Exercise 7

DYNAMIC BALANCING OF RIGID ROTORS

1. Aim of the experiment

The aim of the experiment is to get acquainted with the technique of dynamic balancing of rigid rotors by means of an automatic balancer with an electronic measurement system.

2. Theoretical introduction

2.1. Definition of a rigid rotor

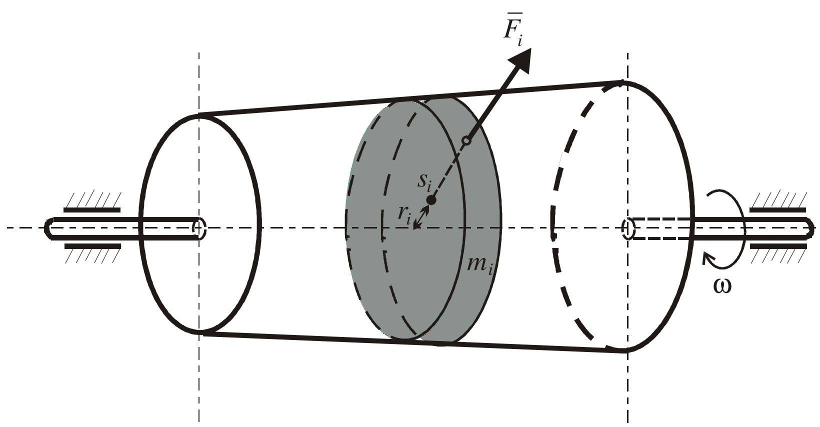

Fig. 7.1. Rigid rotor

In the design of rigid rotors, a circular symmetric mass distribution is assumed. In such a case, the self-balancing of inertia forces is ensured. However, the ideal symmetric mass distribution cannot be achieved in actual rotors due to technological reasons. Thus, during the motion, unbalanced centrifugal inertia forces act on the mass elements (m_i) of the rotor. These forces (F_i) are proportional to the accelerations (a_i) of individual mass centres:

$$\overline{F}_i = -m_i \overline{a}_i \ . \tag{7.1}$$

For the constant angular velocity ω, there is a centripetal acceleration $a_i = \omega^2 r_i$, where r_i is a radius of the mass centre position (see Fig. 7.1).
A model of the system of inertia forces acting on mass elements of an arbitrary rotor is presented in Fig. 7.1. Let us imagine that the rotor under consideration consists of a set of thin discs divided by planes perpendicular to the rotor axis. The inertia force

$$\overline{F}_i = -m_i \omega^2 \overline{r}_i \tag{7.2}$$

is applied to the mass centre of each individual disc and it rotates together with the rotor during its motion. These forces cause the rotor bending, which becomes more intensive when the angular velocity of the rotor increases. During the increase in the velocity ω, positions of mass centres of all imaginary discs vary (as a result of bending), according to the shape of the resonance curve. It causes that the diagram of the total rotor load F as a function of the velocity ω, shown in Fig. 7.2, has also a shape of the resonance curve, even though it would appear that the rotor load should vary according to the parabola resulting from Eq. (7.2). The maximum rotor load occurs at $\omega = \omega_{c1}$, where ω_c is the **first critical speed of the rotor**, equal to the first resonance (natural) frequency ω_{n1}.

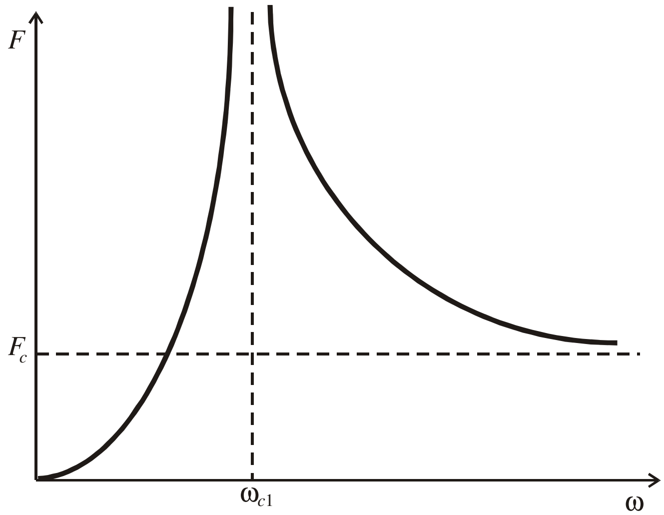

Fig. 7.2. Diagram of the total rotor load F as a function of the velocity ω; F_c is the critical load at the first critical speed of the rotor ω_{c1}

The behaviour of the rotor described above subjected to centrifugal inertia forces shows that no rotor can be treated as a rigid one in advance, i.e., independently of its speed of motion. We can assume that the rotor under consideration is rigid only in the case when its deflection is negligibly small. Such a situation takes place when the working speed ω is sufficiently small in comparison with the first critical speed ω_{c1}. Then, the rotor can be treated as a non-deformable one, i.e., with an invariable mass distribution.

The above condition is determined by the following practical criterion, which allows us to recognize the dynamic features of the rotor: *the rotor can be considered as a rigid one, if its angular (rotational) working velocity does not exceed half of the first critical speed, i.e.*:

$$\omega \leq \frac{\omega_{c1}}{2} \tag{7.3}$$

or

$$n \leq \frac{n_{c1}}{2}. \tag{7.4}$$

If working velocities are higher than $\omega_{c1}/2$, then it is necessary to consider an influence of the rotor deflection due to inertia forces and corresponding dynamical effects. Such a rotor can be considered as a flexible one. A distinction between flexible and rigid rotors is of essential significance from the viewpoint of the selection of a balancing method.

2.2. Balancing of a rigid rotor

Every rotor is unbalanced due to an inaccuracy in the manufacturing process. The rotor is unbalanced if its principal axis of inertia does not overlap with the axis of rotation (see Fig. 7.3a). An occurrence of centrifugal inertia forces during the rotational motion of the rotor, caused by its unbalance, is a reason of the appearance of many unwanted phenomena, such as variable load of bearings, vibrations of the supporting construction and surrounding devices, noise. We can reduce or even eliminate the inertia forces by correcting the mass distribution of the rotor, that is, by balancing.

The correction of the rotor mass in order to move its mass centre S to the axis of rotation is called ***static balancing*** (see Fig. 7.3b). Such balancing tends to reduce the resultant inertia force, acting on the rotor, to zero by means of an appropriate correcting mass. If, as a result of the correction, the rotation axis becomes one of the principal axes of inertia, then we have the so called ***dynamic balancing***. Thus, this kind of balancing reduces the inertia force and the moment of the inertia force (acting on the rotor) to zero and it requires applying two correcting masses.

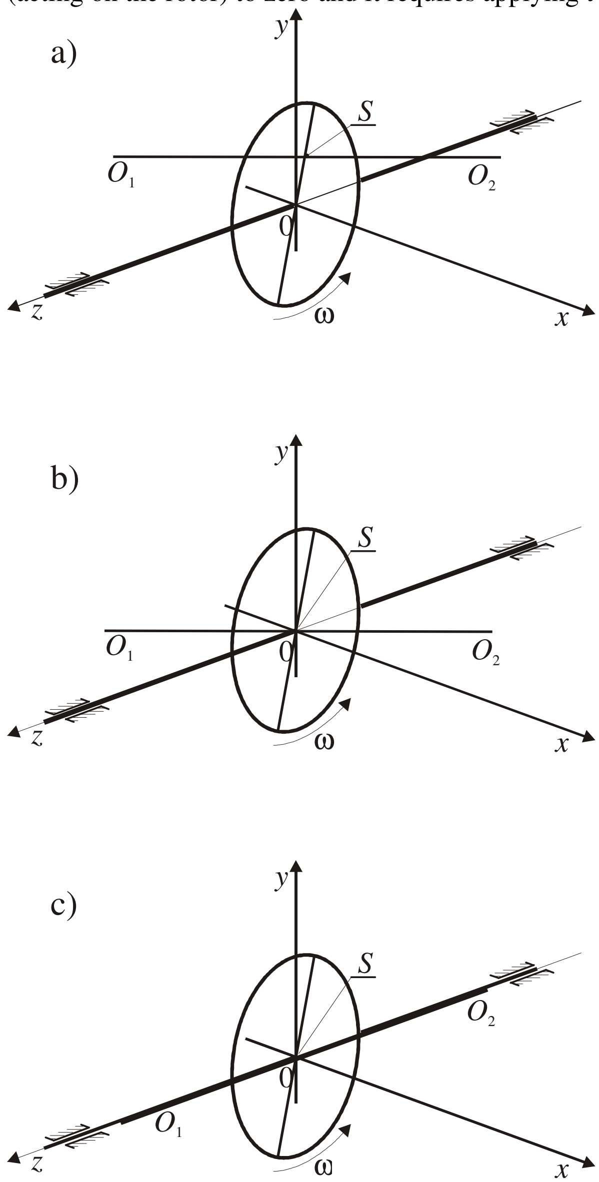

Fig. 7.3. Unbalanced rotor (a), static balancing (b), dynamic balancing (c)

Dynamic balancing

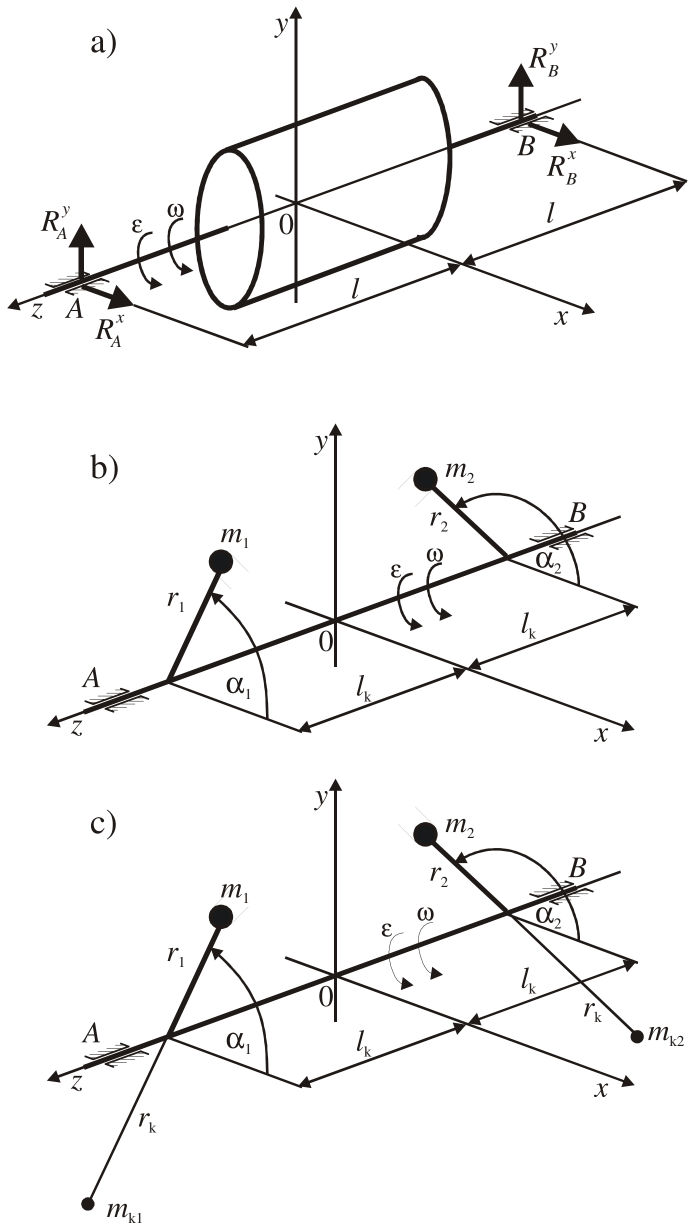

Fig. 7.4. Spatial rigid rotor (a), its equivalent before (b) and after (c) balancing

Consider a general case – a spatial rigid rotor shown in Fig. 7.4a. The rotor rotates around the axis of rigid bearings AB with the angular velocity ω and the angular acceleration ε. In a Cartesian system of coordinates (its coordinate z overlaps with the axis of bearings), we have the following equations of equilibrium of the acting forces:

(a) $$\sum F_i^x = R_A^x + R_B^x + \omega^2 S_x + \varepsilon S_y = 0,$$

(b) $$\sum F_i^y = R_A^y + R_B^y + \omega^2 S_y - \varepsilon S_x = 0,$$

(c) $$\sum F_i^z = 0, \qquad (7.5)$$

(d) $$\sum M_i^x = R_A^y l_A + R_B^y l_B - \omega^2 B_{yz} + \varepsilon B_{xz} = 0$$

(e) $\sum M_i^y = R_A^x l_A - R_B^x l_B + \omega^2 B_{xz} + \varepsilon B_{yz} = 0$

(f) $\sum M_i^z = \varepsilon B_{zz}$

The quantities B_{xz}, B_{yz}, B_{zz} (centrifugal mass moments of inertia) and S_x, S_y are determined by the formulas:

$$B_{xz} = \int_m xz \, \mathrm{d}m, \qquad B_{yz} = \int_m yz \, \mathrm{d}m, \qquad B_{zz} = \int_m (x^2 + y^2) \, \mathrm{d}m,$$

$$S_x = \int_m x \, \mathrm{d}m, \qquad S_y = \int_m y \, \mathrm{d}m.$$

The rotor under consideration can be substituted by two concentrated masses m_1 and m_2. These masses are located on two arbitrarily chosen correction planes (let these be planes at a distance l_k from the plane xy) by means of massless rods of the length r_1 and r_2 sloped to the plane xz at the angles α_1 and α_2, correspondingly (see Fig. 7.4b). In order to reduce the reactions in bearings to zero, it is necessary to apply a pair of skew centrifugal inertia forces to the rotor. These forces act on two correcting masses m_{k1} and m_{k2}. They are fixed to the rotor in the correction planes, at a distance r_k from the axis of rotation z, as depicted in Fig. 7.4c. The unknown values m_{k1} and m_{k2} can be calculated from Eq. (7.5), when the values S_x, S_y, B_{xz}, B_{yz}, or their equivalents $m_1 r_1$, α_1, $m_2 r_2$, α_2, are known.
The balancing of a real rotor of the unknown quantities S_x, S_y, B_{xz}, B_{yz} can be carried out as follows:

1) Place the rotor in the balancer and secure it.
2) After starting the balancer, determine the radiuses r_1, r_2 and the angles α_1, α_2 determining the position of the masses m_1, m_2, where the principal central axis of inertia passes through their centres (see Fig. 7.5a).

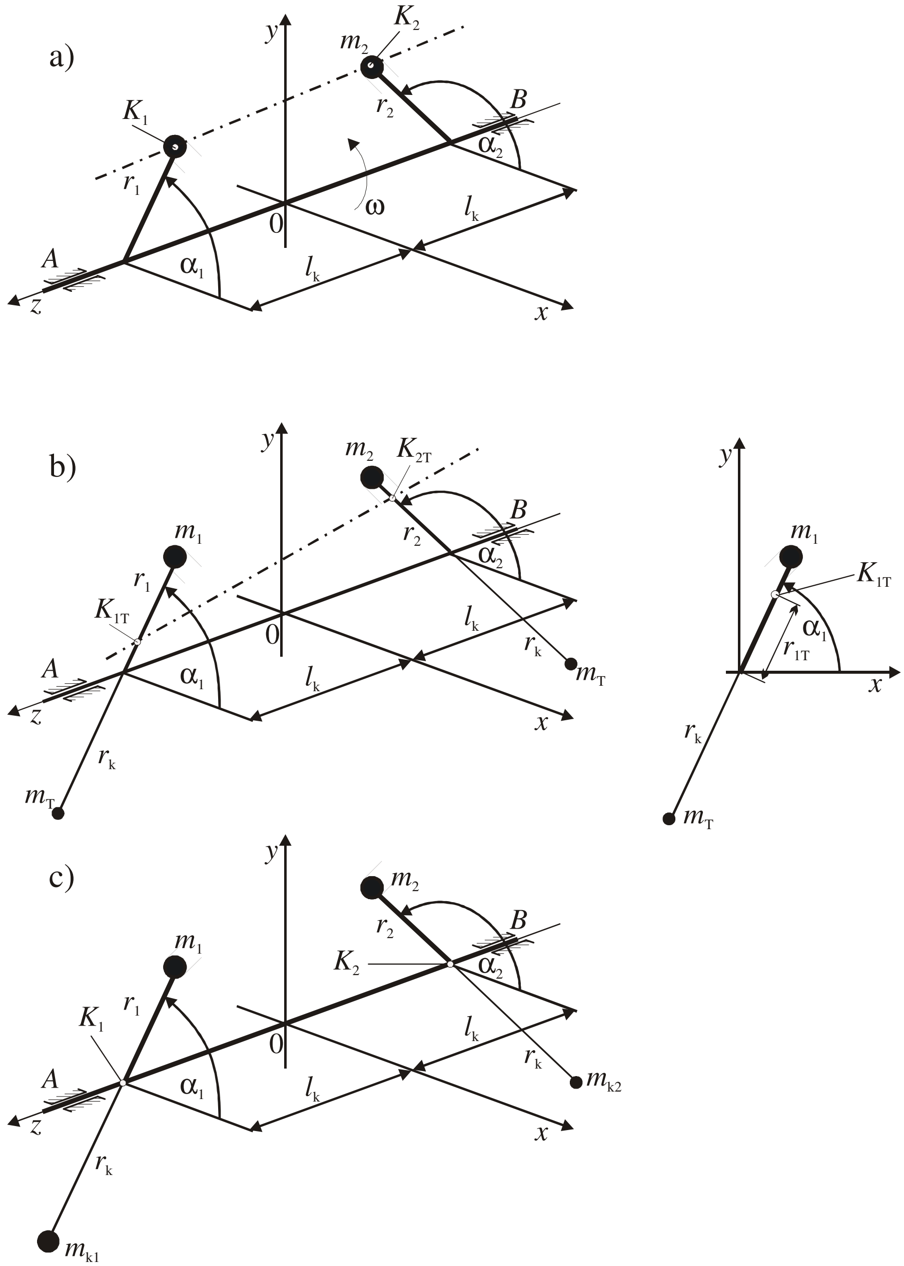

Fig. 7.5. Stages of dynamic balancing

3) Fasten two test weights of the arbitrary chosen masses m_T on the correction planes. The fastening radius r_k is also arbitrary – in practice it is determined by the rotor design. The fastening points of the weights are located opposite the points K_1 and K_2 (Fig. 7.5b). The fastening of the weights causes a change in the position of the principal central axis of inertia. Now, it crosses the correction planes in the points K_{1T} and K_{2T}, being mass centres of systems (m_1 – m_T) and (m_2 – m_T), at the distances being at r_{1T}, r_{2T} from the axis of rotation. The distances r_{1T}, r_{2T} have to fulfil the equations of equilibrium of static moments:

$$\begin{aligned} m_1 r_1 - m_T r_k &= (m_1 + m_T) r_{1T}, \\ m_2 r_2 - m_T r_k &= (m_2 + m_T) r_{2T}, \end{aligned} \qquad (7.6)$$

hence:

$$m_1 = m_T \frac{r_k + r_{1T}}{r_1 - r_{1T}}, \quad m_2 = m_T \frac{r_k + r_{2T}}{r_2 - r_{2T}}. \tag{7.7}$$

4) After restarting the balancer, determine the new radiuses r_{1T}, r_{2T} and calculate the masses of correcting weights m_{k1}, m_{k2} from the following formulas:

$$\begin{aligned} m_1 r_1 &= m_{k1} r_k, \\ m_2 r_2 &= m_{k2} r_k, \end{aligned} \tag{7.8}$$

hence:

$$m_{k1} = m_1 \frac{r_1}{r_k}, \qquad m_{k2} = m_2 \frac{r_2}{r_k}, \tag{7.9}$$

and after putting Eq. (7.7) into Eq. (7.9), we obtain:

$$m_{k1} = m_T \frac{r_k + r_{1T}}{r_1 - r_{1T}} \frac{r_1}{r_k}, \qquad m_{k2} = m_T \frac{r_k + r_{2T}}{r_2 - r_{2T}} \frac{r_2}{r_k}. \tag{7.10}$$

Since $r_{1T} << r_k$ and $r_{2T} << r_k$, Eqs. (7.10) can take the following form:

$$m_{k1} = m_T \frac{r_1}{r_1 - r_{1T}}, \quad m_{k2} = m_T \frac{r_2}{r_2 - r_{2T}}. \tag{7.11}$$

The application of correcting weights leads to a zero value of the balancer indications $r_{1k} = 0$ and $r_{2k} = 0$. Thus, the principal central axis of inertia is reduced to the axis of rotation and the rotor is dynamically balanced (see Fig. 7.5c).

3. Experiment

1) Place the rotor in the balancer and secure it.
2) Start the balancer and read out the lengths of the radiuses r_1, r_2 and the angles α_1, α_2.
3) Stop the balancer and fasten two test weights of the arbitrary chosen masses m_T on the correction planes. Their fixing points should be found opposite the points K_1 and K_2 determining the location of the principal central axis of inertia.
4) Start the balancer again and determine the lengths of the radiuses r_{1T}, r_{2T} and the angles α_{1T}, α_{2T}.
5) Calculate the masses of correcting weights m_{k1}, m_{k2} on the basis of Eqs. (7.11).
6) **Remark:** Since the balancer indicates the absolute values $|r_{iT}|$ (where $i = 1, 2$), so in the case of $\alpha_{iT} = \alpha + 180°$, the relation $r_{iT} = -|r_{iT}|$ should be substituted in Eqs. (7.11).
7) Check the value of residual unbalancing after applying the calculated correcting weights.

References

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