Exercise 5

DYNAMIC ABSORBER

1. Aim of the exercise

The aim of the experiment is to reduce vibrations at particular (resonance) frequency of a one-degree-of-freedom system by means of a dynamic absorber and to find the range of effectiveness of the absorber.

2. Theoretical introduction

A model of the vibrating system with two degrees of freedom is shown in Fig. 5.1. The differential equations of motion for the masses *M* and *m* are:

$$
\begin{aligned}
M\ddot{x} &= -k_1 x - k_t(x-y) - c\dot{x} - k_2(x - e\sin\omega t) \\
m_t\ddot{y} &= -k_t(x-y)
\end{aligned}
\tag{5.1}
$$

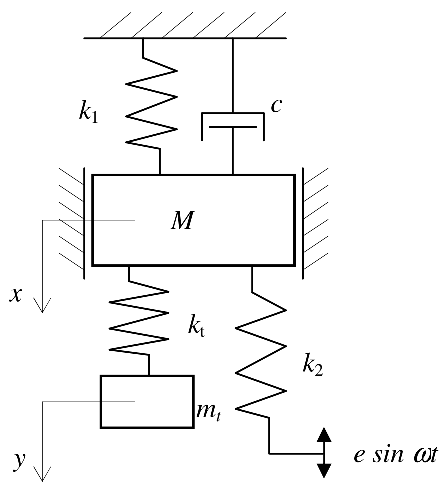


Fig. 5.1. Model of the system

Equations (5.1) can be reduced to:

$$
\begin{aligned}
M\ddot{x} + c\dot{x} + (k_1 + k_2 + k_t)x - k_t y &= k_2 e \sin\omega t \\
m_t\ddot{y} + k_t y - k_t x &= 0.
\end{aligned}
\tag{5.2}
$$

After dividing by *M*, we obtain:

$$
\begin{aligned}
\ddot{x} + \frac{c}{M}\dot{x} + \frac{k_1 + k_2 + k_t}{M}x - \frac{k_t}{M}y &= \frac{k_2 e}{M}\sin\omega t \\
\ddot{y} + \frac{k_t}{m_t}y - \frac{k_t}{m_t}x &= 0
\end{aligned}
\tag{5.3}
$$

The substitution of the following notations:

$$
2h = \frac{c}{M};\ a = \frac{k_1 + k_2 + k_t}{M};\ b = \frac{k_t}{M};\ q = \frac{k_2 e}{M};\ d = \frac{k_t}{M},
\tag{5.4}
$$

gives:

$$\ddot{x} + 2h\dot{x} + ax - by = q\sin\omega t$$
$$\ddot{y} + dy - dx = 0 \tag{5.5}$$

The solutions to the above system of equations are sought in forms:

$$x = A\sin\omega t + B\cos\omega t$$
$$y = C\sin\omega t + D\cos\omega t \tag{5.6}$$

Substituting (5.6) in (5.5), one receives:

$$\begin{aligned} -A\omega^2\sin\omega t - B\omega^2\cos\omega t + 2h(A\omega\cos\omega t - B\omega\sin\omega t) + \\ aA\sin\omega t + aB\cos\omega t - bC\sin\omega t - bD\cos\omega t = q\sin\omega t \\ -C\omega^2\sin\omega t - D\omega^2\cos\omega t + dC\sin\omega t + dD\cos\omega t - \\ dA\sin\omega t - dB\cos\omega t = 0 \end{aligned} \tag{5.7}$$

To fulfil these equations at any time instant, one has to compare the terms with *sin* ωt and *cos* ωt. Then, the following system of 4 equations is obtained:

$$\begin{aligned} (a-\omega^2)A - 2h\omega B - bC + 0D &= q \\ 2h\omega A + (a-\omega^2)B + 0C - bD &= 0 \\ -dA + 0B + (d-\omega^2)C + 0D &= 0 \\ 0A - dB + 0C + (d-\omega^2)D &= 0 \end{aligned} \tag{5.8}$$

To obtain the values of *A*, *B*, *C*, *D*, the determinant *W* is calculated:

$$W = \begin{vmatrix} (a-\omega^2) & -2h\omega & -b & 0 \\ 2h\omega & (a-\omega^2) & 0, & -b \\ -d & 0 & (d-\omega^2) & 0 \\ 0 & -d & 0 & (d-\omega^2) \end{vmatrix} \tag{5.9}$$

In the developed form, we get:

$$W = \left[(a-\omega^2)(d-\omega^2) - bd^2\right]^2 + 4h^2\omega^2(d-\omega^2)^2 \tag{5.10}$$

Additionally:

$$W_B = \begin{vmatrix} (a-\omega^2) & q & -b & 0 \\ 2h\omega, & 0 & 0 & -b \\ -d & 0 & (d-\omega^2) & 0 \\ 0 & 0 & 0 & (d-\omega^2) \end{vmatrix} = -2qh\omega(d-\omega^2)^2 \tag{5.11}$$

Next, the components of the amplitudes of the main mass *M* are expressed as:

$$A = \frac{W_A}{W}; \quad B = \frac{W_B}{W} \tag{5.12}$$

and:

$$A_1 = \sqrt{A^2 + B^2} = \frac{\sqrt{W_A{}^2 + W_B{}^2}}{W} \tag{5.13}$$

Using (5.11) and (5.12) in (5.13), we have:

$$A_1 = \frac{q\sqrt{\left(d-\omega^2\right)^2}\sqrt{\left[\left(a-\omega^2\right)\left(d-\omega^2\right)-bd\right]^2 + 4h^2\omega^2\left(d-\omega^2\right)^2}}{\left[\left(a-\omega^2\right)\left(d-\omega^2\right)-bd\right]^2 + 4h^2\omega^2\left(d-\omega^2\right)^2} \tag{5.14}$$

and:

$$A_1 = q\frac{\left| d-\omega^2 \right|}{\sqrt{\left[\left(a-\omega^2\right)\left(d-\omega^2\right)-bd\right]^2 + 4h^2\omega^2\left(d-\omega^2\right)^2}} \tag{5.15}$$

If we assume that there is no damper (i.e., $b = d = 0$ and $a = \alpha^2$), it results in formula:

$$A_0 = q\frac{\left| -\omega^2 \right|}{\sqrt{(\omega_N^2 - \omega^2)^2\omega^4 + 4h^2\omega_N^2\omega^4}} = \frac{q}{\sqrt{(\omega_N^2 - \omega^2)^2 + 4h^2\omega^2}} \tag{5.16}$$

and the phase angle between excitation and the resulting motion of the mass M is defined as:

$$\varphi_1 = \tan^{-1}\frac{B}{A} = \frac{-2qh\omega\left(d-\omega^2\right)^2}{q\left(d-\omega^2\right)\left[\left(a-\omega^2\right)\left(d-\omega^2\right)-bd\right]} = $$
$$= \frac{-2h\omega\left(d-\omega^2\right)}{\left(a-\omega^2\right)\left(d-\omega^2\right)-bd} \tag{5.17}$$

2.1. Example of the analytic determination of the amplitudes A_1 and the phase angle shift φ_1 for certain parameters of the system

Assuming the following values of the parameters:

$$4\times 2h = \frac{a}{2} = b = d = q = 1 \tag{a}$$

we obtain:

$$A_1 = \frac{\left| 1-\omega^2 \right|}{\sqrt{\left(2-\omega^2\right)\left(1-\omega^2\right) - 1^2 + \frac{10^2}{16}\left(1-\omega^2\right)^2}} \tag{b}$$

$$\varphi_1 = \tan^{-1}\frac{-\frac{\omega}{4}\left(1-\omega^2\right)}{\left(2-\omega^2\right)\left(1-\omega^2\right)-1} \tag{c}$$

For the main system without an absorber:

$$A_0 = \frac{1}{\sqrt{\left(1-\omega^2\right)^2 + \frac{\omega^2}{16}}}; \quad \varphi_0 = \arctan \frac{-\omega}{4\left(1-\omega^2\right)} \tag{d}$$

A dependence of the amplitudes A_1 and A_0 on the excitation frequency is shown in Fig. 5.2. The marked area there represents the effectiveness of the dynamic absorber.

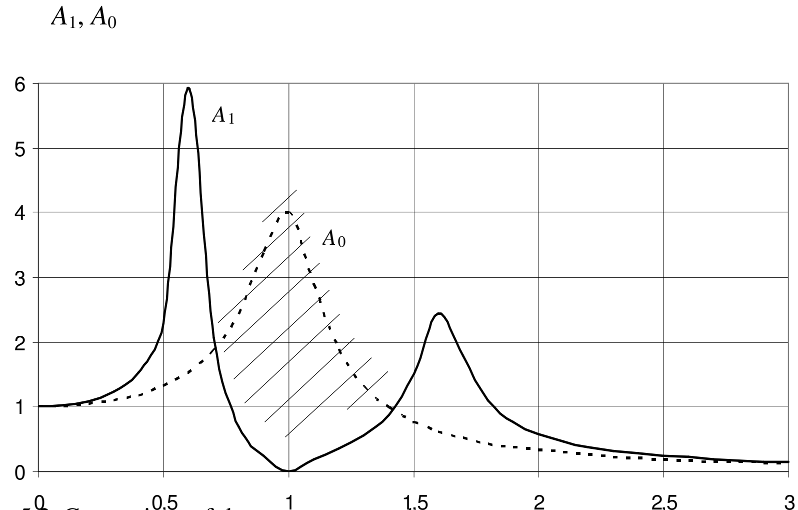


Fig. 5.2. Comparison of the resonance graphs

2.2. Effective range of the dynamic absorber

The frequency values ω, for which the main system vibration amplitudes with an absorber and without it are equal, can be calculated from the equation:

$$A_1 = A_0 . \tag{5.18}$$

When (5.15) and (5.16) are used, it results in:

$$q \frac{\left| d-\omega^2 \right|}{\sqrt{\left[\left(a-\omega^2\right)\left(d-\omega^2\right)-bd\right]^2 + 4h^2\omega^2\left(d-\omega^2\right)^2}} = \\ = \frac{q}{\sqrt{\left(\alpha^2-\omega^2\right)^2 + 4h^2\omega^2}} , \tag{5.19}$$

where:

$$a = \frac{k_1+k_2+k_3}{M} = \frac{k_1}{M} + \frac{k_2}{M} + \frac{k_3}{M} = \alpha^2 + \frac{q}{e} + b \tag{5.20}$$

The comparison of the amplitude values can be written as:

$$\left(d-\omega^2\right)^2 \left[\left(\omega_N^2-\omega^2\right)+4h^2\omega^2\right] = \\ = \left[\left(\omega_N^2-\omega^2+\frac{q}{e}+b\right)\left(d-\omega^2\right)-bd\right]^2 + 4h^2\omega^2\left(d-\omega^2\right)^2 \tag{5.21}$$

The frequency values obtained from Eq. (5.21) give the effective range of the absorber.

3. Measurement system

The main vibration system is shown in Fig. 5.3 and its description is to be found in **Exercise 4**. The dynamic absorber consists of the mass m_1 and a spring of the stiffness k_t.

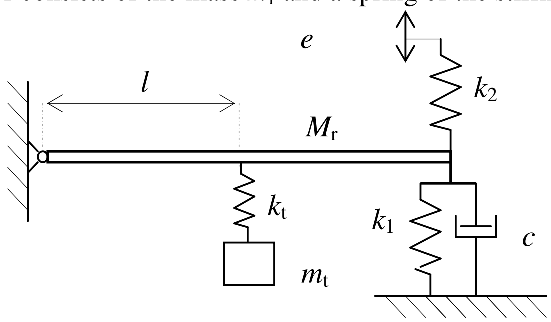


Fig. 5.3. Scheme of the test stand

4. Course of the exercise

1) Experimental data for the main system resonance curve.
   a) For the suggested values of rotational speed (from Table 1), measure vibration amplitude values of the main system without an absorber. Collect more data close to the resonance, trying to estimate their values carefully.
   b) Draw a resonance graph on the basis of the obtained data for the system without an absorber.
   c) Find the resonance graph peak coordinates (A_m,ω_m).

Table 5.1

Excitation frequencies		without an absorber	with an absorber
No	rotational speed [rad/s]	A_0	A_i
1	0		
2	10		
3	20		
4	25		
5	30		
6	32		
7	34		
8	36		
9	38		
10	40		
11	42		

12	45		
13	50		
14	55		
15	60		
16	70		

1. Using the formulae for one-degree-of-freedom systems Eq. 5.22, find the natural frequency for the system under consideration. Use a spreadsheet to do this.

$$q = \frac{\omega_m^2 x_0}{\sqrt{1-\left(\frac{x_0}{A_m}\right)^2}} \ , \quad \omega_N^2 = \frac{q}{x_0} = \frac{\omega_m^2}{\sqrt{1-\left(\frac{x_0}{A_m}\right)^2}} \ ,$$

$$2h = \sqrt{2\left(\omega_N - \omega_m^2 \right)} = \omega_m \sqrt{\frac{1}{2\sqrt{1-\left(\frac{x_0}{A_m}\right)^2}} - 1} \qquad (5.22)$$

   a. Draw a resonance graph employing the data acquired.

3) Calculate the absorber tuned frequency. Let us remember we do need to decrease forced vibrations of the main system with chosen value (its natural frequency ω_n). To do it, we need to determine the active length of the absorber beam (spring) – l. Then, the frequency of natural vibrations of the dynamical absorber should be equal to ω_n. The length l should be chosen to fulfil the condition: $\omega_t = \omega_n$. One should pay attention to the fact that a change in the spring length l of the dynamic absorber is followed by a change in its stiffness k_t and its reduced mass m_t as well. Both k_t and m_t can be calculated using the Rayleigh scheme:

$$k_t = \frac{3EJ}{l^3} \text{ and } m_{red} = m_t + \frac{1}{5}\mu l, \qquad (5.23)$$

where: E – Young modulus, J – moment of inertia of the spring under bending, $\mu = m_b/l$ – unit mass of the beam (spring). Because $\omega_t^2 = k_t/m_t$, then employing the above-mentioned formulae, we get equation which allows to determine proper length of the beam:

$$\frac{1}{5}\omega_n^2 \mu l^4 + \omega_n^2 m l^3 - 3EJ = 0 \qquad (5.24)$$

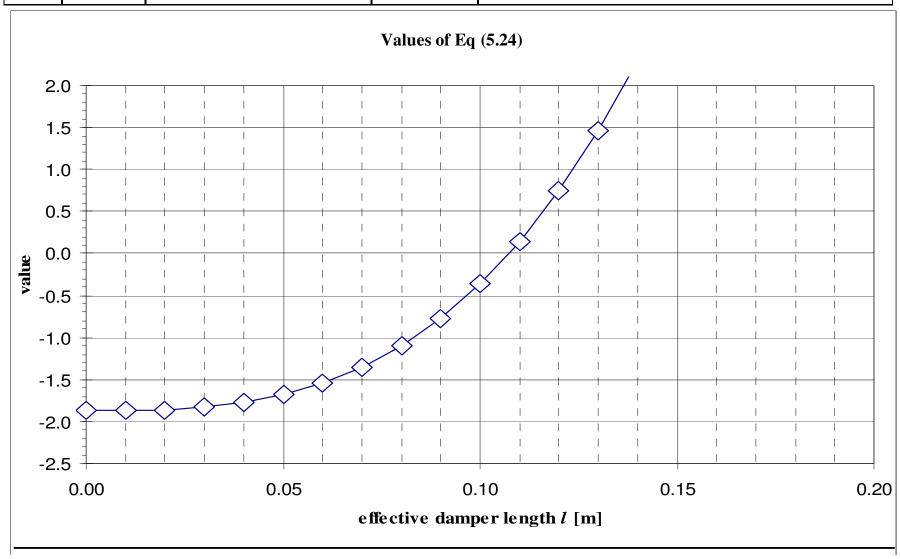
Equation (5.24) is of the fourth order with the unknown value l and it is solved numerically. The spreadsheet function called GOAL SEEK in Excel can be used – an example is shown below. Such a solution can be obtained when we assume the l value in a spreadsheet cell, then relate this value to another cell estimating a value of the expression on the left-hand side of Eq. (5.24) and try to obtain its value as zero. As a result, the l cell presents the desired value. Set then the length of the dynamic absorber leaf spring to this value at the rig.

Example of the experimental data:

DATA:
$b = 0.030$ [m], $h = 0.0015$ [m], $E = 0.74*10^{11}$ [N/m^2], $\rho = 7500 \div 8350$ kg/m^3,
$m_t = 0.94$ [kg], $J = b\,h^3/12$.

Example of the Excel spreadsheet solving Eq. (5.24):

	A	B	C	D
1				1/5 $\omega_n^2 \mu l^4 + \omega_n^2 m l^3 - 3EJ_b = 0$
2	E	74000000000		
3	ω_n	40		
4	μ	=B8*B9*B10	*l-ini*	0.2
5			eq	=0.2*B4^2*B5*D5^4+B4^2*B7*D5^3-3*B3*B8
6	m	0.94		
7	J_b	=B8*B9^3/12		Example result in cell D4: 0.107 m
8	b	0.03		
9	h	0.0015		
10	ρ	8000	<-	7500-8350
11				



2. Collect the data for another experimental resonance curve with a dynamic absorber; this should be done in a similar way as previously.
3. Compare both the graphs. Find the values of the forcing frequencies ω_1 and ω_2 at which the vibration amplitudes of the main system with and without an absorber are equal. The agreement of the results with formula (5.21) should be checked by substituting the obtained values to the formula. As a result, the efficient range of the dynamic absorber is obtained.
4. Present your comments.

5. Laboratory report should contain:

1) Aim of the exercise.
2) Results of the experimental investigations of the system without an absorber.
3) Resonance graph based on the data obtained for the system without an absorber.
4) Results of the computer calculations of the absorber tuned frequency and the length of the absorber.
5) Experimental resonance curve of the system with a dynamic absorber.
6) Theoretical and experimental efficient range of the dynamic absorber – comparison.
7) Conclusions and remarks.

References

1. Rao S.S.: *Mechanical Vibrations*, Addison-Wesley, NY, 1995.
2. Tse F.S., Morse I.E., Hinkle R.T.: *Mechanical Vibrations - Theory and Applications,* Allyn and Bacon Inc., 1978.
3. Kapitaniak T.: *Wstęp do teorii drgań*. Wydawnictwo Politechniki Łódzkiej, Łódź 1992.