Excercise 14

HARMONIC ANALYSIS OF AIR-COMPRESSOR VIBRATIONS

1. Aim of the exercise

Analysis of a complex signal recorded from the machine and the determination of its elementary components.

2. Theoretical introduction

2.1. Measurement signals

In order to make proper measurements, different signals, met in typical kinds of vibrations, should be known. Two groups of signals can be mentioned, namely:

1. **Deterministic signals** – those whose values are predictable in time, their shpes can be unequivocally described by means of mathematical functions, but this description cannot contain random quantities. Such signals contain:

a) periodic vibrations (for each moment of time *t*, $x(t + T) = x(t)$, where *T* - signal period.):

- harmonic, in the time domain, they can be described by a harmonic function: $x(t) = A\cos(\omega_0 t + \varphi)$, where *A* - signal amplitude (e.g. in mm, m/s, m/s^2), $\omega_0 = 2\pi f_0$ - circular frequency (pulsation), φ - phase shift.
- complex (polyharmonic), are the linear combination of at least two harmonic signals, the frequencies of which must be an integer multiple of a certain fundamental frequency.

b) aperiodic signals (signals for which the periodicity condition is not met):

- almost periodic,
- transient (pulse, i.e., impacts starting and finishing with zero values)
- chaotic performed by deterministic system (deterministic chaos).

2. Random, stochastic signals – they have unpredictable, random values at any time instant. Their properties are described with statistic characteristics, i.e., averaging parameters applied to amplitudes, frequencies and time. Here can be found:

a) stationary signals with statistic characteristics (mean, mean square values) which are not functions of time,

b) non-stationary signals - they are not functions of time.

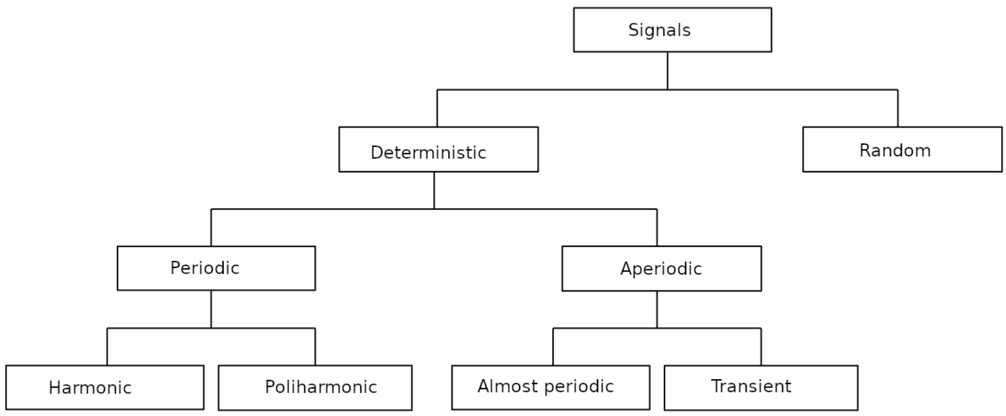

Fig. 1. Classification of signals

As an example of deterministic signals, one can mention vibrations of a gearbox (Fig. 2a) or piston movements in an engine with two frequencies ω and 2 ω (Fig. 2b). Part c) of this figure shows a time history and a power spectrum of the non-continuous signal, i.e., an impact. Typical examples of random vibrations are those resulting from fluid flows, noise, disturbances, and vibrations of a car body moving on a rough road, spatter of rain. They are characterized by random behaviour, have no characteristics of their frequency, and the spectrum seen as a function of frequency is almost evenly distributed.

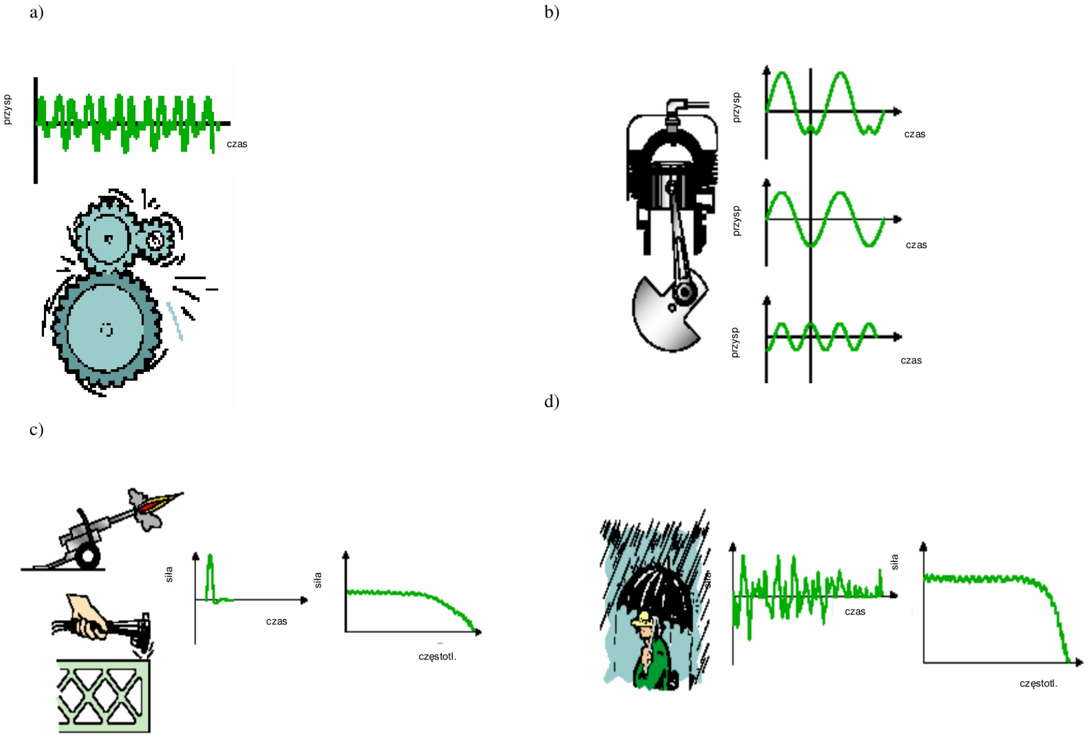

Fig. 2. Examples of signals

If measurements and analysis have to deal with a particular range of frequencies and there are no demands to be fulfilled, say, imposed by standards, then the general rule is to measure the value which has the most flat character as a function of frequency, Fig. 3. This allows us to cover the widest range of signal dynamics of the system under investigation. But in the case when the characteristics are unknown, there is a general suggestion to choose a vibration velocity signal as the basis for analysis. This is important, especially in the case when the characteristic curve is not flat. Then, all components of the level below the average one will have less influence on the final result and, in the case of measurements in the full range of frequencies, the lowest components cannot be detected at all.

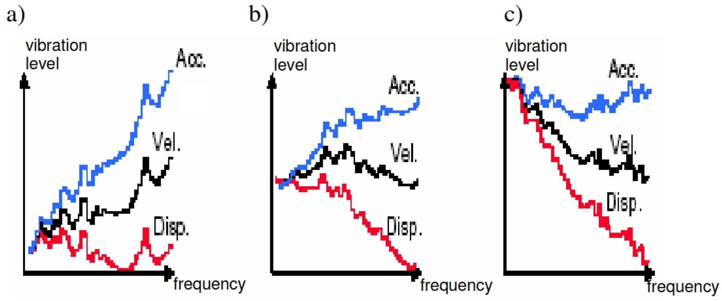

Fig. 3. Choice of parameters of vibrations measured with respect to the power spectrum: a) displacement, b) velocity, and c) acceleration

The demand for flat characteristics means that velocity is to be measured in most cases of machine vibration measurements. In some cases, however, this should be rather acceleration, but for the majority of machines, accelerations of high amplitude are found only at high frequencies. It is rather rare to have flat characteristics of the displacement signal because such high amplitudes take place only at very low frequencies in most real cases. There can be more reasons why we cannot use particular sensors – e.g., a mass of the sensor is too high, compared to the object under investigation, or its measuring range is not wide enough to use it in testing. Analyzing relations between displacement, velocity and acceleration (differentiation or integration), one can see that for a particular level of velocity of vibrations, displacement amplitudes decrease (since they are divided by the ω value) and acceleration amplitudes increase (multiplied by ω) with an increase in frequency – see Fig. 4.

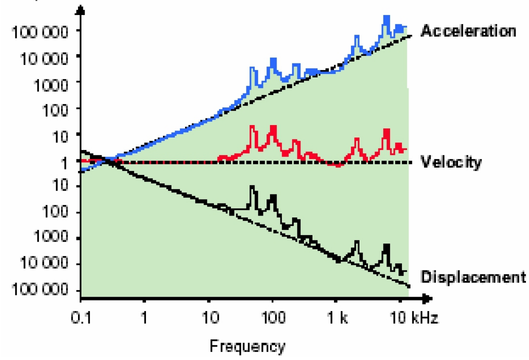

Fig. 4. Example of spectrum characteristics of the vibration signal shown as displacement, velocity and acceleration signals

In some cases of vibration measurements, observed time histories are enough to carry out the analysis. They comprise data to determine the amplitude, frequency (as 1/T) or phase shift of signals. But usually time histories present complex shapes, giving information about the overall level of vibrations only, see Fig. 5.

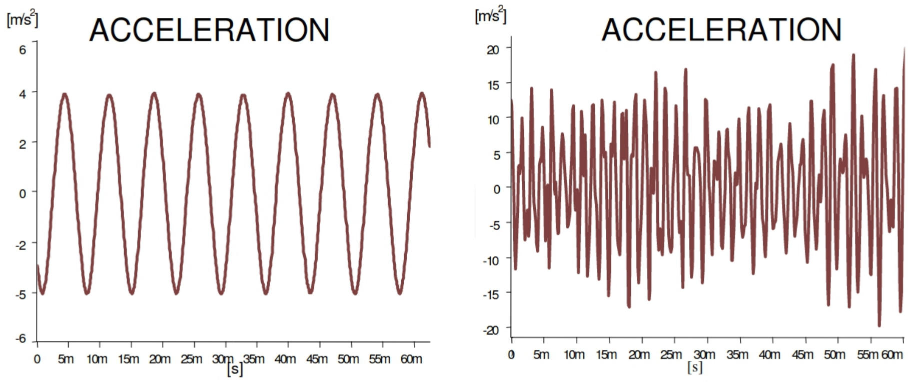

Fig. 5. Example of vibration time histories, time in [ms]

2.2. Spectral analysis of vibrations

In the case when the detailed information about all components of the signal is needed, one has to conduct a power (frequency) analysis of the recorded time history signal. Such an analysis can be done by means of analogue or digital methods. In the first case, power spectrum analysing devices are applied. These include a set of filters with different characteristics of the signals passed or tuned narrow-band filters. In the digital method, the Fast Fourier Transform (FFT) is applied.

A periodic function can be represented with the Fourier series by the following components: a constant part, a_0, and harmonic parts of the frequencies ω_1, $2\omega_1$, ..., $n\omega_1$, where ω_1 is the basic frequency and the terms $n\omega_1$ are harmonic frequencies, whereas n is a natural number. The basic frequency is described as:

$$\omega_1 = 2\,\pi/T$$

where T denotes the period of the function. The complete equation describing all components of a periodic function $x(t)$ with the Fourier series terms has the following form:

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\omega_1 t + b_n \sin n\omega_1 t\right).$$

Let us consider a rectangular wave as an example – it can be represented by an infinite trigonometric series of odd harmonics (1, 3, 5, 7, ...) with diminishing amplitudes, see Fig. 14.5. This is a representation in the time domain.

$$x(t) = \frac{4A}{\pi}\left(\sin\omega_1 t + \frac{1}{3}\sin 3\omega_1 t + \frac{1}{5}\sin 5\omega_1 t + \frac{1}{7}\sin 7\omega_1 t + \ldots\right)$$

Periodic signals can be represented in a graphical form in the frequency domain. The horizontal axis presents the frequency values f [Hz] (or $\omega = 2\pi f$ measured in [rad/s]), whereas the vertical one shows the amplitudes measured in the same way as in time histories or using some relative values. The height of a component is proportional to the values of amplitude of the particular harmonic term, Fig. 6. Such a graph is a power spectrum graph or an amplitude graph. Spectra of periodic signals have a discrete form of separated bars, while non-periodic signals, e.g., pulse or stochastic, have continuous forms, see Figs. 14 and 15.

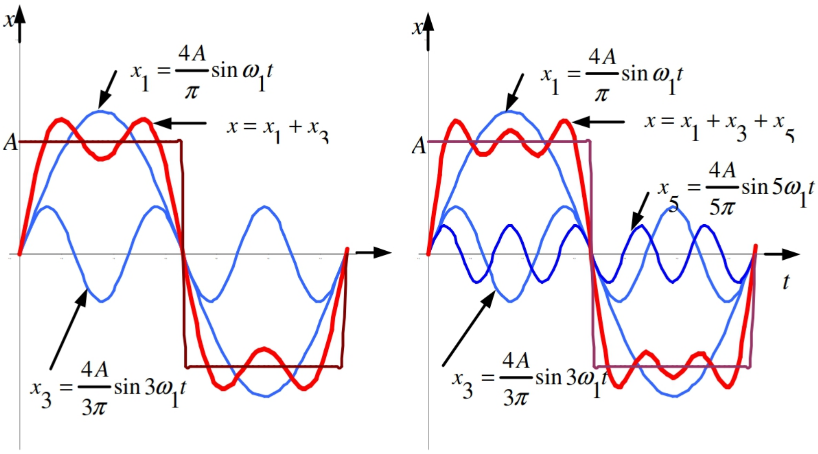

Fig. 6. Approximation of the rectangular wave with a limited number of harmonics

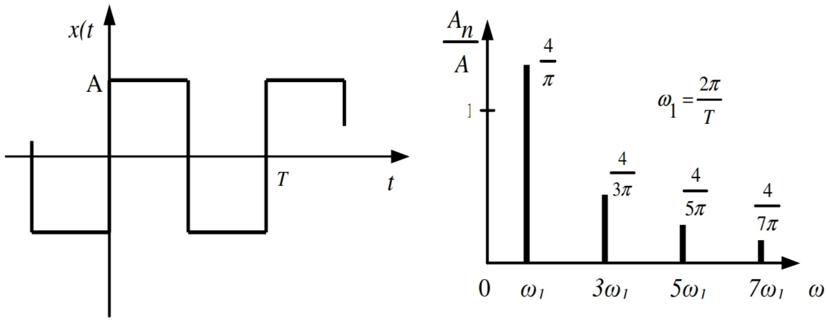

Fig. 7. Power spectrum of the rectangular signal

Variable forces with different frequencies act on the tested object during its operation:

$$P_1 \sin(\omega_1 t + \lnot_1),\ P_1 \sin(\omega_2 t + \lnot_2),\ \dots\ ,P_k \sin(\omega_k t + \lnot_k)$$

and as a result of their operation, the course of object vibrations is complex. In order to determine the origin of the exciting forces and their influence on the object vibrations, the vibration signals x (t) obtained from the vibration sensor should be broken down into harmonic components:

$$x(t) = \sum_{i=1}^{k} A_i \sin(\omega_i t + \beta_i)$$

For this purpose, the spectral (frequency) analysis of the vibration time signals obtained from the measurements of the vibration time signals should be performed. Signal analysis can be analog, digital or mixed.

2.2.1. Filters - band passing, analogue analysis

Analogue signal processing can be performed with the help of electronic spectrum analyzers. This can be a bank of filters with different transmission frequencies or a tunable narrowband filter. In analog analyzers, electric band-pass filters are most often used. These filters pass signal components whose frequencies are in the pass-band B of the filter (Fig. 8).

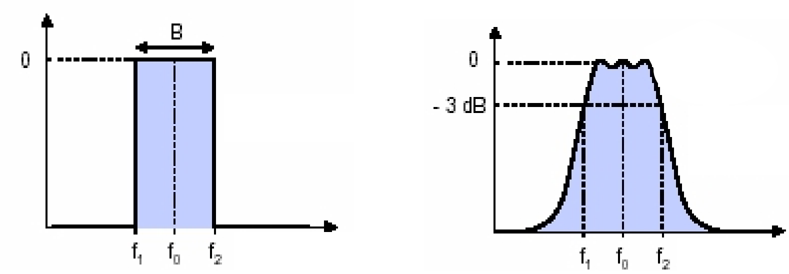

Fig. 8. Characteristics of a) ideal and b) real filters

The figure shows the characteristics of an ideal and real band-pass filter. An ideal band-pass filter should have an attenuation equal to zero in the pass-band and infinitely great beyond that pass-band, so the ideal filter characteristic is rectangular. Filter attenuation is given in decibels as:

$$N[dB]=10\log\left(\frac{U_{wy}^2}{U_{we}^2}\right)=20\log\left(\frac{U_{wy}}{U_{we}}\right)$$

where U_{we} (input) and U_{wy} (output) are input and output filter signals. Real filters characteristics are said to be correct if they are flat in the mid section of the passing band and edge slopes are steep. Band pass filters are characterized by their mid-frequency f_0 and the band width $B = f_2 - f_1$, which is the difference of its limit frequencies, upper and lower, at which damping of the signal diminishes by –3dB, the signal power halves:

$$\frac{U_{wy}^2}{U_{we}^2}=\frac{1}{2}$$

and the amplification factor changes from k=1 to k=1/√2 in comparison with the average level in the passing band, see Fig. 10.

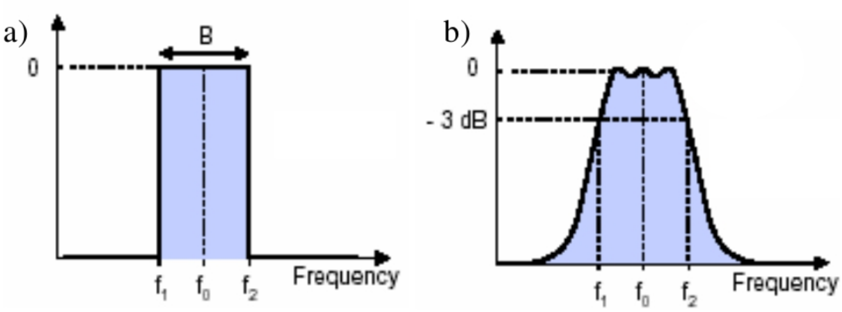

Fig. 9. Characteristics of a) ideal and b) real filters

Then the signal attenuation is

$$N[dB]=10\log\frac{1}{2}\approx-3\,dB$$

and **amplification**:

$$k=\frac{U_{wy}}{U_{we}}$$

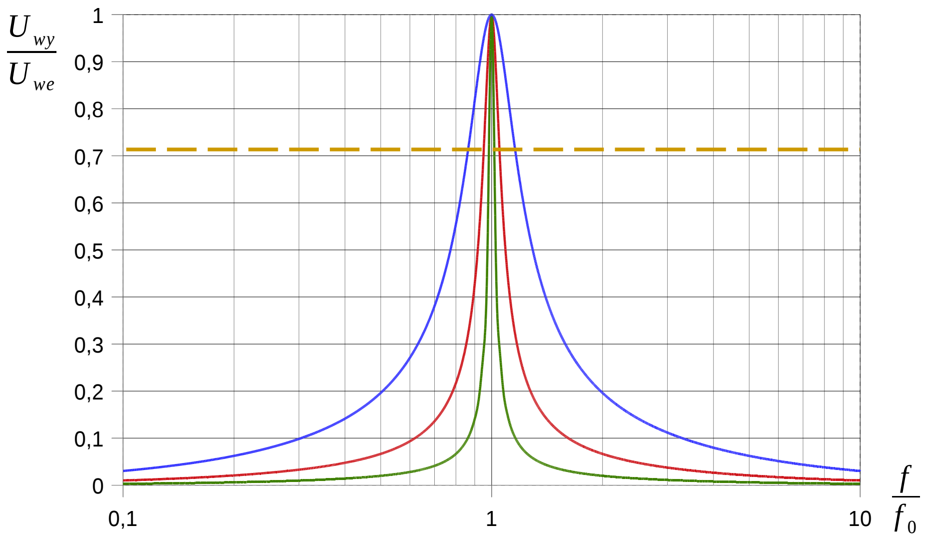

Fig. 10. Narrowband filter characteristics of 3%, 10% and 30% width.

Two types of filters are used for the frequency analysis of vibration signals:

- filters with constant absolute width of the pass-band as 3Hz, 100Hz etc.
- filters width constant percentage width of the pass-band related to its center value f_0, as 3%, 10%, 30%, Fig. 10.

The center frequency of the band and the cutoff frequencies of these filters are related to:

$$f_0 = \sqrt{f_1 \cdot f_2}$$

The narrower the filter bandwidth, the more detailed the information can be obtained from the waveform being analyzed, but the longer the analysis time.

2.2.2. Digital analysis

Fourier series

A periodic waveform that can be physically realized (satisfies the Dirichlet conditions) can be represented in the form of a Fourier series consisting in the general case of a constant component and the sum of trigonometric functions with different pulsations:

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_1 t + b_n \sin n\omega_1 t)$$

a_0 – constant,
a_n , b_n – Fourier series coefficients, possible to be determined in analytical way,
ω_1 – basic frequency, $\omega_1 = 2\pi f = 2\pi/T$, where T – period of the time function
n – natural number

Let us consider a periodic signal in the form of a square wave (Fig. 4). After determining the coefficients of the Fourier series, it can be represented by an infinite trigonometric series of odd harmonics (1, 3, 5, 7, ...), with decreasing amplitudes:

$$x(t) = \frac{4A}{\pi}\left(\sin \omega_1 t + \frac{1}{3}\sin 3\omega_1 t + \frac{1}{5}\sin 5\omega_1 t + \frac{1}{7}\sin 7\omega_1 t + \dots\right)$$

This is a representation of a signal in the time domain. Figure 11 shows the first three components of the above equation and their sum.

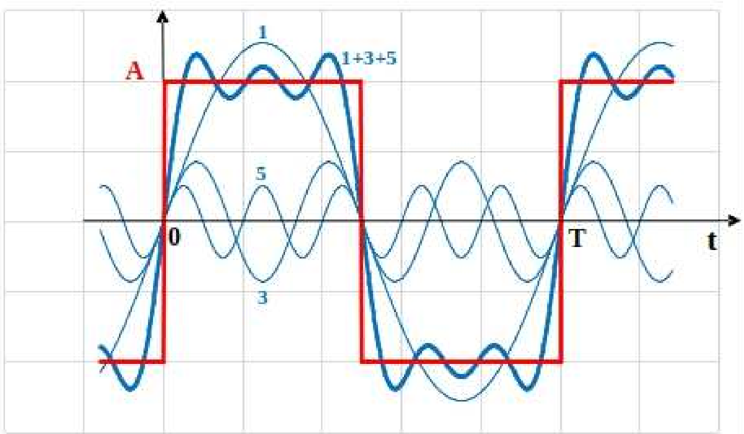

Fig. 11. Square wave approximation using the first three components of the Fourier series (first, third and fifth harmonics)

Examples of approximation of various signals by means of a finite number of components can be found at: https://en.wikipedia.org/wiki/Fourier_series

Periodic signals can also be plotted in the frequency domain. The frequency f or pulsation $\omega = 2\pi f$ is assumed on the abscissa. In contrast, on the ordinate of amplitude or amplitude ratios in the form of fringes (Fig. 12). The length of the fringes is proportional to the amplitudes of the respective harmonics in the analyzed signal. Such a plot is called an amplitude spectrum or a frequency spectrum.
The spectra of periodic signals are discrete, while the spectra of non-periodic signals (e.g. impulse or stochastic signal) - continuous (Fig. 14 and 15).

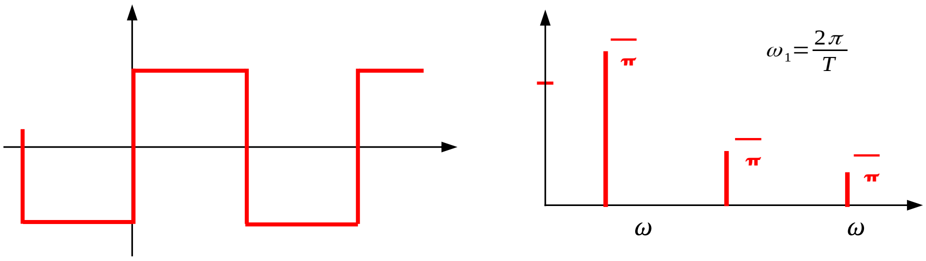

Fig. 12. Amplitude spectrum of rectangular signal

Fourier transform

The Fourier transform is most often used to process digital signals, it is the basic tool in the analysis of stationary signals. The result of the Fourier transform is a function called the Fourier transform. The Fourier transform decomposes a signal (e.g. a vibration signal) into sinusoidal components of different frequencies

$$F(f)=\int_{-\infty}^{\infty} f(t)e^{-j2\pi ft}dt$$

Optimized, computer version of the Fourier transform, the so-called Fast Fourier Transform (FFT) allows for quick and accurate evaluation of harmonic components of a given signal. As a result, a set of data is obtained in the form of frequency values and the corresponding amplitudes in the frequency range selected for the analysis.
This result can be presented on a graph, where the ordinate axis usually denotes the amplitudes of individual components (harmonics), while the abscissa axis gives their frequencies. The Fourier transform thus serves to convert a time-dependent function to a frequency-dependent function, i.e. it enables a transition from the time domain to the frequency domain.

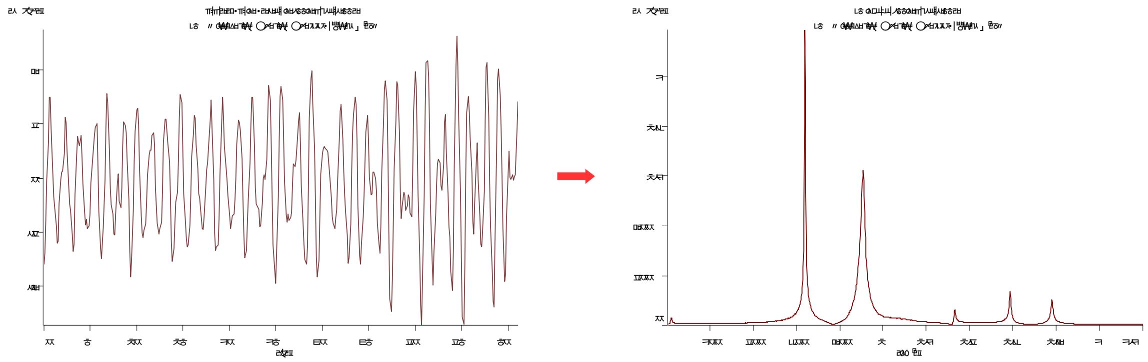

Fig. 13. An example of the Fourier transformation of the vibration acceleration signal - transition from the time domain to the frequency domain

Fourier transforms have many applications - mainly signal processing, but also image processing, sound compression in MP3 format, solving important partial differential equations (Poisson equation), or numerical analysis of FEM (some dynamic problems require switching between the time and frequency domains).
For the pulse signal (Fig. 7), the amplitude spectrum is continuous. The theoretical pulse $\delta(t)$ (Dirac function) contains signals of all frequencies from $-\infty$ to $+\infty$, with the same amplitude equal to 1. At the time $t = 0$ all spectral components are in the same phase. It is this concentration that makes it possible to create an impulse [1]. A short-term impulse caused, for example, by an impact of the so-called measuring hammer (with a built-in force sensor) contains components of the same amplitude in a wide frequency

range. So the system is forced by all frequencies, which allows you to obtain the so-called dynamic characteristics (stiffness or dynamic susceptibility) with an FFT analyzer.

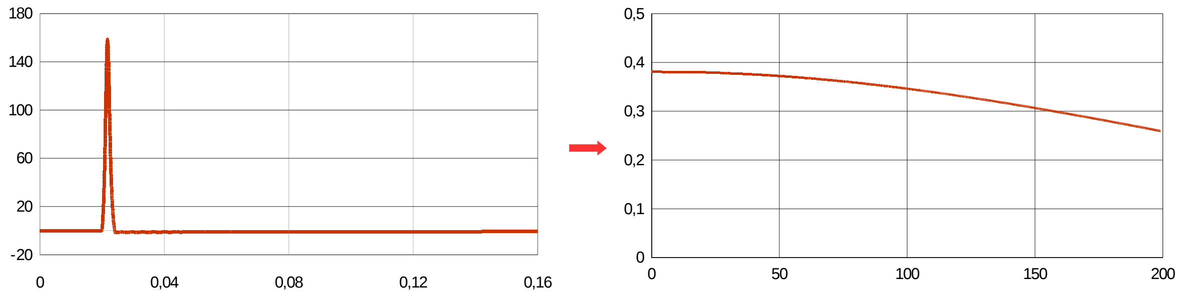

Fig. 14. Time wave and amplitude spectrum of a pulse signal.

A stochastic signal is a signal whose values are random variables at any moment. Of great practical importance is a completely disordered stochastic signal containing all frequencies of the same amplitude. Its energy is evenly distributed over the entire frequency band. By analogy to optical spectra, it is called white noise. If we supply an exciter with a signal from the noise generator (which gives a force proportional to the input signal), we force the tested object with a force containing all frequencies (Fig. 15), which allows to determine the dynamic properties of the tested object.

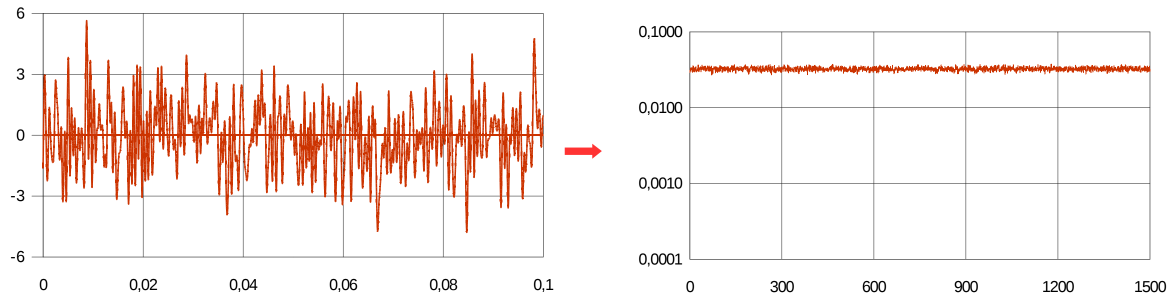

Fig. 15. Time wave and amplitude spectrum of a stochastic signal

3. Experimental rig

One of the objects whose vibrations will be analyzed is a reciprocating compressor. Compressor vibrations during normal operation have a complex course. This is the result of the superimposition of the effects of several forcing forces of different frequencies related to the operation of the device. The analysis of the vibration signal from the measuring sensor will be carried out in the exercise in two ways:
Direct analysis of the course of vibrations of the tested object using the analog narrow-band analyzer SM32 located in the vibration meter SM231 by RFT.
Computer analysis with the use of Fast Fourier transform, with the use of the PULSE measurement system (Bruel & Kjær) or SCADA system (LMS).

4. Experiment

A. Analysis of vibrations of the tested object using an analog narrow-band analyzer.

1. Select the appropriate sensor (piezoelectric, electrodynamic, transformer or laser) depending on the tested object and the conditions of vibration measurement.
2. Place the vibration sensor on the tested object.
3. Set the switch of the type of the measured quantity to the position a, v or ζ (acceleration, velocity or displacement).
4. Set the highest value with the signal level range switch of SM231 meter.

5. Set the BAND WIDTH switch of the SM32 analyzer to the LIN position (the entire frequency spectrum).
6. Connect the vibration meter to the network, start the tested object.
7. While changing the range of the level meter, lead to the deflection of the meter pointer over 1/3 of the range. Make a note of the meter readings.
8. To perform the analysis, set the BAND WIDTH switch on the analyzer to 30% and tune the frequency f0 with the measuring potentiometer until the pointer deflects clearly. Record the levels of the spectrum components of the tested vibrations and the frequencies at which they occur. For precise frequency determination, change the bandwidth to 3%.

B. Spectral analysis of periodic signals, white noise signal and pulse signal.
1. Bring the periodic signals (sine, square, triangle) and the white noise signal from the function generator to the FFT analyzer. Record the waveforms and spectra of these signals.
2. Record the waveform and spectrum of the pulse signal from the hammer.

C. Vibration analysis using the Fast Fourier transform.
1. Place the vibration sensor on the tested object.
2. Start the appropriate calculation program in the PULSE or SCADA system.
3. Start the tested object.
4. Perform vibration analysis, record the results.

5. Literature

1. Hagel R., Zakrzewski J.: Miernictwo dynamiczne, WNT, Warszawa 1984.
2. Otnes R.K., Enochson L.: Analiza numeryczna szeregów czasowych, WNT, Warszawa 1978.
3. Bruel & Kjær company data sheets.

6. Report
1. Enter the results of the measurements from the analog analyzer into the report.
2. Print the recorded waveforms from the digital analyzer.
3. Compare the results from the analog and digital analyzer.
4. On the basis of the conducted tests, determine the sources of vibrations, i.e. the origin of the exciting forces.
5. Compare both methods of vibration analysis.