Excercise 2

EXPERIMENTAL ESTIMATION OF THE MOMENT OF INERTIA OF A MACHINE PART BY MEANS OF THE PENDULUM METHOD

Aim of the exercise

Determine experimentally the moment of inertia of a connecting rod with respect to the axis parallel to its vertical axis and passing through the mass centre (Part I). Determine the moment of inertia of a crankshaft by means of the string support method and check the shear modulus of the string material (Part II).

Part I

Determination of the location of the center of mass and the moment of inertia of the connecting rod.

The connecting rod supported in the point A other than B (Fig. 1) can be considered as a physical pendulum and its motion can be described as:

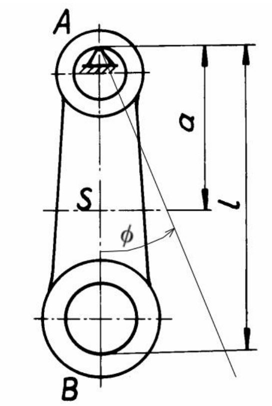

Figure 1 Connecting rod

Equation of motion of such physical pendulm is:

$$B_A \frac{d^2\varphi}{dt^2} = -mga\varphi \qquad (1)$$

where:

B_A – moment of inertia with respect to the point of support A,

m – mass of the connecting rod,

g - gravity,

a – distance of the mass centre from the support axis

φ – angle of rotation.

Find, why such equation is used, where are simplifications and how it has been solved.

The solution of Eq. (1) is as follows:

$$\varphi = \Phi \sin\left(\frac{2\pi}{T_A} + \beta\right) \tag{2}$$

where:

$$T_A = \frac{2\pi}{\sqrt{\frac{mga}{B_A}}} \tag{3}$$

is the period of free vibrations of the rod supported in the point A.

Do you remember how this formula has been obtained?

The connecting rod supported in the point B (Fig. 2b) can also be considered as a physical pendulum and its motion can be described as:

$$B_B \frac{d^2\varphi}{dt^2} + mg(l-a)\varphi \tag{4}$$

where:

*B*B is moment of inertia with respect to the point of support B, and l *is* distance between both axes of rotation.

Period of free oscillations is now:

$$T_B = 2\pi\sqrt{\frac{B_B}{mg(l-a)}} \tag{5}$$

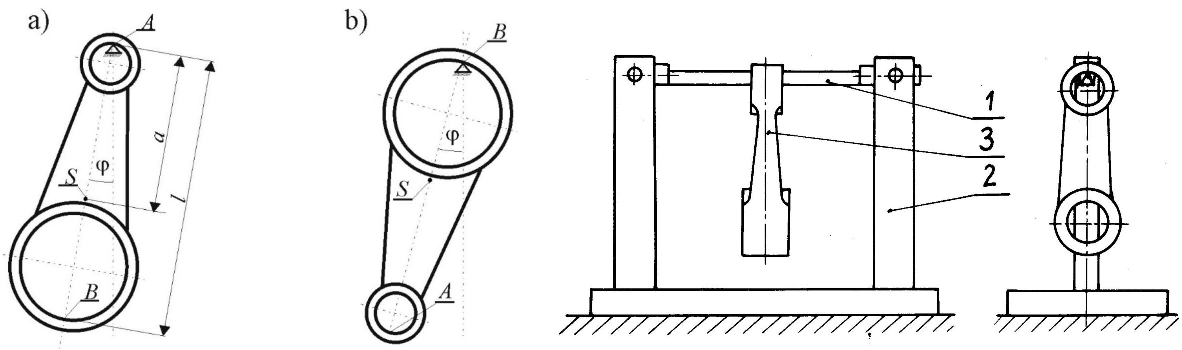

Figure 2 Different support points and rig view

When we apply the Steiner theorem, the moment of inertia B_A can be determined using the B_S value (moment of inertia of the rod through the centre of its mass and parallel to the axis through the point A) as:

$$B_A = B_S + ma^2$$

$$B_B = B_S + m(l-a)^2 \tag{6}$$

Where B_S is mass moment of inertia related to the element mass center of gravity CG.

Using (6) in (3) and (5) we get the following set of equations:

$$T_A = 2\pi\sqrt{\frac{B_S + ma^2}{mga}}$$

$$T_B = 2\pi\sqrt{\frac{B_S + m(l-a)^2}{mg(l-a)}} \tag{7}$$

Now we can determine the unknown values of a and B_S.

$$a = \frac{glT_B^2 - 4\pi^2 l^2}{g\left(T_A^2 + T_B^2\right) - 8\pi^2 l}$$

$$B_S = \frac{mgaT_A^2}{4\pi^2} - ma^2 \qquad (8)$$

How to perform the practical test:

Place the rod to allow oscillations in the point A. Take care of keeping the axis of holes parallel to the prism edge.

Assume: *m,* mass of the connecting rod equal to 1.850 kg, and *l* – distance between the points of support A and B to be 0.270 m.

Allow the rod to oscillate with amplitude lower than 10°, and then measure the time of 50 oscillations three times. These results should be recorded in the report sheet and the mass centre and the moment of inertia are to be determined with the above-mentioned equations.

Repeat the procedure supporting the rod in the point B.

Part II

Aim

Determination of mass moment of inertia of a crankshaft according to its rotation axis and shear modulus of steel spring *G* which acts here as torsional beam.

The crankshaft is supported on a string (Fig. 2.2) and oscillates torsionally. We cannot support it in two ways to make it oscillate along two different axes. Then, we use a supporting string that creates the system a form of torsion physical pendulum.

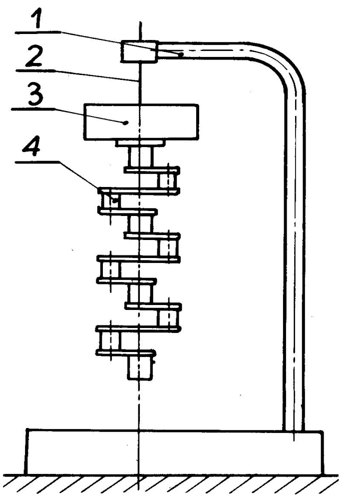

Figure 3 Crankshaft at the experimental rig: 1 - frame, 2 - elastic torsional string of length *l* and diameter *d*, 3 - flywheel, 4 - crankshaft

Here:

$B_c = B + B_o$, B_o – mount moment of inertia,

B – moment of inertia of the element under investigation,

– angle of the string torsion,

G – shear modulus (rigidity of the string), *M* – reaction moment of the string torsion,

$I_0 = \pi d^4/32$ – polar moment of inertia of the string,

d – string diameter,

l – string length.

It can be modelled as

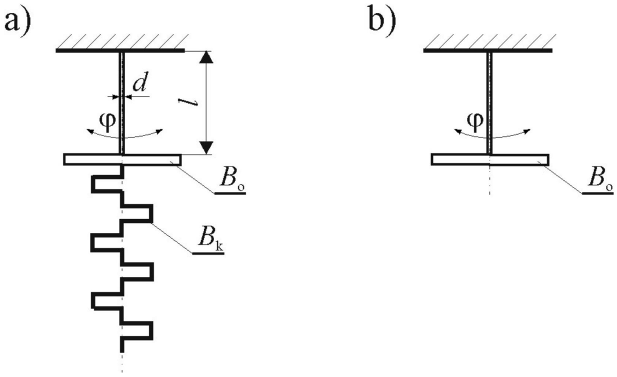

Figure 4 Models of two rig arrangements

Its motion is described by the following equation:

$$(B_K + B_0)\frac{d^2\varphi}{dt^2} = -\frac{GI_0}{l}\varphi \qquad (9)$$

Here, $\frac{GI_0}{l} = k_t$ is elasticity of the torsional beam.

Period of free oscillations is:

$$T_K = 2\pi\sqrt{\frac{B_B + B_0}{k_t}} \qquad (10)$$

When the (heavy) flywheel is supported at the rig alone it can be also treated as torsional pendulum. Now, equation of motion is:

$$\mathrm{B}_0\frac{d^2\varphi}{dt^2} = -\frac{GI_0}{l}\varphi \qquad (11)$$

And period of free oscillations is:

$$T_0 = 2\pi\sqrt{\frac{B_0}{k_t}} \qquad (12)$$

We should recall well known formula for stiffness of torsional beam:

$$k_t = \frac{GI_0}{l} = \frac{G\pi d^4}{32l} \qquad (13)$$

Using (10), (12) and (13) we can determine unknown values of:

$$G = \frac{4\pi^2 B_0 l}{T_0^2 I_0}$$

and

$$B_K = B_0\left(\frac{T_K^2}{T_0^2} - 1\right)$$

How to perform the practical part:

Assume: l – length of the supporting string equal to 0.590 m, and d – string diameter 0.005 m.

Fix the investigated element in the mount in such a way that its axis is in line with the string axis.

Start torsional oscillations with angular amplitude lower than 10°, then measure the time of 20 periods of oscillations (3 times) and determine its average value and the period T_K.

Dismount the device from the flywheel, then measure the time of 20 periods' oscillations of the flywheel alone (3 times) and determine its average value and the period T_0.

Calculate the value of B_K and G, and then check how the G values relates to steel property data.

Assume value of flywheel mass moment of inertia as B_0=0.0707 kgm^2.

Your report should contain:

√ Aim of the exercise.
√ Results of the experimental investigations.
√ Calculation results.
√ Conclusions and remarks (compare numerical values of both obtained mass moments if inertia).

References

1. Rao S.S.: Mechanical Vibrations, Prentice Hall, NY, 1995.
2. Tse F.S., Morse I.E., Hinkle R.T.: Mechanical Vibrations - Theory and Applications, Allyn and Bacon Inc., 1978.
3. Kapitaniak T.: Wstęp do teorii drgań. Wydawnictwo Politechniki Łódzkiej, Łódź 1992.
4. Parszewski Z.: Drgania i dynamika maszyn. PWN, Warszawa 1982.