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Performance Analysis of Gough Stewart Platform with 6 Limbs

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This paper concentrates on widespread study of parallel manipulator. It focuses on optimal designing of manipulator which has a large number of application fields. Optimal design is an important criterion to improve the accuracy of a robot. Through optimal design a robot can achieve isotropic configurations where the condition number of its jacobian matrix equals one. In this we are also concentrating on transmission index and stiffness index along with their plots, which can affect the kinetostatic performance of the robot. In this the singularity of Gough Stewart platform is also studied.

Keywords: parallel manipulator, optimal design, isotropic, condition number, kinetostatic performances.

1. Introduction

The disadvantages of conservative robot arms have made researchers to think for an alternative manipulator. Amid other manipulator architectures, parallel manipulators have been given considerable attention. They consists of several kinematic chains connecting the base to the end-effector (Fig. 1), which allows the actuators to be located on or near the base of the mechanism, thereby increasing the load-carrying capacity and leading to high accuracy, high stiffness and very good dynamic properties. Because of their best performance characteristics parallel manipulators possesses wide range of applications where these properties are of primary importance while a limited workspace is acceptable. The most common application of this type is undeniably in flight simulation. Flight simulation is originally proposed in Stewart (1965). This mechanism is commonly denoted as the "Stewart platform"

and it was first proposed by Gough. Therefore, this mechanism is known as the Gough-Stewart platform.
The limitation of parallel manipulators is that they may lead to singular configurations in which the stiffness of the
Mechanism is lost. This has drawn the attention of several researchers and now this is a major field of research now. This paper also focuses on parallel manipulator and develops it in such a way as to make its working condition accurate.
In this paper a six degrees of freedom parallel mechanism is considered for which the equations of their motion are considered and analysis is carried out.

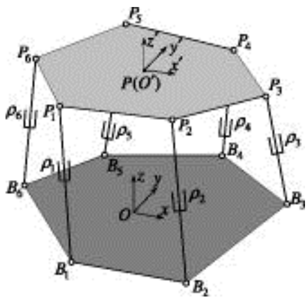

Figure 1 Gough Stewart Platform with notation

The notations in this paper are followed based on the notations shown in the above figure. Here, Bi where i = 1, 2, 3, 4, 5, 6 is the base and Pi where i = 1, 2, 3, 4, 5, 6 is the movable pod.
In a Gough-Stewart platform usually there is a fixed base and a mobile platform. These booth are connected with the help of legs, here there are six legs connecting the two bases. They are connected via prismatic actuators. The attachments to the platform, at points A_i, i = 1,...,6, are spherical joints, while those at the base, at points B_i, i = 1,...,6, are Hooke joints. Hence, the mechanism has six degrees of freedom. The position and orientation of the platform in space are controlled by adjusting the length of the six legs.
As shown in Fig. 1, a reference frame $R\,(Oxyz)$ is fixed to the base and a moving frame R‘ (O‘x‘y‘z‘) is attached to the platform. Furthermore, the position of the ith joint on the base point B_i is denoted by vector $b_i = [b_{ix},\, b_{iy},\, b_{iz}]T$, i = 1,...,6 and the position of the ith joint on the platform point A_i by vector a_i‘ = $[a_{ix}$‘, a_{iy}‘, a_{iz}‘$]T$, i = 1,...,6. Vector b_i is a constant vector when expressed in frame R, while vector a_i‘ is a constant vector when expressed in frame R‘. Let vector $k = [x,\, y,\, z]T$ denote the position of point O‘ with respect to point O expressed in frame R and let Q be the matrix representing the rotation from frame R to frame R‘. Further the position vector of point A_i expressed in frame R, noted M_i, is given by:

$$M_i = k + Q\,a_i \tag{1}$$

where $M_i = [M_{ix},\, M_{iy},\, M_{iz}]T$. Subtracting vector bi from both sides of Eq. (1), we get:

$$M_i - b_i = k + Q\,a'_i - b_i \tag{2}$$

Here the left hand side represents a vector connecting point Bi to point Ai, along the ith leg. Taking the Euclidean norm of both sides of this equation gives us:

$$\rho_i^2 = ||a_i - b_i||^2 = (k + Q * a_i' - b_i)T(s + Q * a_i' - b_i) \qquad i = 1, ..., 6. \tag{3}$$

When above equation is differentiated with respect to time, a set of linear equations relating the joint rates to the Cartesian velocities is obtained. Two Jacobian matrices A and B are obtained and the velocity equations can be written as:

$$AT = B\infty \tag{4}$$

where T is the six-dimensional twist of the platform and ∞ is the vector of joint velocities. These vectors are defined as:

$$T = [kT', wT] \quad \infty = (\infty 1.........\infty 6) \tag{5}$$

in which the angular velocity of the platform is defined as w and k' = [x', y', z']T is the velocity of point O'. The above-mentioned Jacobian matrices can then be written as:

$$\mathrm{B} = \mathrm{diag}(\infty 1.........\infty 6) \tag{6}$$

$$\mathrm{A} = \begin{matrix} C_1^T \\ . \\ . \\ . \\ C_6^T \end{matrix} \tag{7}$$

with:

$$\mathrm{C_i} = \begin{pmatrix} d_i \\ Q_x a_i \end{pmatrix} \qquad i = 1, ..., 6, \tag{8}$$

where di is the vector connecting point B_i to point A_i, i.e.,

$$d_i = a_i b_i \qquad \mathrm{i} = 1, ..., 6 \tag{9}$$

The rotation matrix Q representing the orientation of the platform with respect to the base:
$Q =$

$$\begin{bmatrix} \cos\theta\cos\psi & \cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi & \cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi \\ \cos\theta\sin\psi & \sin\psi\sin\theta\sin\varphi + \sin\psi\sin\theta\cos\varphi & \sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\theta\cos\varphi \end{bmatrix} \tag{10}$$

where ψ, θ, φ are three Euler angles defined according to the convention (Q_z, Q_y, Q_x).

2. Optimal design

Optimal design is class of experimental designs in the design of experiments that are optimum relating to some statistical criterion.
Advantages of optimal design:

1. Optimal designs reduce the experimental cost by permitting statistical models to be estimated with less number of experimental runs.

2. Optimal designs can lodge various types of factors, such as discrete factors, process and mixture.

3. Optimization of designs can be done when we constrain the design space.

To make a design optimized we need to make the condition number fed into the surface mesh and obtain the surface plot. This is one of the methods to get optimized design which gives an accurate workspace. The surface plot gives us the optimized design.

3. Performance analysis

3.1. *Manipulability*

A quality measure for redundant manipulator is called Manipulability index. This index describes the distance to singular configurations. The approach is based on analyzing the manipulability ellipsoid that is spanned by the singular vectors of the Jacobian. we use an extended manipulability measurement in order to consider constraints that limit the maneuverability in workspace. Such constraints are introduced by joint boundaries and workspace (self-) collisions, but any other constraints can be incorporated as long as the derivation with respect to joint movements can be built.
The concept of manipulability of a manipulator was introduced by Yoshikawa. The manipulability is defined as the square root of the determinant of the product of the manipulator Jacobian by its transpose. The manipulability is equal to the absolute value of the determinant of the Jacobian in case of square Jacobian. Let matrix be K:

$$\mu = \sqrt{(det(K))} \tag{11}$$

3.2. *Condition number*

When the determinant of the Jacobian is equal to zero, it means that the manipulator approaches singularities. However, the actual value of the determinant cannot be used as a practical measure of the degree of ill-conditioning. For this purpose it is convenient to use the condition number of the Jacobian. It is well known from the singular value decomposition theorem Condition number of nonsingular square matrix M defined by:

$$\text{condM} = ||\text{M}|| \cdot ||\text{M}||$$

By convention:

$$\text{cond(M)} = \alpha \quad \text{if M singular}$$

The condition number of an n x n matrix numerical value depends on the specific norm used (indicated by the corresponding subscript), but because of the equivalence of the underlying vector norms, these values can differ by at most a fixed constant (which depends on n), and hence they are equally useful as quantitative

measure of conditioning. Condition number of the matrix measures the ratio of the maximum relative stretching to the maximum relative shrinking that matrix does to any non-zero vectors. Another way to say that the condition number of a matrix measures the amount of distortion of the unit sphere (in the corresponding vector norm) under the transformation by the matrix. The larger the condition number, the more distorted (relatively long and thin) the unit sphere becomes when transformed by the matrix.
Properties of the condition number:

1. For any matrix M, cond $(M) \geq 1$
2. For identity matrix, cond $(I) = 1$
3. For any matrix M and scalar δ, cond (δA) = cond (A)
4. For any diagonal matrix D = Diag (d_i), cond $(D) = (\max |d_i|)/(\min |d_i|)$.

3.3. *Minimum singular value*

In maximum cases the minimum singular value is used efficiently for indicating whether the determinant is near to zero. The minimum singular value changes more radically near singularities than the other singular values. In a minimum singular value graph the changes of the minimum singular values (msv) of the Jacobian over the workspace with constant orientation. The orientation is the same as for the obtained reciprocal condition numbers.

3.4. *Transmission index*

This is another criterion to know the performance whether it might be kinematic or dynamic. This index is important and it depends on the identity matrix combined with the norm movable pad matrix of the parallel manipulator in a matrix form.

3.5. *Stiffness index*

The deformations or compliant displacements in the geometry of a body are caused due to application of load on the body. Stiffness can be defined as the capacity of a mechanical system to sustain loads without excessive changes of its geometry (Rivin, 1999). Moreover, the stiffness of a body can be defined as the amount of force that can be applied per unit of compliant displacement of the body (Nof, 1985), or the ratio of a steady force acting on a deformable elastic medium to the resulting displacement. Compliant displacements in a multibody robotic system allow for mechanical float of the end-effector relative to the fixed base. This produces negative effects on static and fatigue strength, efficiency (friction losses), accuracy, and dynamic stability (vibrations). In this paper stiffness index is taken and plotted showing the performance.

4. Methodology

First of all the robot motion is analyzed and taken into consideration. Then equations are developed based on how the manipulator motion should be. The base values of the robot geometric position are given as the input values for the program code written in MATLAB. A matrix C is given using which we can find the position of movable pod. These are fed into an equation along with the rotation angles and a vector. Using these given inputs in the equations for Manipulability analysis, Minimum singular value, Condition number, Transmissibility index and stiffness index are found. Here we get a very large matrix as we get the whole points through which the robot passes. The above obtained values are fed into the code written to produce plots showing different indices.

5. Result

All the results are obtained when the coordinates of the tool are x = 2; y = 3; z = 10. Here we are considering only few performance analysis factors like Manipulability index, Condition number and minimum singular value.

5.1. *Manipulability index*

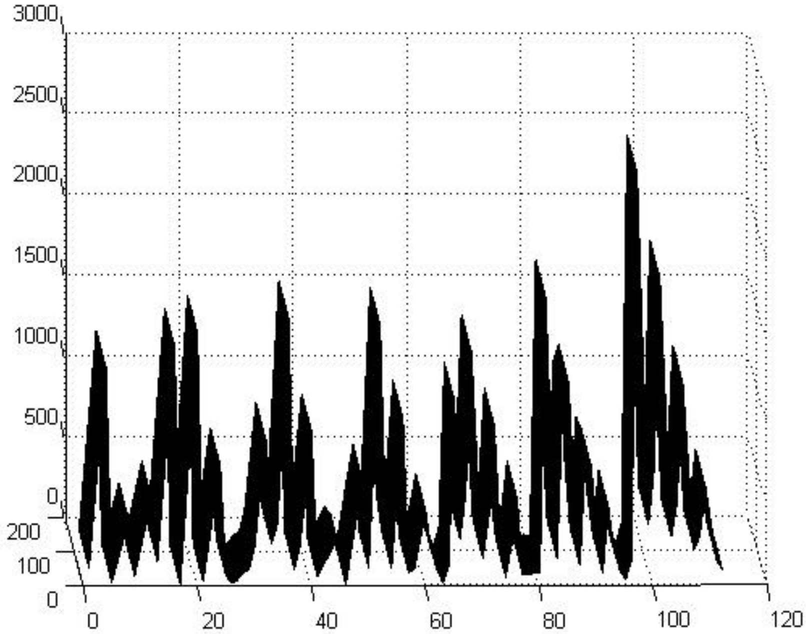

Figure 2 3D plot of Manipulability index of the manipulator

5.2. *Condition number*

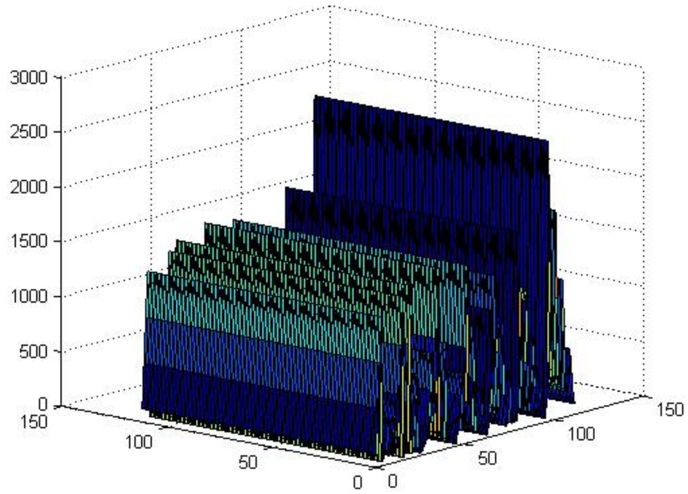

Figure 3 3D plot of Manipulability index of the manipulator

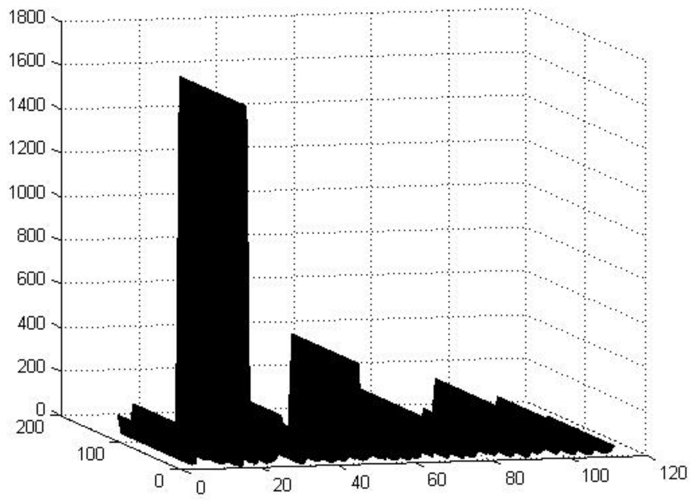

Figure 4 Plot showing the condition number

5.3. *Minimum singular value*

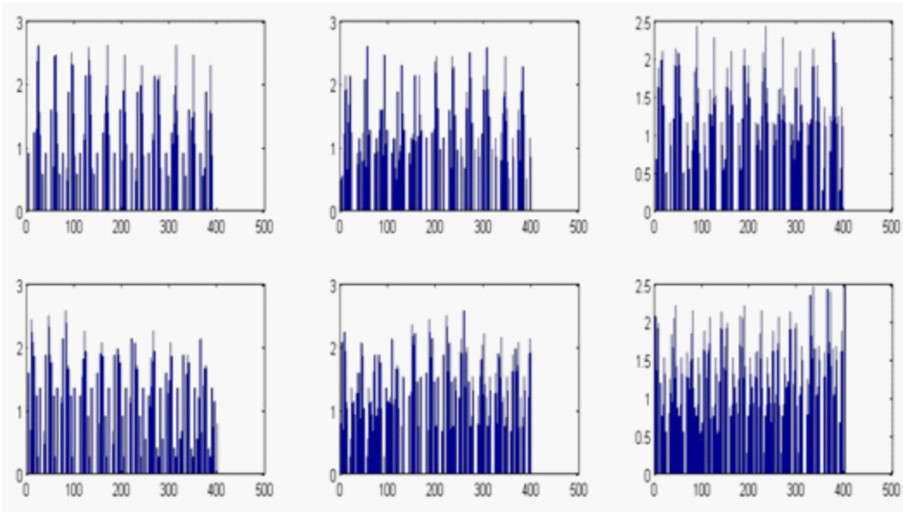

Figure 5 Plots of minimum singular value

6. Conclusion

In this paper, performance indexes are discussed as measures of kinematic capabilities of manipulators. Dexterity and manipulability of manipulators are considered. All obtained results for the performance indexes are graphically visualized. The presented graphs allow a comparison of the different performance indexes, i.e., what kind of similarities exist. Some of the known indexes are considered and some new indexes are introduced. The presented graphical examples for the performance of a manipulator can be easily interpreted and also they can help in application and design of manipulators.

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