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Buckling of Rectangular Plates with Different Central Holes

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The behavior of thin rectangular perforated plates under the action of uniform compressive deformation is studied using finite element analysis. The central holes are either circular holes or square holes. The effects of plate-support conditions, plate aspect ratio, hole geometry, and hole size on the buckling strengths of the perforated plates was studied. The results show that for the same plate weight density, the buckling strengths of the plates with square holes generally surpass those of the plates with circular holes over the range of hole sizes.

Keywords: buckling, Finite-Element analysis, rectangular plates, holes, Abaqus, elastic properties.

1. Introduction

Problems of initial and postbuckling represent a particular class of bifurcation phenomena; the long history of buckling theory for structures begins with the studies by Euler [1] in 1744 of the stability of flexible compressed beams. Although von Karman formulated the equations for buckling of thin, linearly elastic plates which bear his name in 1910 [2], a general theory for the postbuckling of elastic structures was not put forth until Koiter wrote his thesis [3] in 1945. General theories of bifurcation and stability originated in the mathematical studies of Poincaré [4], Lyapunov [5], and Schmidt [6] and employed, as basic mathematical tools, the inverse and implicit function theorems, which can be used to provide a rigorous justification of the asymptotic and perturbation type expansions which dominate studies of buckling and postbuckling of structures. Accounts of the modern mathematical approach to bifurcation theory, including buckling and postbuckling theory, may be found in many recent texts, most notably those of Keller and Antman [7], Sattinger [8], Iooss and Joseph [9].

Buckling of square plates with central circular holes subjected to compression in the plane was studied theoretically and experimentally by several authors [10-14]. Methods of theoretical analysis used by most researchers [10-13] are the minimum energy method Rayleigh-Ritz and method of Timoshenko [15]. Thus, most previous solutions buckling are limited to small hole sizes, and are not able to study the effects of different boundary conditions of the plate on the strengths of buckling plates with holes of arbitrary size.

With the availability of more powerful tools, such as computer programs for structural finite element analysis, it is now possible to calculate the stresses of post-buckling for rectangular plates with any aspect ratio, any shapes and sizes of cuts, and in different boundary conditions.

This work is based on the numerical evaluation of the critical buckling load of a perforated rectangular plate by manipulating the size of the plate and the size of the hole. The study is done using the computer code Abaqus 6.10.

2. Description of the problem

The geometry of the perforated rectangular plates and different boundary conditions used in the finite-element analysis are described as follows.

2.1. *Geometry*

We can see in Fig. 1 that we have two types of plate, one with square hole with side a, and the other with circular one with diameter d.

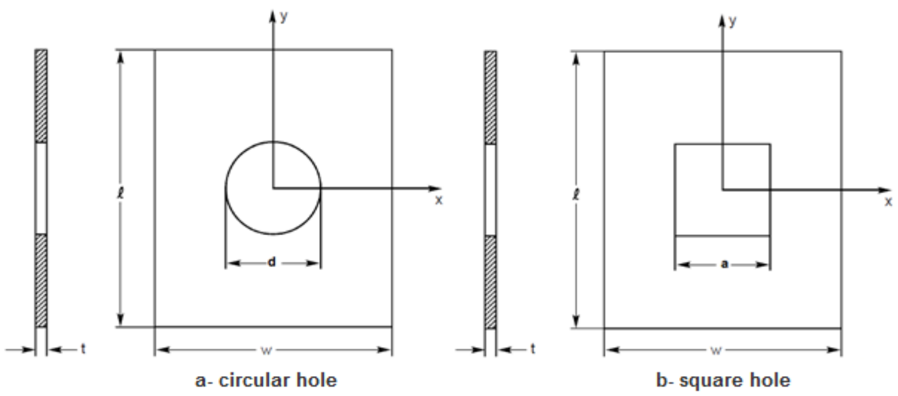

Figure 1 Rectangular plates with central cuts

All the plates have the same thickness (t = 0.1 cm), same width (w = 20 cm), and we manipulate the other dimensions given on Tab. 1.
Where a is the side of square hole in cm, d Diameter of the circular hole cm, t is the thickness of plate in cm, L the length of plate in cm and w is the width of the plate, cm.

Table 1 Dimensions of the performed rectangular plates

w	t	l/w	d/w	a/w
20	0.1	1.0	0∼0.7	0∼0.7
20	0.1	1.5	0∼0.7	0∼0.7
20	0.1	2.0	0∼0.7	0∼0.7

In this table, 0 ∼ 0.7 design 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6 and 0.7.

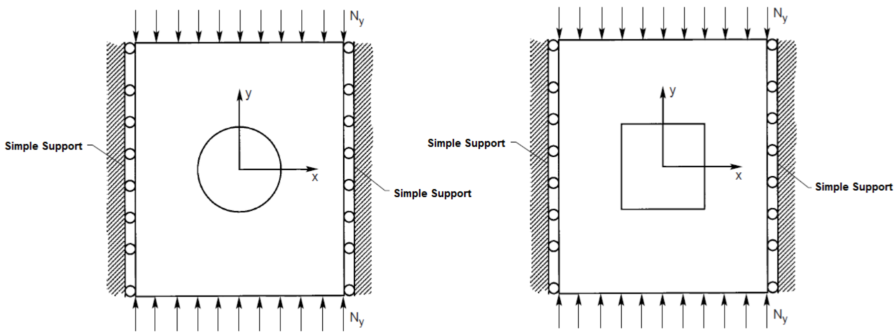

Figure 2 Simply supported plates

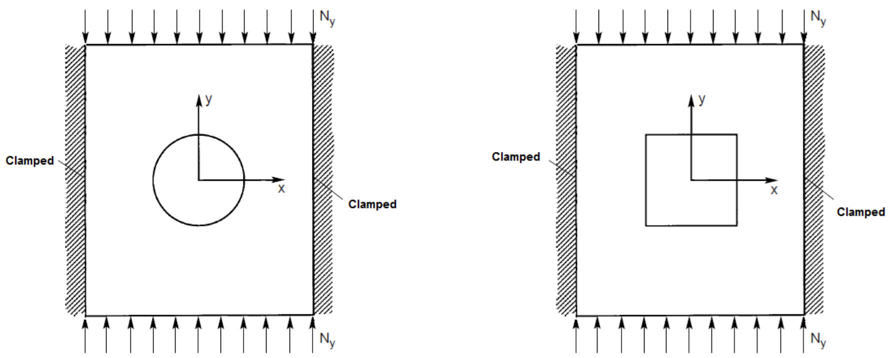

Figure 3 Clamped plates

2.2. *Boundary conditions*

In this case, we let two edges of the plate free and the two other are either simply supported (Fig. 2) or clamped (Fig. 3).
The plates are subjected to uniform compression along the widths.

2.3. *Finite element analysis*

The modelling is done, using the Finite Element code Abaqus [16]. The modeling is performed following these steps:
Geometry: realization of the model.
Choice of material: Cast Iron has been considered, his properties are shown in Tab. 2.
Creation of Step: in this level, we use linear perturbation then buckle, and we introduce the buckling mode.
Mesh: The simulation results are related to the mesh of the plate, thus we have chosen a special mesh very refined at the hole (Figs. 4 and 5)
Finally, analysis and results.

Table 2 NF A32-702 properties at room temperature

	E (GPa)	G (GPa)	ν	α (10^{-6}K^{-1})
NF A32-702	170	68	0.26	12

where E is the Young's modulus, G is the Shear modulus, ν is the Poisson's ratio and α is the Thermal expansion coefficient.

3. Results

3.1. *Plates with circular holes*

Figs. 6 to 8 show the buckling loads plotted as functions of hole size d/w for rectangular plates with circular holes.

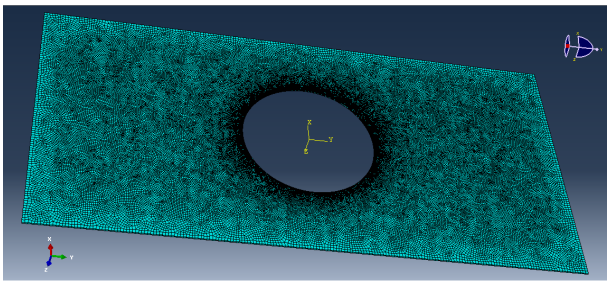

Figure 4 Plate with circular hole

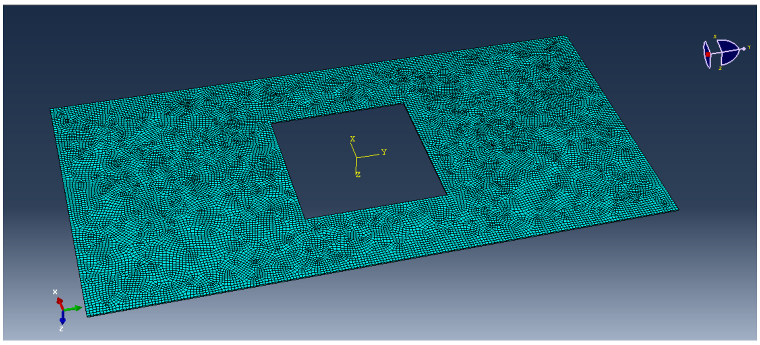

Figure 5 Plate with square hole

For the simply supported plates (SS), the critical load decrease when the hole size (d/w) increase. But in the other case, when the plates are clamped, the critical load decrease first to reach its lowest value at about d/w = *0.25*, then when the hole size increase the critical charge increase too.

We can add that the dimensions of the plates affect the critical load: when l/w increases the critical force increase.

3.2. *Plates with square holes*

Figs. 9 to 11 show the buckling loads plotted as functions of hole size d/w for rectangular plates with square holes.

The behavior of these plates is similar to that of the plates with circular holes.

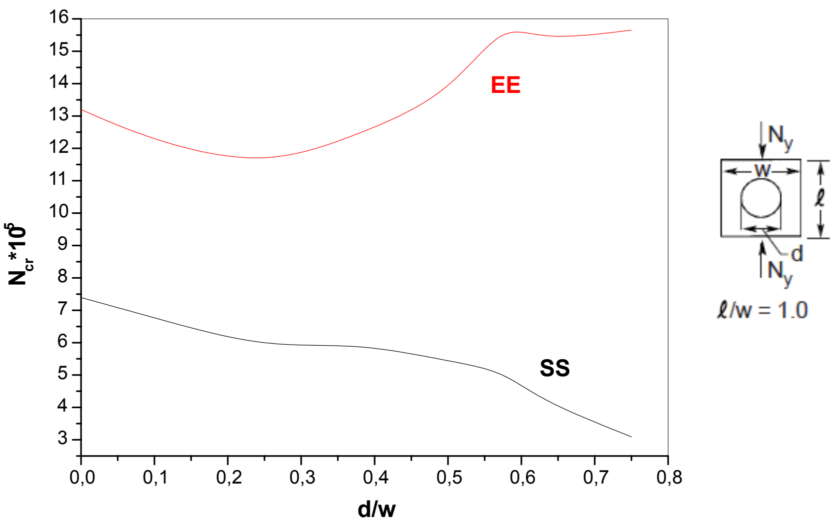

Figure 6 Critical buckling load as functions of hole size; circular holes with l/w=1

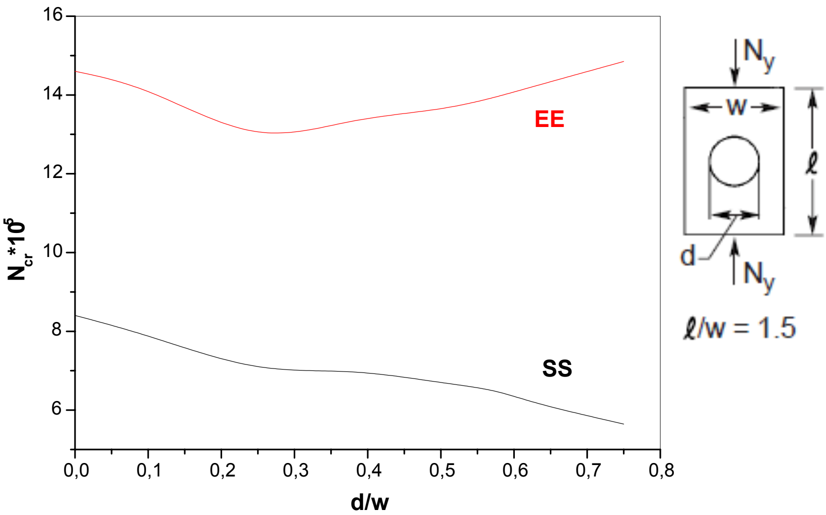

Figure 7 Critical buckling load as functions of hole size; circular holes with l/w=1.5

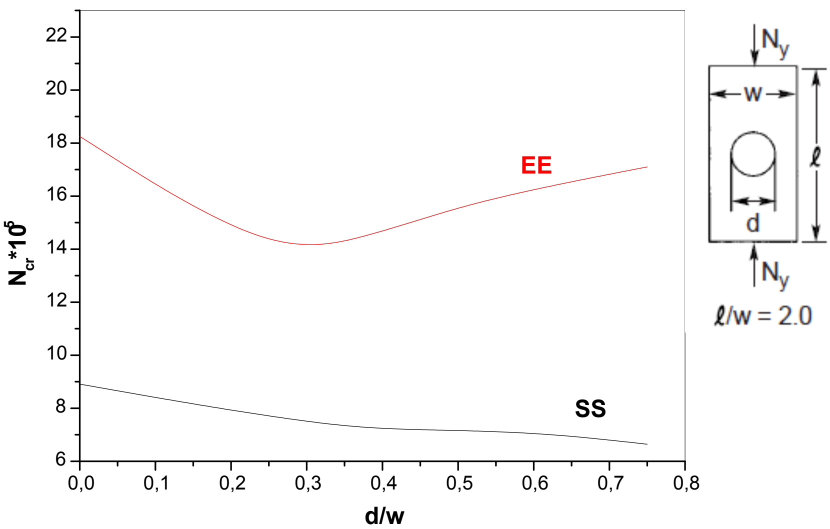

Figure 8 Critical buckling load as functions of hole size; circular holes with l/w=2

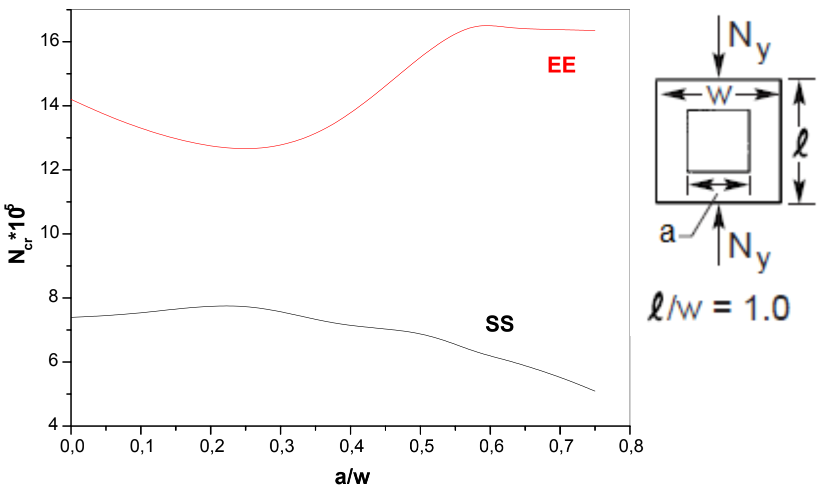

Figure 9 Critical buckling load as functions of hole size; square holes with l/w=1.0

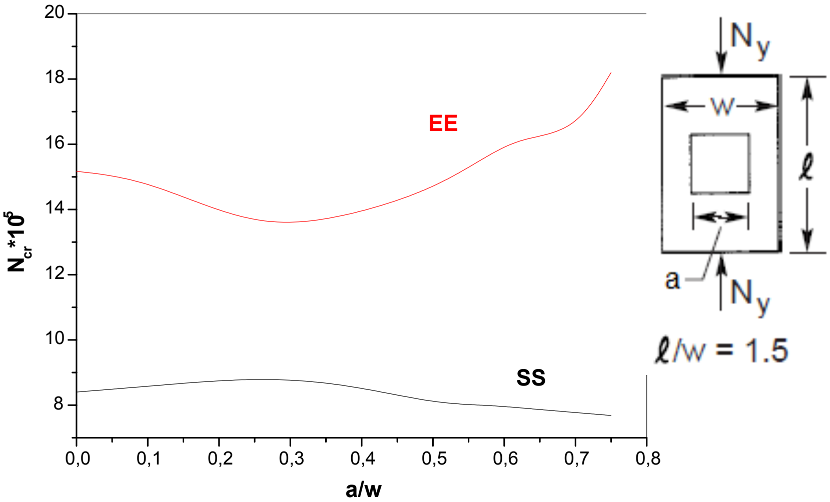

Figure 10 Critical buckling load as functions of hole size; square holes with l/w=1.5

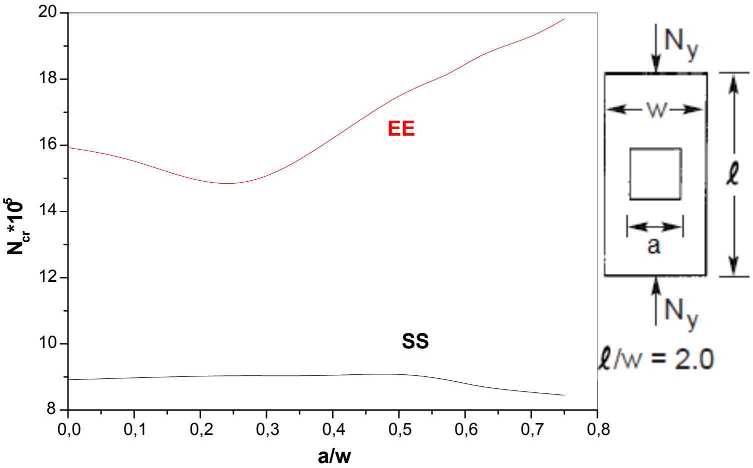

Figure 11 Critical buckling load as functions of hole size; square holes with l/w=2

3.3. *Comparison between the force of bucking*

In this section, we present the comparison of buckling charges of rectangular plates with different geometrical cutouts under the same weight density conditions: the area of the square hole was equal to that of the circular hole by adjusting the side a of the square hole according to the relationship (1):

$$a = \frac{d}{2}\sqrt{\pi} \qquad (1)$$

Figs. 12 to 14 compare the buckling loads of the two types of perforated plates (simply supported–free and clamped-free).We can see that the square hole cases exhibit higher buckling strengths than the respective circular hole case.

4. Discussion

The mechanical analysis of buckling using the finite element was performed on plates containing central circular and square holes. The effects of aspect ratio of the plate, the geometry of the hole, the hole size and plate support conditions on the critical buckling loads were studied. We can see that this comportment is particular, cause when the hole size increases the critical load increases too. We except that as hole size augments, the plate lose more material and become weak then the critical charge decreases but it wasn't the case. We can explain this as follows:

When the hole size becomes considerably large, most of the compressive load is carried by the lateral narrow strips of material along the plate boundary. It's clear that, a stronger plate boundary condition increases the buckling resistance, while the higher stress concentration reduces the buckling resistance. Thus, the effects become dominant which will determine the increase or decrease in buckling strength of the perforated plates.

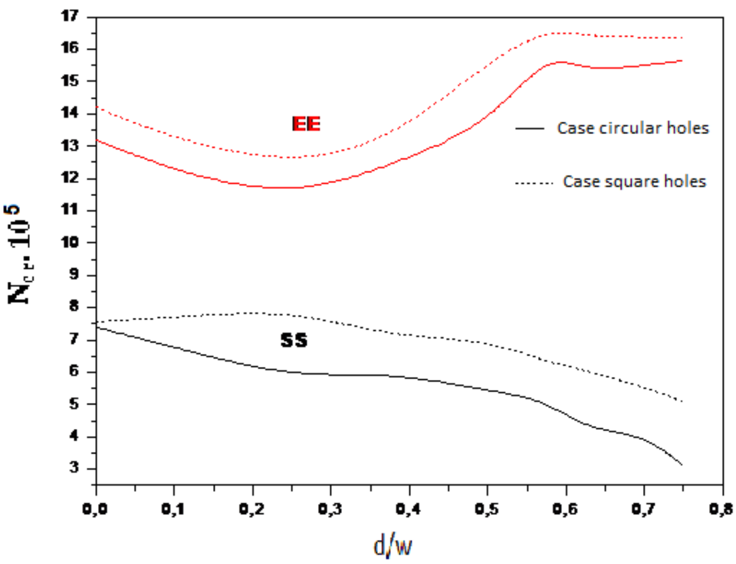

Figure 12 Comparison of the critical load plates with circular holes and those plates with square holes, for l/w=1

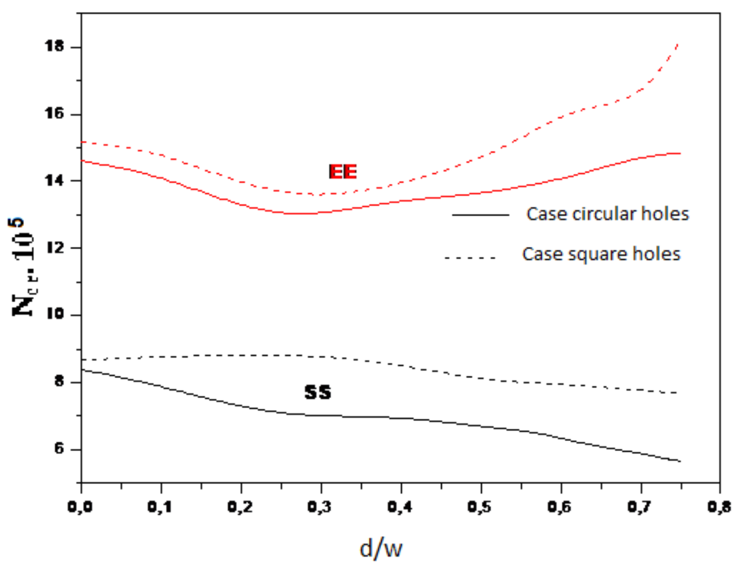

Figure 13 Comparison of the critical load plates with circular holes and those plates with square holes, for l/w=1.5

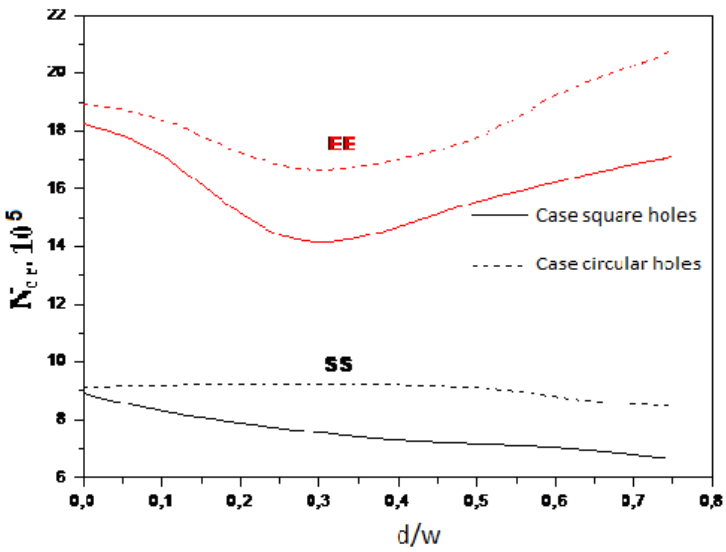

Figure 14 Comparison of the critical load plates with circular holes and those plates with square holes, for l/w=2

For the square-hole cases, the load-carrying narrow side strips along the plate boundaries are practically under uniform compressive stress fields. For the circular-hole cases, the narrow compressed side strips are under stress concentration, which reduces the buckling strengths. That's why the buckling strengths of the plates with square holes is greater than the buckling strengths of the plates with circular holes having the same weight density.

5. Conclusion

The principals' conclusions of the analysis are:
The increase in the size of the hole does not necessarily lead to the reduction of the critical buckling load of a perforated rectangular plate. For some aspect ratio and boundary conditions, this charge increases as the size of the hole increases.
For most cases and in the same density, the buckling resistance of the plates with square holes is higher than that of the plates with circular holes.
The clamped plates have a higher resistance to buckling than the simply supported plates.

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