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Fourier Variant Homogenization Treatment of Single Impulse Boundary Effect Behaviour

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Boundary effect behavior understood as near-boundary suppression of boundary fluctuation loads is described in various ways depending on the mathematical representation of solutions and the type of the center. In the case of periodic composites, the homogenization method is decisive here. In the framework of the *Tolerance Averaging Approach*, developed by prof. Cz. Woźniak leading to an approximate model of phenomena related to periodic composites this effect is described by a homogeneous part of differential equation for fluctuation amplitudes and usually this approximate description of the boundary effect behavior is restricted to a single fluctuation. In this paper, contrary to the previous elaborations, the boundary effect is developed in the variant of the tolerance thermal conductivity model in which the temperature field is represented by the Fourier expansions composed by an average temperature with infinite number of Fourier terms imposed on the average temperature as tolerance fluctuation suppressed in the framework of the boundary effect.

Keywords: even temperature fluctuations, homogenization, tolerance modelling.

1. Introductory concepts

The derivation of a single *conductivity equation* for the average temperature field is a crucial problem in any homogenization technique approach of thermal behaviors in periodic composites. In this paper this procedure is finalized as a result of the *Tolerance Modelling Technique* adopted to the investigation of thermal solutions of Heat Transfer Equation in periodic composites in the form of Fourier expansions. The direct consequence of *Equations* presented in [9,15] in the special two-dimensional case with one-directional periodicity for single fundamental odd fluctuation. This problem includes a counterpart to the approximate description of Boundary Effect Behavior developed by Cz. Woźniak, cf. [3], and illustrated by many continuators in the form of examples in heat conduction as well as in the linear elasticity areas, [2–7]. However description proposed in this paper includes a certain seemingly small but qualitative correction of the coefficient in term free of the spatial derivatives in single equation for Fourier amplitude. Mentioned correction seems to eliminate near-boundary problems in original tolerance description developed by Cz. Woźniak et. al., [3,11], and continuators, [13,14].

All known tolerance *Boundary Effect Behavior* descriptions takes into account the *Tolerance Model Equations* reduced to the special of two-dimensional case with one-directional periodicity and for single saw-like shape function. However, this approach to the modelling of near-boundary behaviors in the layered composites takes into account model equations which as a rule lead to the approximate description of physical behaviors. In this paper we are to investigate the *tolerance boundary effect behavior description* but instead of using model equations obtained by original *Tolerance Model Equations* we are to apply the *Extended Tolerance Model* equations as a fundamental tool of modelling. Considerations are restricted to the heat transfer behaviors and one-directional composite periodicity. There are also known approaches to the investigation of the near-boundary phenomena based on the asymptotic homogenization, [1,2], and modifying the original homogenization equations by introducing so-called correctors. In this work, we will not use them.

Following approach, originally suggested by Cz. Woźniak [3] equations of the *Extended Tolerance Model* we restrict to the case in Fourier expansion being a Fourier representation of the temperature field contain exclusively one fluctuation term φ. Fourier amplitude related to this single fluctuation will be denoted by q. Denoting by φ the number of repetitive cells in the periodic composite consider:

$$\begin{aligned}&\Omega = (0, L) \times (0, \delta)\\ &\Omega_B = \bigcup\nolimits_{k=1,\ldots,M} \{(y, z) : -\eta\lambda < y - k\lambda < 0,\ z \in (0, \delta)\}\\ &\Omega_W = \bigcup\nolimits_{k=1,\ldots,M} \{(y, z) : 0 < y - k\lambda < (1-\eta)\lambda,\ z \in (0, \delta)\}\end{aligned} \tag{1}$$

as the region occupied by the composite, as the regions occupied by the *BLACK* composite component and as the regions occupied by the *WHITE* composite component.

The type of the *composite periodicity*, as a certain geometrical support unequivocally picked from the composite, is determined by the saturation $\eta = \eta(z)$, $0 < \eta(z) < 1$, $\eta = \eta(z)$, and the length λ of the repetitive cell $\Delta \equiv (-\eta\lambda, \lambda - \eta\lambda)$. Situations in which λ depends in the periodic y and nonperiodic z variables, i.e. in which $\frac{\partial\lambda}{\partial y} \neq 0$ or $\frac{\partial\lambda}{\partial z} \neq 0$ will not be egzamined here. In the mentioned special case *Extended*

Tolerance Model equations, are reduced to the form:

$$\begin{aligned}
&\langle c\rangle\dot{u} - [\tfrac{\partial}{\partial y},\tfrac{\partial}{\partial z}][\langle k\rangle(z)[0,\tfrac{\partial u(z)}{\partial z}]^T - \tfrac{d}{dz}[\langle k\tfrac{\partial\varphi}{\partial z}\rangle q(z)] - \\
&\qquad -[\tfrac{\partial}{\partial y},\tfrac{\partial}{\partial z}]\langle k[\tfrac{\partial g}{\partial y},\tfrac{\partial g}{\partial z}]^T\rangle\psi(z) = -\langle b\rangle \\
&\langle\nabla^T g^A k\nabla g^B\rangle\psi_B + \langle k\nabla^T g^A\rangle\nabla u = 0 \\
&\lambda^2(\langle c\varphi^2\rangle\dot{q} - \tfrac{d}{dz}(\langle k\varphi^2\rangle\tfrac{dq}{dz}) + \lambda\langle k\varphi\tfrac{\partial\varphi}{\partial z}\rangle\tfrac{dq}{dz} + \langle k(\tfrac{\partial\varphi}{\partial y})^2\rangle q + \\
&\qquad +\langle k\tfrac{\partial\varphi}{\partial y}\tfrac{\partial g}{\partial y}\rangle\psi + \langle k\tfrac{\partial\varphi}{\partial y}\rangle\tfrac{du}{dz} = \lambda\langle\varphi b\rangle
\end{aligned} \tag{2}$$

Eliminating tolerance amplitude ψ from the second of Equations (1) we arrive at:

$$\begin{aligned}
&\langle c\rangle\dot{u} - \tfrac{d}{dy}(k^\perp\tfrac{du}{dy}) - \tfrac{d}{dz}(k^\perp\tfrac{du}{dz}) = -\langle b\rangle + (\tfrac{\partial}{\partial y}\langle k\tfrac{\partial\varphi}{\partial y}\rangle + \tfrac{\partial}{\partial z}\langle k\tfrac{\partial\varphi}{\partial z}\rangle)q(z) \\
&\lambda^2(\langle c\varphi^2\rangle\dot{q} - \tfrac{d}{dz}(\langle k\varphi^2\rangle\tfrac{dq}{dz}) + \lambda\langle k\varphi\tfrac{\partial\varphi}{\partial z}\rangle\tfrac{dq}{dz} + \langle k(\tfrac{\partial\varphi}{\partial y})^2\rangle q = \lambda\langle\varphi b\rangle - [k]^\perp\tfrac{du}{dz}
\end{aligned} \tag{3}$$

in which:

$$\begin{aligned}
k^\perp &= \langle k\rangle - \frac{\langle k\frac{\partial g}{\partial y}\rangle^2 + \langle k\frac{\partial g}{\partial z}\rangle^2}{\langle k[(\frac{\partial g}{\partial y})^2 + (\frac{\partial g}{\partial z})]^2\rangle} \\
[k]^\perp &= \langle k\frac{\partial\varphi}{\partial z}\rangle - \frac{\langle k\frac{\partial\varphi}{\partial y}\frac{\partial g}{\partial y}\rangle\langle k\frac{\partial g}{\partial z}\rangle}{\langle k[(\frac{\partial g}{\partial y})^2 + (\frac{\partial g}{\partial z})]^2\rangle}
\end{aligned} \tag{4}$$

is used as the projection of the *Effective Conductivity Constant* onto the z–variable direction periodicity.

2. Boundary effect behavior

As a *Boundary Effect Equation* we understand every equation allowing to diagnose the influence of a material structure on the behavior of a single impulse imposed on a field describing a certain physical phenomenon in in a short time interval following the occurrence of this impulse and/or in the immediate neighborhood of the boundary on which this impulse occurred.

In the *Tolerance Averaging Technique* (*TAT*) approach to the investigation of near-boundary phenomena the *Thermal Boundary Effect Equation* is disengaged from the original tolerance model equations, which are related to (3), by the special selecting the boundary (or initial-boundary) conditions and the geometrical shape of the region occupied by the composite leading to the situation in which obtained boundary problem is solved by the reference the average temperature, usually having the form $u = \alpha z + \beta$ for a certain $\alpha, \beta \in R$, together with fluctuation amplitude $q = q(z)$ satisfying the homogeneous part of the "equation for tolerance amplitudes" being a counterpart to the second from Equation (1):

$$\lambda^2(\langle c\varphi^2\rangle\dot{q} - \frac{d}{dz}(\langle k\varphi^2\rangle\frac{dq}{dz}) + \lambda\langle k\varphi\frac{\partial\varphi}{\partial z}\rangle\frac{dq}{dz} + \langle k(\frac{\partial\varphi}{\partial y})^2\rangle q = 0 \tag{5}$$

(or a counterpart to the stationary of the second from Equation (1)) together with appropriate initial-boundary condition (or boundary condition) attached to this counterpart. That is why Equation (5) will be referred to as the *single-impulse Thermal Boundary Effect Equation.*

Equation (5) is an ordinary differential equation with variable or constant coefficients depending on whether or not the saturation η changes with the change of the non-periodic z- variable. In the previous tolerance descriptions of boundary effect behavior (not only thermal boundary effect behavior), cf. [11–14], for the unidirectional multicomponent composite on this description is usually imposed additional condition related to $[k]^{\perp}\frac{du}{dz}=0$ (resulting in the disposal of the term $[k]^{\perp}\frac{du}{dz}$ in the second from Equations (3)). In this paper we will not take into account this imposition of treating the boundary effect problem of as a certain problem of controlling the equation (5) by boundary conditions attached to this equation in order to obtain the expected characteristics of impulses transport through the composite. That is why we also allow the interpretation of Equation (5) in which $q=[q_1,....,q_n]^T$ and $\varphi=[\varphi_1,....,\varphi_n]^T$(also including $n\to\infty$) and in the boundary effect problem we have n-th boundary values of Fourier amplitudes $q_1,....,q_n$ as control parameters. That is why, in this paper Equation (5) will be a priori recognized as a properly description of near-boundary phenomena treated as a result of "cooperation" of the impulses imposed on the average temperature and the material-geometrical structure of the composite.

3. Formulation of the problem

It must be emphasized that Equation (5) includes the damping term $\lambda\langle k\varphi\frac{\partial\varphi}{\partial z}\rangle\frac{dq}{dz}$ does not appear in the original tolerance description of the effect of the boundary effect behavior. Since *Tolerance Model Equations* usually are not an exact but approximate equivalent of the original heat transfer equation for composite materials and the *Extended Tolerance Model Equations* are the exact equivalent (in the class of solutions given by appropriate Fourier basis) of these equations the question arises whether skipping in the equation (5) of component $\lambda\langle k\varphi\frac{\partial\varphi}{\partial z}\rangle\frac{dq}{dz}$, responsible for an additional damping behavior is justified. Such omission is generally used in the *TAT* approach. In this paper we illustrate this problem by two basic examples regarding one-way *FGM*-periodicity.

4. Boundary Effect Equation for odd boundary single impulses

Let $j=j(\xi)$ be an arbitrary periodic and even real function, $j(\xi)=j(-\xi)$. The basic period of j is taken as 2. Hence $j(\xi+2)=j(\xi)$ and if $j(\xi+\xi_1)=j(\xi)$ for $0<\xi_1<2$ then $\xi_1=2$. Function j will be identified as *generating function.* Now we are to consider $\phi(y)=\lambda\omega(\frac{y}{\lambda})$ as the *p-th j-based odd impulse* iff $\omega(\xi,z)=j((2p-1)v(\xi,z))$ for:

$$v(\xi,z)=\begin{cases}\frac{\xi}{\eta(z)}+1 & for \quad -\eta(z)\le\xi\le 0\\ \frac{\xi}{1-\eta(z)}-1 & for \quad 0\le\xi\le 1-\eta(z)\end{cases}\tag{6}$$

as a single Fourier fluctuation φ in (4). Note that:

$$\frac{\partial\varphi}{\partial z}(\frac{y}{\lambda},z)=y\omega'(v(y,z))\frac{\partial v}{\partial\xi}(\frac{y}{\lambda},z)\eta'(z)=y\frac{\partial\varphi}{\partial y}(y,z)\eta'(z)\tag{7}$$

and under:

$$\frac{\partial v}{\partial\xi}(\frac{y}{\lambda},z)=\frac{\chi_{(-\eta\lambda,0]}(y,z)}{\eta}+\frac{\chi_{(0,(1-\eta)\lambda)}(y,z)}{1-\eta}\tag{8}$$

where χ_A stands for the characteristic function of $A\subset R$, we conclude that:

$$\frac{\partial\varphi}{\partial z}(\frac{y}{\lambda},z)=y[\frac{\chi_{(-\eta\lambda,0]}(y,z)}{\eta}-\frac{\chi_{(0,(1-\eta)\lambda)}(y,z)}{1-\eta}]\frac{\partial\varphi}{\partial y}(y,z)\eta'(z)\tag{9}$$

Finally:

$$\langle k\varphi\frac{\partial\varphi}{\partial z}\rangle = 0.5(k^B\langle y\frac{\partial(\varphi^2)}{\partial y}\rangle_B - k^W\langle y\frac{\partial(\varphi^2)}{\partial y}\rangle_W)\eta'(z)$$
$$= 0.5\eta'(z)(k^B - k^W)\langle y\frac{\partial(\varphi^2)}{\partial y}\rangle_W \neq 0 \tag{10}$$

where $\langle\cdot\rangle_B$ and $\langle\cdot\rangle_W$ are integral mean values taken over repetitive cell parts occupied by *BLACK* and *WHITE* composite component, respectively. Hence Boundary Effect Equation (5) reduces to the form:

$$\lambda^2(\langle c\varphi^2\rangle\dot{q} - \frac{d}{dz}(\langle k\varphi^2\rangle\frac{dq}{dz}) - 0.5\eta'(z)(k^B - k^W)\langle y\frac{\partial(\varphi^2)}{\partial y}\rangle_W\frac{dq}{dz} + \langle k(\frac{\partial\varphi}{\partial y})^2\rangle q = 0 \tag{11}$$

including not vanish damping term and hence different from counterparts of (11) investigated in [12, 10, 15] as well as a counterparts investigated in the framework linear elasticity in [11, 13].

5. Boundary Effect Equation for even boundary single impulses

Let:

$$\alpha \equiv \frac{1}{\eta+1} \tag{12}$$

Now we are to consider the *p-th j-based odd impulse*, $p \in R$:

$$f(\xi) = \begin{cases} \frac{1}{2}[1 - \frac{1}{2(1+\eta)} - \frac{1}{2(1+\eta)}\cos[2p(\frac{\xi}{\eta}+1)] & \text{for} \quad -\eta \le \xi \le 0 \\ \frac{1}{2}[1 - \frac{1}{2(1+\eta)} - \frac{1}{2(1+\eta)}\cos[2p(\frac{\xi}{\eta}+1)]|_{\bar{\xi}=0} & \text{for} \quad 0 \le \xi \le 1-\eta \end{cases} \tag{13}$$

supported on the *BLACK* material regions as a single Fourier fluctuation φ in (4). Note that:

$$\frac{\partial\varphi}{\partial z}(\frac{y}{\lambda}, z) = \frac{y}{\lambda}\eta'(z)\frac{\chi_{(-\eta\lambda,0]}(y,z)}{\eta}\frac{\partial\varphi}{\partial y}(y,z) \tag{14}$$

Finally:

$$\langle k\varphi\frac{\partial\varphi}{\partial z}\rangle = 0.5\langle ky\frac{\partial(\varphi^2)}{\partial y}\rangle_B\eta'(z) = 0 \tag{15}$$

Hence in Boundary Effect Equation (5) reduces to the form:

$$\lambda^2(\langle c\varphi^2\rangle_B\dot{q} - k^B\frac{d}{dz}(\langle\varphi^2\rangle_B\frac{dq}{dz}) + 0.5\eta'(z)\langle ky\frac{\partial(\varphi^2)}{\partial y}\rangle_B\frac{dq}{dz} + \langle k(\frac{\partial\varphi}{\partial y})^2\rangle_B q = 0 \tag{16}$$

not including damping term and investigated in [11, 16].

6. Longitudinal and transversal gradation of material properties

Following [12] we accept that we deal with composites with longitudinal gradation of material properties if $\frac{\partial\eta}{\partial z} = 0$ and with composites with transversal gradation of material properties if $\frac{\partial\eta}{\partial y} = 0$. For longitudinal linear gradation we assume that

$\eta(y,z) = \eta(y) = \frac{y}{L}$ or $\eta(y,z) = \eta(y) = 1 - \frac{y}{L}$. For transversal linear gradation $\eta(y,z) = \eta(z) = \frac{z}{\delta}$ or $\eta(y,z) = \eta(z) = 1 - \frac{z}{\delta}$ will be assumed. Note that:

$$\langle c\varphi^2\rangle = \frac{1}{2}[\eta k^B + (1-\eta)k^W], \quad \langle k\varphi^2\rangle = \frac{1}{2}[\eta k^B + (1-\eta)k^W] \tag{17}$$

and

$$\langle k(\frac{\partial\varphi}{\partial y})^2\rangle = \frac{(2p-1)^2\pi^2}{2}(\frac{k^B}{\eta} + \frac{k^W}{1-\eta}), \quad \langle k(\frac{\partial\varphi}{\partial y})^2\rangle = 2p^2\pi^2\frac{k^B}{\eta} \tag{18}$$

for odd and for *BLACK* even single impulse, respectively, we conclude that the Bessel-like differential equation:

$$\lambda^2(\langle c\varphi^2\rangle\dot{q} - \frac{d}{dz}(\langle k\varphi^2\rangle\frac{dq}{dz}) + \frac{1}{2}\eta'(z)(k^B - k^W)\langle y\frac{\partial(\varphi^2)}{\partial y}\rangle_B + \langle k(\frac{\partial\varphi}{\partial y})^2\rangle q = 0 \tag{19}$$

in the odd impulse case and:

$$\lambda^2(\langle c\varphi^2\rangle\dot{q} - \frac{d}{dz}(\langle k\varphi^2\rangle\frac{dq}{dz}) + \frac{1}{2}\eta'(z)k^B\langle y\frac{\partial(\varphi^2)}{\partial y}\rangle_B + \langle k(\frac{\partial\varphi}{\partial y})^2\rangle q = 0 \tag{20}$$

in the *BLACK* even impulse case is the mathematical support of the Boundary Effect Equation for single odd and single even impulses. Equation (20) includes the damping term.

7. Multi-impulse boundary effect equation

Allowing the interpretation of Equation (5) mentioned at the end of Section 2. we intend to refer to the considerations made in [16], corresponding to the situation in which $q = [q_1, q_2]$ and $\varphi = [\varphi_1, \varphi_2]^T$ are related to the pair of single odd and single even fluctuations and in which term $\lambda\langle k\varphi\frac{\partial\varphi}{\partial z}\rangle\frac{dq}{dz}$ should not be dropped out. Allowing the interpretation of Equation (5) mentioned at the end of Section 2. we also intend to refer to the considerations made in [8,10,11], corresponding to the situation in which $q = [q_1, q_2]$ and $\varphi = [\varphi_1, \varphi_2]^T$ are related to the pair of even [8,9] and the pair of odd [11] fluctuations. In investigations presented in [10,11] the term $\lambda\langle k\varphi\frac{\partial\varphi}{\partial z}\rangle\frac{dq}{dz}$ also should not be dropped out.

8. Final remarks

In the exemplified examples, with the exception of that indicated in Section 7., the omitting the component $\lambda\langle k\varphi\frac{\partial\varphi}{\partial z}\rangle\frac{dq}{dz}$ in (5) is not justified. This remark is also deal the saw-lake impulse used to set *TAT* approach to the description of *Boundary Effect Behaviors.* Allowing the interpretation of Equation (5) mentioned at the end of Section 2. leading to the *multi-impulse Boundary Effect Equation* control problem indicated in Section 7. for the pair of single odd and single even impulses produce situation in which term $\lambda\langle k\varphi\frac{\partial\varphi}{\partial z}\rangle\frac{dq}{dz}$ should not be dropped out. Control problem signaled in this way is an open mathematics and engineering problem due to the mutual cooperation of even and odd fluctuations during their transport by the composite as two impulses imposed on the average temperature. It is difficult to study, especially in the case of transversal and longitudinal grading of the composite journal leading to the Bessel-type system of two differential equations with constant coefficients. The study of this system is an open mathematical and engineering problem.

The paper contains an answer to the question how to transfer the methodology for describing the damping phenomenon observed while the temperature boundary loadings are superimposed by an additional impulse. The paper takes into account equations of the Extended Tolerance Model of Heat Conduction developed in [15]. This new model of thermal conductivity, being an extension of the related Tolerance Model developed originally by Professor Czesław Woźniak, produces the analytical formula for the error made in the approximate solutions proposed in TAT and hence this new model as well as resulted Boundary Effect Equation can be treated as descriptions equivalent to that given by the parabolic Heat Transfer Equation. Proposed in the paper upgrade of the tolerance Boundary Effect Equation contain an important correction in the form of the additional term responsible for the special damping not taken into account in the previous tolerance approaches.

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