 Influence of the number of ribs on the effective mechanical behaviour of the beams under torsion load: deflection*mass vs. thickness [mm]

4. Results discussion

4.1. *Bending*

There is a small variation for number of ribs equal to or higher than 3 in the study of this value in bending. This can be explained with the fact that solid sections are less effective in terms of deflection multiplied by the mass than hollow-sections, according to the presented criterion. When adding a number of ribs equal to or higher than 3, there is a very good improvement of the moment of inertia with a small increment of mass involved. This leads to an improvement in the effective mechanical behaviour. However, when adding more reinforcements, the effectiveness is much lower because the behaviour approaches closer a solid section one. When studying the thickness in bending, the variation is small, with only a few exceptions. This is due to the fact that when increasing the thickness, lowering of the deflections is compensated by the increase in mass in a similar amount. This happens because the reinforcements of the parts are kept the same, and therefore, the variation in thickness originates linear variations in the mass and deflections. The presented studies are only valid considering that the global stiffness of the part is high enough for the linear elastic bending laws to be valid.

4.2. *Torsion*

Under torsion loading, the increase in the number of reinforcements is ineffective for values of 1 or more. This behaviour happens because the reinforcements are oriented along the longitudinal direction of the beam, therefore, adding reinforcements will not have a great influence on the relevant inertia moment. This results in an improvement in the mechanical behaviour, but with an increase in the mass in a similar amount as lowering in the deflections. However, when adding one rib, the geometry gets stiffer, because the rib is connecting the upper and lower faces of the beam, increasing the rotational stiffness under torsion loading. On the study of thickness under torsion loading, there is a very small variation in the effective mechanical behaviour, for the same reasons already presented in bending: the increase in thickness originates a linear decrease in deflections and a linear increase in mass with close absolute values. The exception is the specimen a4. Due to the fact that, in this geometry, the reinforcements have contact with neighbours, the angle is sharper, therefore increasing the mean distance between the reinforcements. This causes the mechanical behaviour to be worse for lower value of thickness. However, when increasing the thickness, the difference in the distances between the geometry a4 and the other geometries is lower, and, therefore, its behaviour gets closer.

5. Conclusions

5.1. *Bending*

For the study of deflection dependence on number of ribs, there is a strict decrease in the deflection with the adding of reinforcements, as expected. The best models considering low rib numbers are those marked as b1, b2 and b3. For higher number of ribs, the best models seem to be the same but without transversal ribs: a1, a2 and a3. However, the a4 and b4 models also give interesting results, except a4 with one rib only.

For the study of deflection: thickness, there is a strict decrease in the deflection with increasing thickness, as expected. Overall, the models a1, a2, and a3 are the ones which behave best. There is a pattern with some changes in results when comparing deflection vs. rib number for different models and several rib numbers.

In case of effective mechanical behaviour vs. number of ribs, there is a slight increase in the deflection multiplied by mass for most models. The best behaviour exists for a1, a2, a3 and a4, except a1 with a single rib only.

Effective mechanical behaviour in function of thickness, the best results occur for the model a3 with 3 mm thickness. For 0.5, 1 and 2 mm, the best models are a1 and a4. For 1.5 mm, the best models are a3 and a4. For $t = 2.5$ mm, the model a3 and b3 behave the best, and for $t = 3.0$ mm, the model a3 and c3 are the best.

5.2. *Torsion*

Study of deflection vs. rib number under torsion loading suggests the best models are b4 and d4, although the models c4, d3, b1, b2, b3 also present interesting results.

When studying deflection while thickness varies, it is possible to see that there is a strict decrease in the deflection with increasing thickness, such as in bending loads as expected. The models show less differences in results when compared with each other when the thickness is higher. The best models are b4 and d4, but the models d3, b1, b2 and b3 also show good results, although not as good as b4 and d4.

In the study of deflection multiplied by mass vs. rib number, the best models are: d3, b1, b2, b3. The models b4 and d4 also show good results. The model d4 is good, especially for high values of thickness. On the chart of deflection multiplied by mass vs. thickness, the worst models are clearly a1, a2, a3 and c1 and the best are: d3, b1, b2 and b3. Those marked as b4, c4 and d4 are also worth considering in the modelling process.

References

[1] **Ashby, M.F., Evans, A.G., Fleck, N.A, Gibson, L.J, Hutchinson, J.W and Wadley, H.N.G**.: Metal foams: A design guide, Butterworth Heinemann, Boston, **2000**.

[2] **Evans, A.G., Hutchinson, J.W., Fleck, N.A., Ashby, M.F. and Wadley, H.N.G**.: The topological design of multifunctional cellular metals, *Progress in Materials Science*, 46, 309–327, **2001**.

[3] **Wadley, H.N.G., Fleck, N.A. and Evans, A.G**.: Fabrication and structural performance of periodic cellular metal sandwich structures, *Composites Science and Technology*, 6316, 2331–234, **2003**.

[4] **Hutchinson, R.G. Wicks, N., Evans, A.G., Fleck, N.A. and Hutchinson, J.W**.: Kargone plate structures for actuation, *International Journal of Solids and structures,* 4025, 6969–6980, **2003**.

[5] **Qiu, X., Deshpande, V.S. and Fleck, N.A**.: Finite element analysis of the dynamic response of clamped sandwich beams subject to shock loading, *European Journal of Mechanics A- Solids,* 226, 801–814, **2003**.

[6] **Xue, Z.T., Hutchinson, J.W**: Preliminary assessment of sandwich plated subject to blast loads, *International Journal of Mechanical Sciences*, 454, 687–705, **2003**.

[7] **Fleck, N.A., Deshpande, V.S**.: The resistance of clamped sandwich beams to shock loading, *Journal of Applied Mechanics,* 713, 386–401, **2004**.

[8] **Williams, D., Leggett, D. and Hopkins, H**.: Flat sandwich panels under compressive end loads, *Technical Report, R.A.E.*, **1987**.

[9] **Allen, H**.: Analysis and Design of Structural Sandwich Panels, Robert Maxwell, MC, MP, **1969**.

[10] **Dai, G.M., Zhang, W.H**.: Size effects of basic cell in static analysis of sandwich beams, *International Journal of Solids and Structures*, 459, 2512–253, **2008**.

[11] **Léotoing, L., Drapier, S. and Vautrin A.**: Nonlinear interaction of geometrical and material properties in sandwich beam instabilities, *International Journal of Solids and Structures,* 39, 3717–3739, **2002**.

[12] **Valdevit, L.,Wei, Z.,Mercer, C., Zok, F.W. and Evans, A.G**.: Structural performance of near-optimal sandwich panels with corrugated core, *International Journal of Solids and Structures,* 43, 4888–4905, **2006**.

[13] **Silva, H. M., De Meireles, J. F**.: Effective Mechanical Behavior of Sandwich Beams under Uncoupled Bending and Torsion Loadings, *Applied Mechanics and Materials*, 590, 58-62, **2014**.

[14] **Silva, H. M., De Meireles, J. F**.: Feasibility of Internally Reinforced Thin-Walled Beams for Industrial Applications, *Applied Mechanics and Materials*, 775, 119–124, **2015**.

[15] **Silva, H.M**.: Determination of material/geometry of the section most adequate for a static loaded beam subjected to a combination of bending and torsion, MSc. Thesis, University of Minho, **2011**.

[16] **Silva, H.M., De Meireles, J.F**.: Determination of material/geometry of the section most adequate for a static loaded beam subjected to a combination of bending and torsion, *Materials Science Forum,* Vols. 730-732, 507–512, **2013**.

[17] **Liu, Y**.: Design enhancement of thin-walled steel beams with improved stiffness and reduced weight*, International Journal of Design Engineering*, 1(2), **2008**, 149–165.

[18] **Silva, H. M., De Meireles, J. F**.: Effective Stiffness Behavior of Sandwich Beams under Uncoupled Bending and Torsion Loadings, 852, 469–475, **2016**.