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Numerical Study of Natural Convection in a Vertical Cylindrical Partially Annular

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A study of free convection in a vertical cylinder partially annulus is conducted numerically. Uniform temperature is imposed cross a vertical wall, while the top and bottom walls are adiabatic. The governing equations are solved numerically by using a finite volume method. The coupling between the continuity and momentum equations is effected using the SIMPLER algorithm. Solutions have been obtained for Prandtl numbers equal to 7.0, Rayleigh numbers of 10^3 to 10^6 and height ratios of 0 to 1. The influence of physical and geometrical parameters on the streamlines, isotherms, average Nusselt has been numerically investigated.

Keywords: natural convection, Finite Volume Method, partially annular, Nusselt number.

1. Introduction

The problem of natural convection of fluids confined in annulus cylinder has been the subject of many studies. This is due to the important role it plays in many technical and engineering applications such as, solar energy collectors, nuclear engineering, cooling of electrical and electronic components, and so on. Many researchers focused their investigation on the heat transfer and fluid flow behavior in a vertical annulus cylinder. One of the first, well documented studies the natural convection in vertical annular enclosures was presented by Vahl Davis et al. [1] numerically studied the natural convection in annulus, and their results were further extended by other authors, including Prasad et al. [2], Kumar et al. [3] and Prasad [4]. It was found that the annulus curvature can strongly affect the flow structure and the heat transfer. Y.L. He et al. [5] studied the natural convection heat transfer and fluid flow in a vertical cylindrical envelope with adiabatic lateral wall. They founded that, for the small cases $\Delta T_W = 25$ K, studied (Ra $= 1.1 \times 10^5 \sim 4 \times 10^7$), there is a global circulation between the hot and cold ends, accompanied by some local recirculations in the envelope. For the large temperature difference case $\Delta T_W = 220$ K, with the decrease in L/D, the average heat transfer rate increases.

In experimental studies, A. H. Malik et al. [6] investigated experimentally and numerically the buoyancy driven flow within bottom heated vertical concentric cylindrical enclosure. The experimental and numerical study of the axial temperature gradient and the heat transfer mechanism within the enclosure were performed. They showed that the numerical results of the streamlines within the enclosure depicted the real picture of the buoyancy effects. Nilesh B. Totala et al. [7] studied experimentally natural convection through vertical cylinder. The local heat transfer coefficient along the length of cylinder is determined experimentally and is compared with theoretical value obtained by using appropriate governing equations. They concluded that the heat transfer coefficient is having maximum value at the beginning length of cylinder and decrease in upward direction. R. Hosseini et al. [8] experimentally studied the natural convection heat transfer from a long heated vertical cylinder to an adjacent air gap of concentric and eccentric conditions. The aspect and diameter ratios of the cylinder are 55.56 and 6.33, respectively. The experimental measurements were obtained for a concentric condition and six eccentricities from 0.1 to 0.92 at five different heat fluxes. The surface temperature of the heated rod is measured at different heights, and the Nusselt number is calculated at the temperature measurement locations. A correlation is suggested to determine the Nusselt number based on the variation of the eccentric ratio values. They showed that the heat transfer rate is greater for eccentric pipes. At low heat flux, the temperature along the rod is almost constant, and when heat flux increases, temperature increases.

In contrast with steady natural convection, transient analysis of natural convection in a vertical cylindrical cavity has received much attention in the literature, K. Choukairy et al. [9] studied numerically and analytically the transient laminar free convection in a vertical cylindrical annulus filled with air Pr $= 0.71$. The result showed that the required time to reach steady state decreases with the convection intensity and is independent of the curvature. Anil Kumar Sharma et al. [10] studied numerically the transient turbulent natural convection heat transfer from a volumetric energy generating source placed inside a cylindrical enclosure filled with low Prandtl number fluid Pr $= 0.005$. Amitesh Kumar et al. [11] numerically investigated unsteady axi-symmetric laminar natural convection in a vertical cylindrical enclosure laterally heated at the vertical wall for different aspect ratios 2-4, and different Prandtl numbers of $0.01 \leq \Pr \leq 10$. They showed that for low Prandtl number fluids, $\Pr < 1.0$, critical Rayleigh number increase with increase in aspect ratio while for high Prandtl number fluids, $\Pr > 1.0$, decrease with increase in aspect ratio.

To investigate the stability of natural convection in an annular cavity, K. Choukairy et al. [12] studied numerically the effect of an obstacle on the heat transfer. The obstacle modifies the flow structure and affects local transfers. This change is more pronounced in a cylindrical configuration given the asymmetry resulting flows. The configuration considered is that of a cavity annular where the vertical walls are differentially heated and the horizontal walls are adiabatic. The effect of the size and conductivity of the obstacle on the transfer are analyzed when it has a low conductivity (insulating) or high conductivity. The obstacle is placed near the inner cylinder and its thickness variable. Numerical results show that the inclusion of obstacle modifies the overall flow and therefore local transfers. P. Venkata Reddy et al. [13] numerically investigated the natural convection in a vertical annulus driven by a central heat generating rod. The primitive equation is solved using a pressure-correction algorithm. They varied the heat generation based Grashof number, aspect ratio and the solid-to-fluid thermal conductivity ratio over wide ranges with the Prandtl number fixed at 0.7. They founded that the average Nusselt numbers on the inner and outer boundaries increase with the Grashof number. M. Sankar et al. [14] numerically investigated the natural convection in a vertical annulus with a localized heat source. A discrete heater is placed at the inner wall, while the top and bottom walls as well as the unheated portions of the inner wall are kept adiabatic, and the outer wall is maintained at a lower temperature. They founded that the placement of heater near the middle portion of inner wall yields a maximum heat transfer and minimum near the top and bottom portions of the inner wall. Further, they founded that the rate of heat transfer is an increasing function of radii ratio of the annulus.

A part of research has shifted their attention to understanding the mechanism of heat and mass transfer in annular enclosures, Sheng Chen et Jonas Tölke [15] numerically investigated the double-diffusive convection in vertical annuluses with opposing temperature and concentration gradients for higher Rayleigh numbers up to 10^7 using a simple lattice Boltzmann model for buoyancy ratio forces $0.8 \leq N \leq 1.3$, the aspect ratio $0.5 \leq A \leq 2$ and the radius ratio $1.5 \leq K \leq 3$. The results showed that the convective flows with $Ra = 10^6$ and $N<1.0$, there is only one large clockwise thermal recirculation in the enclosure, independent of K and A. On the contrary, the number of vortices varies depending on K and A when $N>1.0$. The average Nusselt number Nu and the average Sherwood number Sh are monotonic increasing functions of K. Jiten C. Kalita et Anoop K. Dass [16] numerically studied the Double-diffusive natural convection in a vertical porous annulus using the Higher Order Compact (HOC) algorithm. The flow is investigated in the regime $-50 \leq N \leq 50$, $1 \leq A \leq 10$, $1 \leq k \leq 50$, $0 \leq Ra \leq 10^3$ and $1 \leq Le \leq 500$, where N, A, k, Ra and Le are the buoyancy ratio, aspect ratio, radius ratio, thermal Rayleigh number and Lewis number respectively. The results revealed that the strength of the general convective movement increases with increasing Ra and the development of boundary layers is also seen with increase in N, for larger values of N, a large portion of the core of the annulus is seen to be stagnant because of the blocking effect of the combined effect of vertical. M. Sankar et al. [17] analyzed the double-diffusive convection in a fluid saturated vertical porous annulus subjected to discrete heat and mass fluxes from a portion of the inner wall. The physical model for the momentum equation is formulated using the Darcy law, and the resulting governing equations are solved using an implicit finite difference technique. Rajesh Sharma et al. [18] have studied the coupled momentum and heat transfer in unsteady, incompressible flow along a semi-infinite vertical porous moving plate adjacent to an isotropic porous medium with viscous dissipation effect. The finite element method (FEM) is utilized to simulate the unsteady free convection flow of a viscous fluid, B. V. Rathish Kumar et al. [19] numerically studied the mixed convection flow by Galerkin finite element method in vertical square porous enclosure filed with a concentration-stratified fluid-saturated, The forced flow conditions are imposed by providing an inlet at the bottom wall and an outlet with a suction on the top wall. Numerical results are presented by tracing the cumulative heat and mass fluxes, streamline and isotherms of the fluid for a wide range of governing parameters

Other investigations have been conducted to study the effect of magnetic field in different enclosure. Kewei Song et al. [20] numerically have investigated on the Magnetothermal free convection of air in a square enclosure under a nonuniform magnetic field provided by a permanent neodymium-iron-boron magnet, they have focused their work on the heat transfer and flowing characteristic of the magnetothermal convection of air driven by a permanent magnet field, and the comparison between the convection driven by magnetic field and gravity field.

The Natural convection of a nanofluid has also attracted considerable attention. Recently Hicham Salhi et al. [21] investigated Natural convection of a nanofluid (NF) consisting of water and (Ag or TiO_2) in an inclined enclosure cavity; the left and right walls of the cavity have a complex-wavy geometry and are maintained at a low and high temperature (random temperature, based on the random function). Results are presented in the form of streamlines, isotherms, and average Nusselt number. S. Bazi et al. [22] numerically analysised the heat transfer and fluid flow in a partially heated horizontal annulus filled with nanofluids. A heat source owing constant temperature is placed along the outer cylinder of the annular region. The temperature of the inner cylinder is lower than that of the outer cylinder, while the remaining parts are kept insulated. Four nanoparticles (Au, Cu, CuO, Al_2O_3) and three base fluids (water, ethylene glycol, oil) are selected to examine potential heat transfer enhancement in the annulus.

Based on the above-mentioned works, and published results according to our best knowledge that there is no works has been published yet, devoted to studying features of natural convection heat transfer in such geometry: the inner cylinder height is different than the outer cylinder one. Therefore, we focus our attention to study the steady laminar natural convection heat transfer in a cylindrical cavity partially annular filled with water Pr = 7.0. The main objective of the present study is to determine the influence of the height ratio X = h/H, ($0 \leq X \leq 1$) on the natural convection.

2. Problem statement and boundary conditions

The configuration under investigation is represented in Figure 1 and consists of a vertical annulus of height outer cylinder H, outer radius r_e , and height inner cylinder h, inner radius r_i. The vertical walls are maintained at different uniform hot and cold temperatures, T_h and T_c, respectively. The top and bottom walls of the enclosure are thermally insulated expect the horizontal wall of inner cylinder is maintained at hot temperature. It is assumed that the flow is incompressible and laminar, and the fluid is Newtonian, no viscous heat dissipation. The thermophysical properties of the fluid are assumed to be constants, except density in the buoyancy term, which depends linearly on the local temperature, i.e., the Boussinesq approximation is assumed to be valid. The annular gap width $\Delta r = r_o - r_i$ is taken as a reference length for the spatial coordinates (R, Y) = (r, y)/Δr. The references for velocity, pressure, and temperature are defined as (U, V) = (u, v)×Δr /υ, P = p×Δ$r^2/\rho\upsilon^2$, and θ = (T-T_0)/(T_h-T_c), with T_0 = (T_h+T_c)/2. u, v, υ, ρ, g presente the r and y velocity coordinate, fluid viscosity, fluid density, and gravitational acceleration, respectively.

The governing dimensionless equations for continuity, momentum, and energy can be written as:

$$\frac{1}{R}\frac{\partial (RU)}{\partial R} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial R} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial R} + \nabla^2 U - \frac{U}{R^2} \tag{2}$$

$$U\frac{\partial V}{\partial R} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \nabla^2 V + \frac{R_{aT}}{\mathrm{Pr}}\theta \tag{3}$$

$$U\frac{\partial \theta}{\partial R} + V\frac{\partial \theta}{\partial Y} = \frac{1}{\mathrm{Pr}}\nabla^2 \theta \tag{4}$$

where:

$$\nabla^2 = \frac{\partial}{R\partial R}(R\frac{\partial}{\partial R}) + \frac{\partial^2}{\partial Y^2}$$

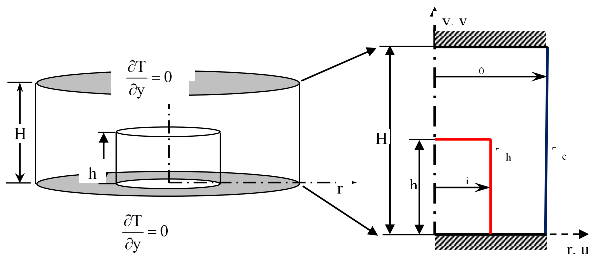

Figure 1 Physical model

The nondimensionalization process results in the following control parameters:

$$A = \frac{H}{(r_0 - r_i)} = \frac{H}{\Delta r} \qquad a = \frac{h}{(r_0 - r_i)} \qquad X = \frac{h}{H} = \frac{a}{A} \qquad K = \frac{r_0}{r_i} \tag{5}$$

$$\mathrm{Ra} = \frac{g\beta\Delta T\Delta r^3}{\nu\alpha} \qquad \mathrm{Pr} = \frac{\nu}{\alpha} \tag{6}$$

A is the aspect ratio of the cavity, K is the radii ratio of the curvature, Ra is the thermal Rayleigh number, and Pr is the Prandtl number.

The dimensionless boundary conditions for the physical system considered are:

$$R = \frac{1}{K-1} \quad \text{and} \quad 0 < \mathrm{Y} < \mathrm{a} \quad \theta = 0.5 \quad \text{and} \quad \mathrm{U} = \mathrm{V} = 0 \tag{7}$$

$$R = \frac{K}{K-1} \quad \text{and} \quad 0 < \mathrm{Y} < \mathrm{A} \quad \theta = -0.5 \quad \text{and} \quad \mathrm{U} = \mathrm{V} = 0 \tag{8}$$

$$\mathrm{Y} = 0 \quad \text{and} \quad \frac{1}{K-1} \leq R \leq \frac{K}{K-1} \quad \frac{\partial \theta}{\partial Y} = 0 \quad \text{and} \quad \mathrm{U} = \mathrm{V} = 0 \tag{9}$$

$$\mathrm{Y} = \mathrm{A} \quad \text{and} \quad 0 \leq R \leq \frac{K}{K-1} \quad \frac{\partial \theta}{\partial Y} = 0 \quad \text{and} \quad \mathrm{U} = \mathrm{V} = 0 \tag{10}$$

$$\mathrm{Y} = \mathrm{a} \quad \text{and} \quad 0 \leq R \leq \frac{1}{K-1} \quad \theta = 0.5 \quad \mathrm{U} = \mathrm{V} = 0 \tag{11}$$

As our study area is symmetrical about the vertical axis (OY) (Fig. 1), the calculation is based on half of the area containing the fluid. The boundary conditions at the axis of symmetry (OY) become:

$$\mathrm{R} = 0 \quad \text{and} \quad a < \mathrm{Y} < \mathrm{A} \quad \frac{\partial \theta}{\partial R} = 0 \quad \frac{\partial V}{\partial R} = 0 \quad \text{and} \quad \mathrm{U} = 0 \tag{12}$$

The average rate of heat transfer across the inner (r_i) and external (r_0) cylinders are expressed by using the respective Nusselt numbers.

$$\begin{aligned}
&\mathrm{Nu_{i1}} = \left.\frac{\partial \theta}{\partial R}\right|_{R=\frac{1}{K-1}} \partial Y \quad \text{and} \quad \mathrm{Nu_{i2}} = \left.\frac{\partial \theta}{\partial Y}\right|_{Y=a} \partial R \\
&\overline{\mathrm{Nu_i}} = \frac{1}{a}\int_0^a \left.\frac{\partial \theta}{\partial R}\right|_{R=\frac{1}{K-1}} \partial Y + \frac{1}{R}\int_0^{R=\frac{1}{K-1}} \left.\frac{\partial \theta}{\partial Y}\right|_{Y=a} \partial R \\
&\overline{\mathrm{Nu_0}} = \frac{1}{A}\int_0^A \left.\frac{\partial \theta}{\partial R}\right|_{R=\frac{K}{K-1}} \partial Y
\end{aligned} \tag{13}$$

3. Numerical procedure

A finite-difference numerical solution technique based on integration over a control volume is used to solve Eqs. (1)–(4) P. Venkata Reddy [23]. The SIMPLER algorithm is used for the pressure- velocity coupling in the momentum equation. The convergence of the algorithm is reached when the residual of the momentum equations and the average quadratic residues of each governing equation evaluated on the whole computational domain is less than 10^{-7}.

4. Validation

The computer code was validated with published numerical results on natural convection. The steady-state results have been compared with the benchmark results given by R. Kumar et al. [3] and G. Vahl Davis et al. [1] in the natural convection case corresponding to X = h/H =1.
The comparison is good and the relative error on the average Nusselt number compared to the values proposed by the different authors is around 1% (Tab. 1).

Table 1 Comparison of the average Nusselt number A=10, K=2, X=1.0 and Pr=0.7

Nombre de Rayleigh	Présent travail	R.Kumar et al	De Vahl Davis et al
10^4	2.361	2.355	2.333
5.10^4	3.702	3.718	3.758
10^5	4.535	4.558	4.568

5. Result and discussion

In the present work, a parametric study was conducted and computations were carried out for a wide range of: $10^3 \leq Ra \leq 10^6$, $0 \leq X \leq 1$, A = 1, Pr = 7.0 and K = 2. The heat results along with isotherms and flow fields have been also obtained.

5.1. *Effects of Rayleigh numbers*

In Fig. 2, the streamlines and isothermal contours are shown for $10^3 \leq Ra \leq 10^6$, and X = 0.5. As it can be observed, the flow field is characterized by simple recirculating zone turning in the clockwise direction. The fluid is almost stagnant within the core of the cavity (Fig. 5, Fig. 6). At moderate Rayleigh numbers 10^3 to 10^4. A horizontal stratification of temperature is observed within the cavity showing that flow is mainly dominated by conduction heat transfer. As the Rayleigh number increases from 10^5 to 10^6, the pattern exhibits a strong upward flow at the inner (hot) cylinder and downward flow at the outer (cold) cylinder, due to the increase of the mode of convection heat transfer. It can be seen that the fluid in the middle of the annular space undergoes a strong centrifugal acceleration, and the formation of thermal boundary layers at the side walls indicates the presence of fairly strong convection (Fig. 2 d).

The temperature distribution at Y = 0.8 of the annulus space for various Rayleigh numbers are shown in Fig. 4. For low Rayleigh number (Ra $\leq 10^{4)}$. The temperature profile is almost linear in both zones 0<R<1 and 1<R<2, since the heat is mainly transferred by conduction. However, when Ra increases (Ra $>10^4$), the temperature gradient is weakened in the annular zone (1<R<1.9) and a vertical thermal stratification is observed. In the vicinity zone near the active walls (R$\sim$1.9 and R$\sim$1.95), the fluid motion is increased considerably owning to the presence of an intense temperature gradient.

The horizontal thermal gradients are associated with the fluid flow which is predominantly in the boundary layers near the vertical wall, this can be observed from the both profile, vertical velocity (V) and horizontal velocity for Ra = 10^5 to 10^6 (Fig. 5, Fig. 6).

Figure 3 show the resulting velocity fields for 10^3-10^6. At Ra = 10^3 and 10^4, the dotted line shows clearly single vortex in the centre of annular space and diminishing velocity field above the inter cylinder indicating that the most heat transfert is by heat condution. As Rayleigh number increase Ra = 10^5 the centre velocity fleild distorted and the effet of convection is more pronounced in the velocity feild. The flow accelarated across the vertical walls because of the temperature gradient witch is more severe near the vertical walls, but diminish in the centre.

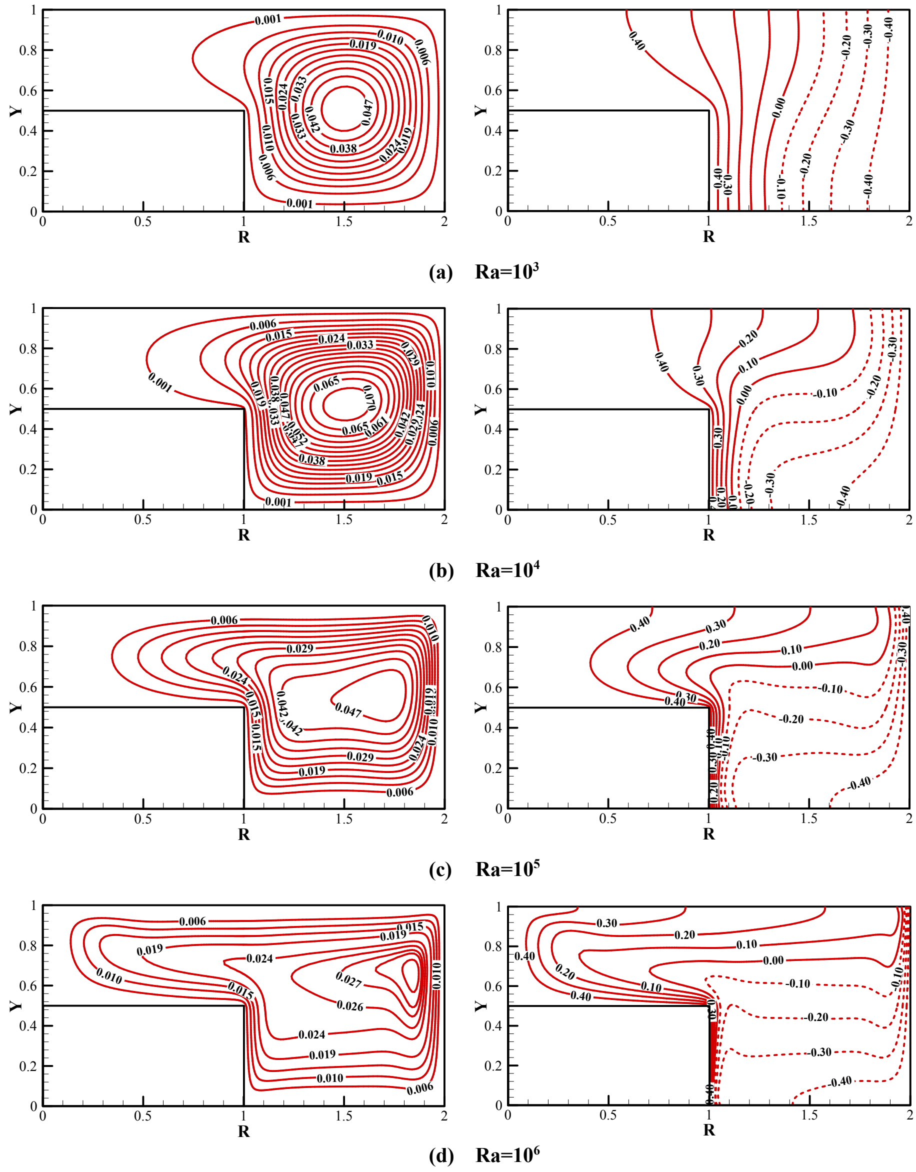

Figure 2 Streamlines and isotherms for a) Ra = 10^3, b) Ra = 10^4, c) Ra = 10^5, d) Ra = 10^6 (A = 1, K = 2, X = 0.5, Pr = 7.0)

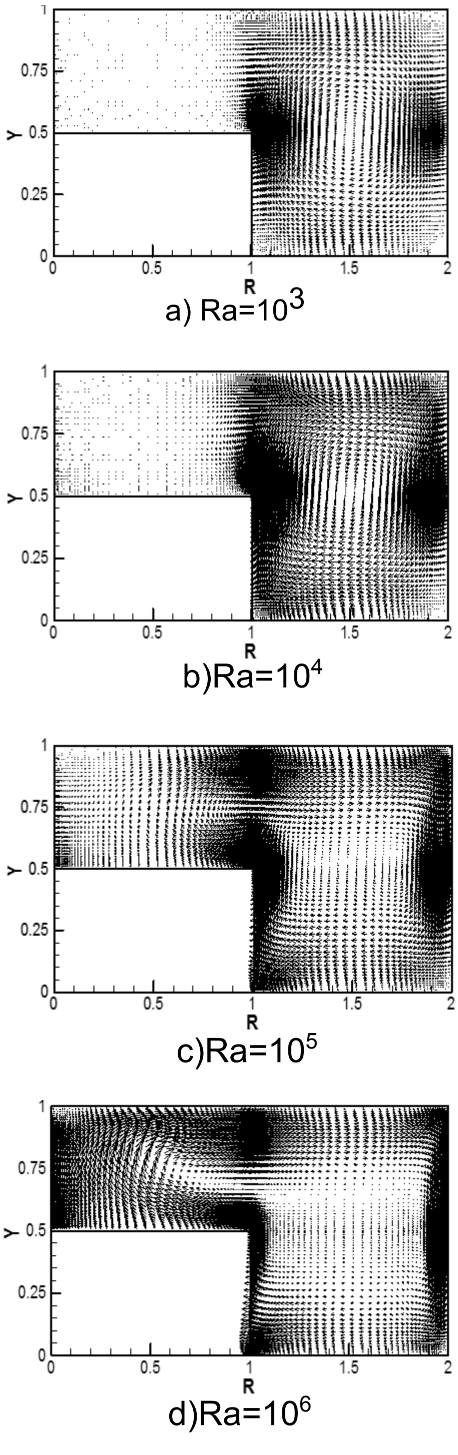

Figure 3 Velocity field for a) Ra = 10^3, b) Ra = 10^4, c) Ra =10^5, d) Ra = 10^6 (A = 1, K = 2, X = 0.5, Pr = 7.0)

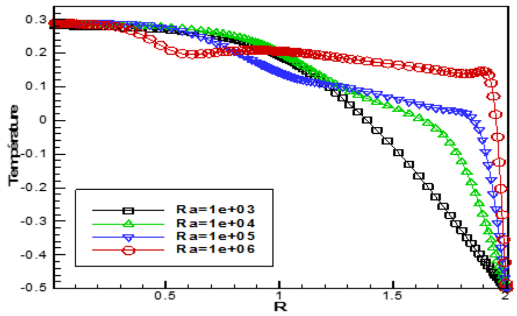

Figure 4 Temperature profiles at Y = 0.8 for different Rayleigh number (A = 1, K = 2, X = 0.5)

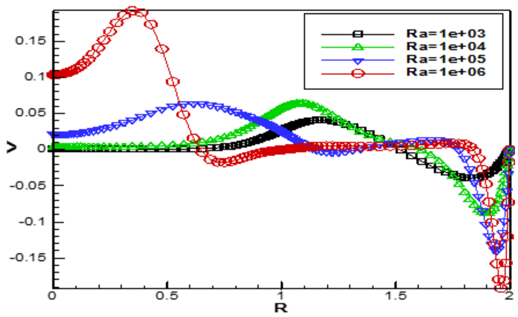

Figure 5 Vertical velocity profiles at Y = 0.8 for different Rayleigh number (A = 1, K = 2, X = 0.5)

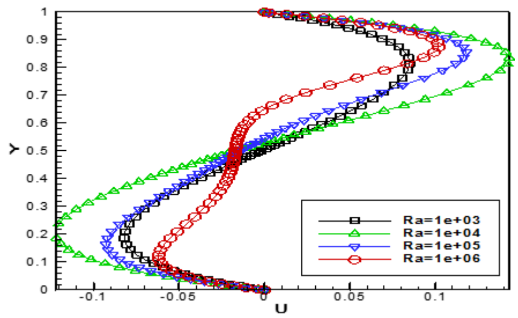

Figure 6 Horizontal velocity profiles at R = 1.65 for different Rayleigh number (A = 1, K = 2, X = 0.5)

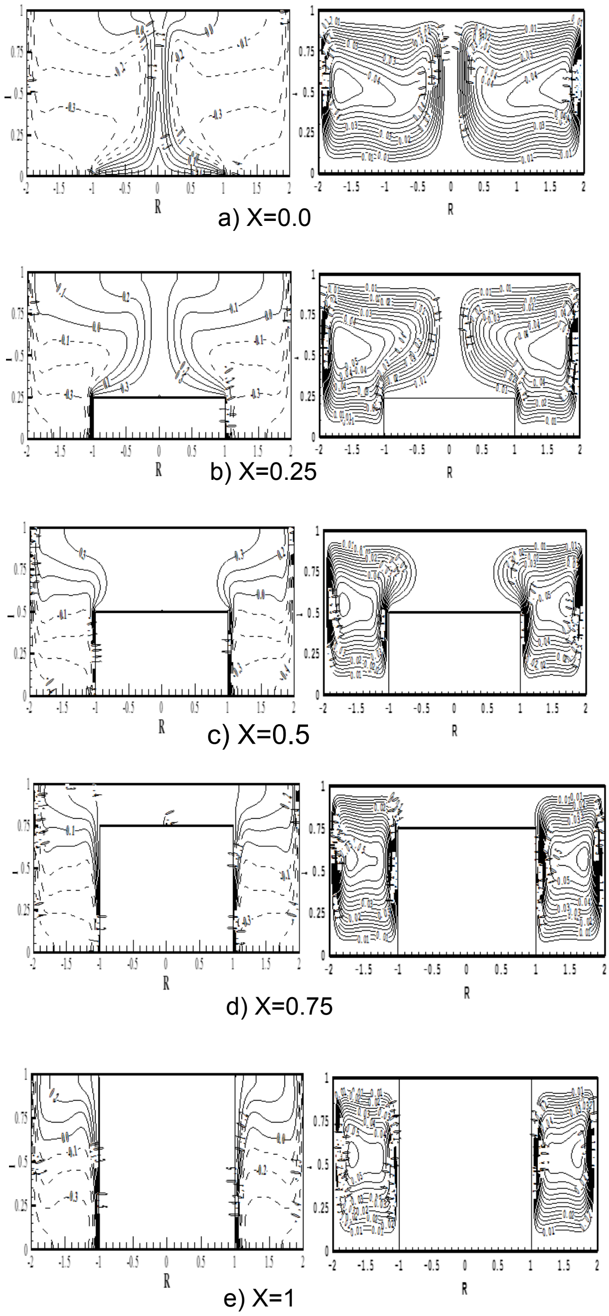

Figure 7 Streamlines and isotherms for Ra = 10^5, A = 1, K = 2 at different values of X

The behaviour continues to Ra = 10^6, the central velocity feild are futher elongated and speed flow increase in top portion of inter cylinder.

5.2. *Effect of the height ratio*

Figure 7 shows the effect of the height ratio (X) on the development of isotherms and streamlines for Ra = 10^5.

For X = 0, a significant vertical thermal gradient adjacent to the horizontal heater surface of the inner cylinder is observed. The fluid is rising along the horizontal hot wall, moving toward the outer wall, and then the flow descends down along the cold wall as a plume. The corresponding streamlines form a single cellular pattern filling the entire annular space. When height ratio increase for X = 0.25, we observe a significant heat transfer in the upper region of the inner cylinder ($-1 \leq R \leq 1$) due to the flow acceleration in the upper region of the hot inner cylinder. But for $0.25 \leq X \leq 0.5$, the isotherms move away from the upper region of the inner cylinder. For $X \geq 0.75$, the fluid motion is fully accelerated in the boundary layers near the vertical wall of the annulus gap ($-2 \leq R \leq -1$, $1 \leq R \leq 2$), and it is stagnated at above of the inner cylinder $1- \leq R \leq 1$.

Fig. 8 shows the temperature distribution at Y = 0.8 of the annulus space for various height ratio and Ra = 10^5. For ($X \leq 0.25$), the temperature gradients near the hot wall increase in the middle of the cavity ($-1 < R < 1$) indicating a high heat transfer rate. When the height ratio increases, shifting up toward the upper adiabatic side walls ($0.5 \leq X \leq 1$), a stagnant zone in that region ($-1 < R < 1$) is observed due to the absence of temperature gradient. The temperature profiles exhibit a strong temperature gradient near the active vertical walls for $X \geq 0.75$, indicating that the flow is fully accelerated in annular space.

The vertical velocity (V) profile at Y = 0.8 for various height ratio is shown in the Fig. 9. Peak velocities are observed only near the horizontal heater surface of the inner cylinder for $X \leq 0.25$. When X increase ($0.25 \leq X \leq 0.5$), the vertical velocity decrease in upper zone of the inner cylinder and stagnated for X = 0.75. For $X \geq 0.75$, the velocities are observed only near the vertical boundaries active walls.

The horizontal velocity (U) profile at R = 1.65 for various height ratio is shown in the Fig. 10, we observe no change with various height ratio.

5.3. *Heat transfer*

Figure 11 shows computed values of the average Nusselt number plotted against Rayleigh number and different height ratios. It was observed that the average Nusselt number increases with the Rayleigh number and the height ratios. This behavior can be explained by an augmentation of the heat exchange area due to an increasing of the height ratio, except for X = 0.5, the heat transfer is maximal, this reveals the fact that, the maximum heat exchange contributed between the heater vertical wall (inner cylinder) and the surrounding fluid, but also by the heater horizontal circular surface of the inner cylinder.

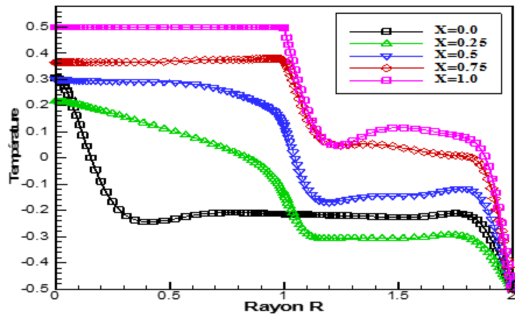

Figure 8 Temperature profiles at Y = 0.8 for various height ratio X (Ra = 105, A = 1.0, K = 2.0)

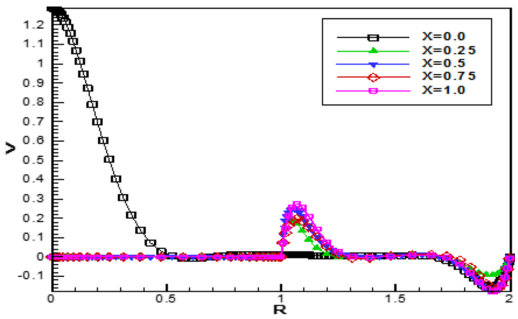

Figure 9 Vertical velocity profile at Y = 0.8 for various height ratio X (Ra = 10^5, A = 1.0, K = 2.0)

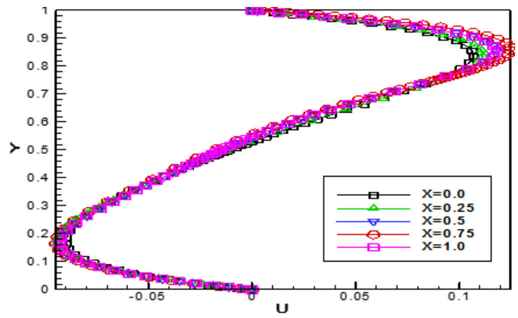

Figure 10 Horizontal velocity profile at R = 1.65 for various height ratio X (Ra = 10^5, A = 1.0, K = 2.0)

The annulus height ratio does not influence the heat transfer rate when $10^3 \leq \text{Ra} \leq 10^4$, because the fluid motion is very slow, and the heat is mainly transferred by conduction, which explains the almost constant value of the Nusselt number. The heat transfer rate is greater at the Rayleigh number is important $\text{Ra} \geq 10^5$.

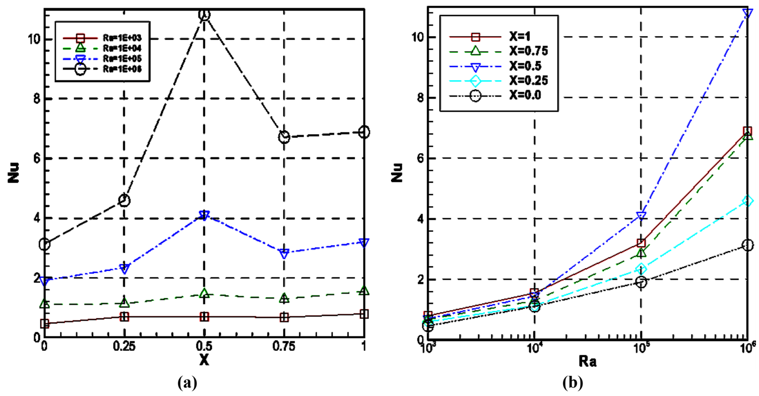

Figure 11 Variation of average Nusselt number for, a) various height ratios (X), b) various Rayleigh number (Ra)

6. Conclusion

A numerical study has been performed to investigate the effects of buoyancy, and height ratio on the nature convection in a vertical cylinder partially annular. The major results of the study can be summarized as follows: the flow field is characterized by simple recirculating zone turning in the clockwise direction. An examination of the effects of height ratio (X), and Rayleigh numbers (Ra) on the flow and isotherm patterns as well as temperature and velocity profiles has been conducted. The results indicate that the changes in the Rayleigh number and height ratio (X) have a major influence on the flow structure and isotherm patterns. Two flow regimes were observed for lower Rayleigh number values and a dominance of conduction heat transfer. At higher values of Rayleigh number, heat transfer rate is increased and are rather dominated by convection mode. Also, this results show that the average Nusselt numbers increase with increasing of the Rayleigh numbers and height ratio. This behavior can be explained by an augmentation of the heat exchange area due to the important buoyancy force and an increasing of the height ratio except for X = 0.5, the heat transfer is maximal. The fluid motion is fully accelerated in the boundary layers near the vertical wall of the annulus gap ($-2 \leq R \leq -1$, $1 \leq R \leq 2$), and it is stagnated at above of the inner cylinder $1- \leq R \leq 1$, for $X \geq 0.5$ and $\text{Ra} \geq 10^5$.

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