Exercise 11

ISOLATION OF MACHINE VIBRATIONS

1. Aim of the exercise

To isolate environment and objects around of a working machine from vibrations generated during its activity.

Experimental determination of the measure of isolation in form of transfer factor of vibrations.

2. Theoretical introduction

Rotating machines with unbalance are typical sources of vibrations. If these devices are mounted rigidly on their foundation, vibrations will transfer to the environment. Transferred vibrations may cause noise, a harmful influence on the human body, or overloading of the instruments which are nearby. Isolation of vibrations can minimize such undesirable interactions. Usually, visco-elastic elements are placed between the vibrating machine and its foundation. This is the so-called **active isolation**, or forced isolation when we want to prevent the force from being transferred to the foundation. In the case when we want to isolate a device from the vibrating foundation, we say this is **passive isolation**, or displacement isolation.

2.1. Admissible vibrations acting on the human body

To provide suitable conditions for people who work nearby machines, the isolation of machine vibrations is carried out among others. Technical standards describe the rules of mechanical vibration measurements to estimate their influence on the human body. Admissible values of vibration parameters are presented also in these standards. Admissible accelerations of total vibrations (acting on the whole body), local vibrations (acting on hands) and admissible work time when these limits are exceeded are these vibration parameters.

Polish Standard PN-91/N-01354 shown in Table 11.1, presents the admissible values of acceleration of vibrations which act on the human body during 480 minutes. The spectral method has been used to estimate the risk. The spectral analyses of acceleration of vibrations in 1/3-octave frequency bands in the ranges 1 ÷ 80 Hz (for total vibrations) and 6.3 ÷ 1250 Hz (for local vibrations) are carried out in this method.

Table 11.1. Admissible values of acceleration of vibrations

Middle frequency of the 1/3-octave band [Hz]	Admissible effective value of acceleration [m/s^2]	
	Vertical component	Horizontal component
1.0	0.63	0.224
1.25	0.56	0.224
1.5	0.50	0.224
2.0	0.45	0.224
2.5	0.40	0.280
3.16	0.355	0.355
4.0	0.315	0.450
5.0	0.315	0.560
6.3	0.315	0.710
8.0	0.315	0.900
10.0	0.40	1.12

12.5	0.50	1.40
16.0	0.63	1.80
20.0	0.80	2.24
25.0	1.00	2.80
31.5	1.25	3.55
40.0	1.60	4.50
50.0	2.00	5.60
63.0	2.50	7.10
80.0	3.15	9.00

2.2. Rubber as a vibro-isolating material

Rubber is widely used as a visco-elastic isolating element. In comparison with steel, rubber has high internal damping, an ability of sound absorption and very good shape elasticity. The stiffness coefficient of the rubber vibro-isolator depends on its hardness and changes in a non-linear way with an increase in the static load. The stiffness coefficient is defined as:

$$k = \frac{dQ}{dy} \quad \left[\frac{N}{m}\right], \qquad (11.1)$$

where: Q – static load of vibro-isolator, y – deflection of vibro-isolator.

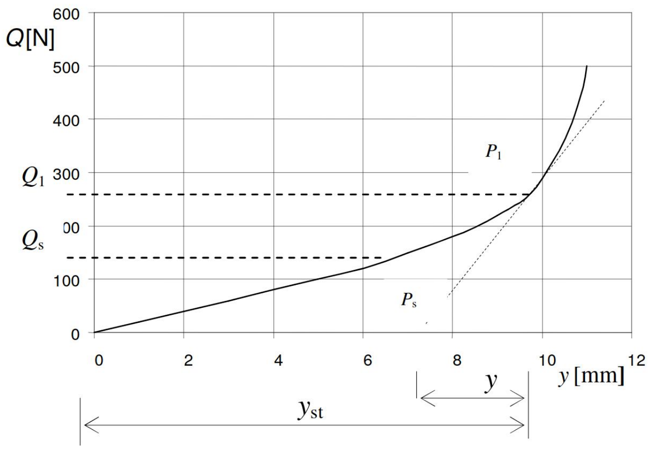


Fig. 11.1. Non-linear characteristic curve of rubber

Figure 11.1 shows the so-called hard characteristic curve. In this case, the k value increases when the loading enlarges. If the static load does not exceed a certain value Q_s, (point P_s in Fig. 11.1), the characteristic curve is linear. In this loading range, the stiffness coefficient k has a constant value. If the static load enlarges, for example to the value Q_1 (point P_1), the stiffness coefficient increases in a non-linear way. The slope in the point P_1 determines the k value in this loading range.

For the isolation purposes, vibration damping properties of rubber are very important. They cause the absorption of vibration energy and facilitate a gentle passage through resonance. Depending on the rubber type, the value of dimensionless damping coefficient h/α is in the range from 0.04 to 0.1 (where: h – viscous damping coefficient referred to the mass in [rad s^{-1}], α - natural frequency in [rad s^{-1}]). Contrary to steel, rubber does not corrode and has high fatigue resistance. This does not apply under low temperatures, because rubber pads may be used in temperatures from -30^o to +75^{o}C.

2.3. Transfer factor

A one-degree-of-freedom vibrating system is considered. The exciting harmonic force $F \sin\omega t$ acts on the mass m (Fig. 11.2). Between the vibrating mass and its foundation, there is a spring with the stiffness coefficient k and a viscous dashpot with the damping coefficient c.

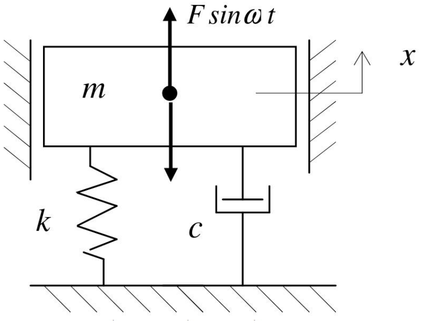


Fig. 11.2. Model of the vibrating system

The equation of motion of the considered model can be expressed in the following form:

$$m\ddot{x} + c\dot{x} + kx = F\sin\omega t\ . \qquad (11.2)$$

After some transformations, equation of motion (11.2) takes the form:

$$\ddot{x} + 2h\dot{x} + \omega_n^2 x = q\sin\omega t\ , \qquad (11.3)$$

where:

$$\frac{c}{m} = 2h;\ \ \frac{k}{m} = \omega_n^2;\ \ \frac{F}{m} = q. \qquad (11.4)$$

The particular solution to Eq. (11.3) can be predicted as:

$$x = A\sin(\omega t - \varphi)\ , \qquad (11.5)$$

where the amplitude of displacement A is expressed by:

$$A = \frac{q}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 + 4h^2\omega^2}}\ , \qquad (11.6)$$

and the phase shift angle φ between excitation and displacement is determined by the following dependence:

$$\varphi = arc\,tg\,\frac{2h\omega}{\omega_n^2 - \omega^2}\ . \qquad (11.7)$$

The force which acts on the foundation consists of a force in the spring and a force in the damper. The force in the spring is as follows:

$$S = kx = kA\sin(\omega t - \varphi). \qquad (11.8)$$

The force in the damper is equal to:

$$R = c\dot{x} = cA\omega\cos(\omega t - \varphi). \qquad (11.9)$$

The maximal total force acting on the foundation is expressed as:

$$P_{\max} = \sqrt{S_{\max}^2 + R_{\max}^2} = \sqrt{(kA)^2 + (c\omega A)^2}. \qquad (11.10)$$

Substituting Eq. (11.4) and Eq. (11.6) into Eq. (11.10) gives:

$$P_{\max} = \frac{F\sqrt{1 + 4\dfrac{h^2}{\omega_n^2}\dfrac{\omega^2}{\omega_n^2}}}{\sqrt{\left(1 - \dfrac{\omega^2}{\omega_n^2}\right)^2 + 4\dfrac{h^2}{\omega_n^2}\dfrac{\omega^2}{\omega_n^2}}} \qquad (11.11)$$

The ratio of the maximal total force acting on the foundation $P_{\max}$ to the exciting force amplitude F is called the **transfer factor** and will be denoted by ν in further considerations. Using Eq. (11.11), the transfer factor can be determined in the form:

$$\nu = \frac{P_{max}}{F} = \frac{\sqrt{1 + 4\frac{h^2}{\omega_n^2}\frac{\omega^2}{\omega_n^2}}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\frac{h^2}{\omega_n^2}\frac{\omega^2}{\omega_n^2}}} \tag{11.12}$$

Figure 11.3 shows plots of the transfer factor ν versus the exciting frequency $\gamma = \omega / \omega_n$ (the ratio of the dimensional exciting frequency ω to the natural frequency ω_n) for different values of the dimensionless damping coefficient h/ω_n.

Depending on the rubber type, the value of the dimensionless damping coefficient h/ω_n is in the range from 0.04 to 0.1. To receive the transfer factor plot value of about 0.1 on the basis of Fig. 11.3 for these damping values, the dimensionless exciting frequency $\gamma = \omega / \omega_n$ should be higher than 3-5.

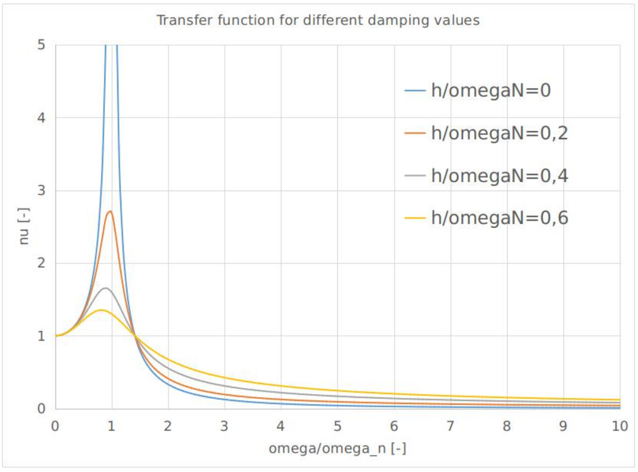


Fig. 11.3. Plot of the transfer factor for different ration of damping/natural frequency values

2.4. Example of the isolator selection for active vibro-isolation on the basis its static characteristics

In this numerical example, a system consisting of a fan and a driving motor is considered. The total mass of the system is m_1 = 200 kg. The rotational speed of the motor is n = 1450 rev/min. The system is mounted in a supporting structure of the mass m_2 = 100 kg. The foundation should be isolated from vertical vibrations of the system by means of four rubber isolators. After the isolation, the transfer factor value should not be higher than 0.1.

Solution

The exciting frequency is equal to:

$$\omega = \frac{2\pi n}{60} = 152.05 \text{ [rad/s]} \tag{a}$$

The gravitational force acting on one isolator is as follows:

$$Q = \frac{m_1 + m_2}{4} g = 735.75 \text{ [N]} \tag{b}$$

It is assumed that rubber isolators will work in the linear part of their static characteristics (Fig. 11.1). Two kinds of M200-type isolators denoted by A and B, depending on their hardness, have been selected to isolate the foundation of the system. The static characteristics of M200-type isolators are shown in Fig. 11.4.

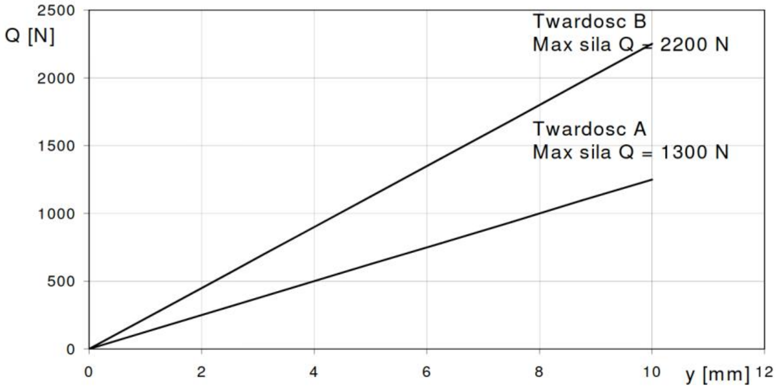


Fig. 11.4. Static characteristics of M200-type isolators

From the characteristic curve of the M200-type isolator of the hardness A, one can read the static deflection under the *Q* load. This deflection is y_{st} = 5.89 mm. Then, one can determine the value of the natural frequency of such an isolator:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{y_{st} m}} = \sqrt{\frac{g}{y_{st}}} = \sqrt{\frac{9.81}{5.89 \cdot 10^{-3}}} = 40.81 \text{ [rad/s]} \qquad \text{(c)}$$

The ratio of the exciting frequency ω to the natural frequency ω_n is equal to:

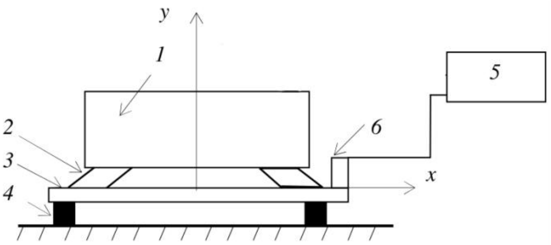
$$\gamma = \frac{\omega}{\omega_n} = \frac{152.05}{40.81} = 3.72 \qquad \text{(d)}$$

Still, one ought to check the transfer factor value for the limiting value of the h/ω_n ratio of rubber isolators.

$$\nu = \frac{\sqrt{1 + 4\left(\frac{h}{\omega_n}\right)^2_{\lim} \frac{\omega^2}{\omega_n^2}}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\left(\frac{h}{\omega_n}\right)^2_{\lim} \frac{\omega^2}{\omega_n^2}}} = \frac{\sqrt{1 + 4 \times 0{,}1^2 \times 3{,}72^2}}{\sqrt{\left(1 - 3{,}72^2\right)^2 + 4 \times 0{,}1^2 \times 3{,}72^2}} = 0{,}097 < 0{,}1 \qquad \text{(e)}$$

3. Measurement device

A scheme of the measurement device is shown in Fig. 11.5. Loudspeaker device (inside the yellow box) *1* is mounted on its foundation, represented by a (blue) plate *3.* Such a model is then set on the floor with support of another set of four rubber isolators *4. These elements stay out of further considerations, simply isolate the test rig from the outside world.* Vibrations of the foundation plate *3* are measured with piezoelectric sensors *6*. The sensors are connected to the measuring device with a narrow-band analyser *5.*



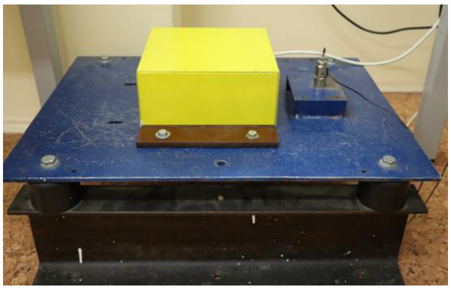


Fig. 11.5. Scheme of the measurement device

4. Course of the exercise

The active isolation of loudspeaker device vibrations should be performed. After the isolation, the transfer factor between the maximal total force acting on the plate and the exciting force amplitude should not be higher than 0.1. The following operations are to be done:

1) mount rigidly the loudspeaker device on the plate modelling the foundation,
2) turn on function generator and amplifier feeding the loudspeaker, set frequency at the value when the accelerometer responds in visible form with amplitude of vibrations of a_{exc}=1 m/s^2. This appears in bast form in range of 70-80 Hz.
3) note the vibration frequency and the acceleration amplitude in the vertical direction. This will be the reference value a_{ref} [m/s^2] to which we will relate further measurements and expressed as $\nu = a_i/a_{ref}$ [-].
4) unmount the yellow box and put rubber element under it while mounting again,
5) maesuer acceleration level and calculate the amplitude of the exciting force. Try finding out what is result of stronger/weaker bolts fixing.
6) replace rubber with cork, repeat measurement,
7) remove cork, select four rubber isolators, mount the to the foundation (blue plate) and set the machine (yellow box) at them properly fixing with 4 srew/nut sets,
8) measure acceleration amplitude and calculate the value of the theoretical transfer factor,
9) switch off the device,
10) calculate values of the transfer factors in each case. The first one should be simply 1.

References

1. Goliński J.: *Wibroizolacja maszyn i urządzeń*, WNT Warszawa 1979.
2. Parszewski Z.: *Drgania i dynamika maszyn*, WNT Warszawa 1982.
3. Information on isolating materials of the Swedish company Trelleborg AB.
4. PN-91/N-01354. *Admissible values of accelerations of vibrations which act on the human body*.
5. Rao S.S.: *Mechnical Vibrations*, 3rd ed., Prentice Hall, NY, 1995.
6. http://www.epsginc.com/data/engineering_guide_rev1.pdf

Laboratory report on *Isolation from machine vibrations*

Date Course

Name(s) and Index No(s)

Experimental and calculation results

Measurements

machine mass m **= 3.85 kg**

Machine rigidly mounted		
Excitation frequency *(measured)*	f [Hz]	
Excitation frequency ω	[rad/s]	
Acceleration amplitude a_{ref} *(measured)*	[m/s^2]	
Force amplitude $F_{ref} = m*a_{ref}$	[N]	
Transfer factor value ν	[-]	**1**
Machine mounted on RUBBER plate		
Excitation frequency *(measured)*	f [Hz]	
Excitation frequency ω	[rad/s]	
Acceleration amplitude a_R *(measured)*	[m/s^2]	
Force amplitude $F_R = m*a_R$	[N]	
Transfer factor $\nu_R=F_R/F_{ref}=a_R/a_{ref}$	[-]	
Machine mounted on CORK plate		
Excitation frequency *(measured)*	f [Hz]	
Excitation frequency	ω [rad/s]	
Acceleration amplitude a_C *(measured)*	a_C [m/s^2]	
Force amplitude $F_C = m*a_C$	F_C [N]	
Transfer factor $\nu_C=F_C/F_{ref}=a_C/a_{ref}$	[-]	
Machine mounted on Vibration Isolators		
Excitation frequency *(measured)*	f [Hz]	
Excitation frequency	ω [rad/s]	
Acceleration amplitude a_{VI} *(measured)*	a_{VI} [m/s^2]	
Force amplitude $F_{VI} = m*a_{VI}$	F_{VI} [N]	
Transfer factor $\nu_{VI}=F_{VI}/F_{ref}=a_{VI}/a_{ref}$	[-]	
Theoretical transfer factor ν_T *(see next page)*	[-]	

Other calculations

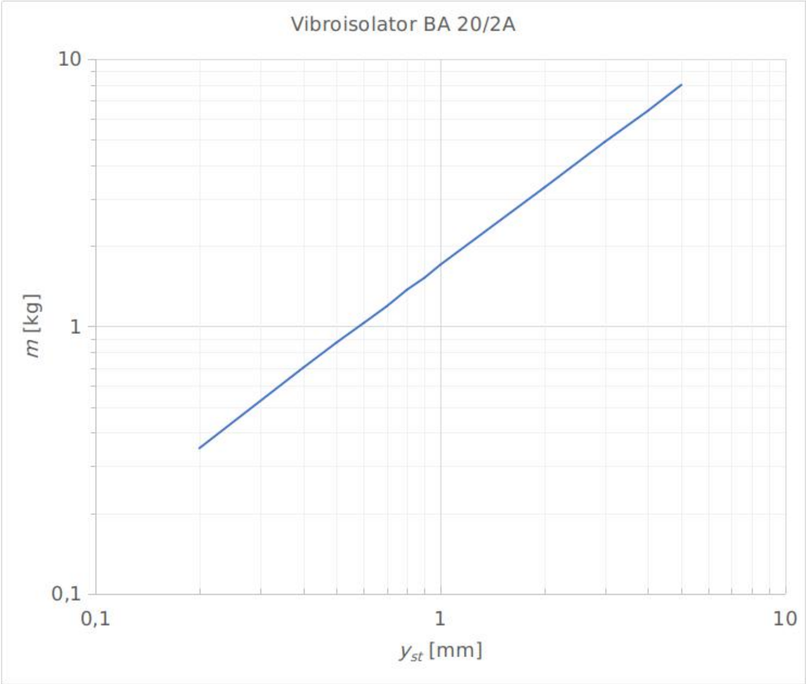
mass per single VI m	[kg]	
Static deflection at single VI y_{st}	[m]	
Natural frequency of the machine located at the set of 4 isolators ω_N	[rad/s]	
Frequency ratio ω/ω_N *auxillary* ω^2/ω_N^2	[-] [-]	
Damping ratio of rubber isolators based on literature data: $\xi = c/c_{critical}$ (assumed)	[-]	0.05
Calculated ratio h/ω_N *auxillary* h^2/ω_N^2	[1/s] [1/s^2]	
All above to be used in theoretical transfer factor calculations		

Other data

m = 3.85 kg (yellow box)

Assumption: $\xi = c/c_{critical}$ = 0.05 (rubber) - include below also your symbolic estimation of *h.*

Vibroisolator manufacturer data chart.



Compare both vibroisolator transfer factor values:

ν_{VI} =

and

ν_T =

Calculate error

err = [(exp-theory)/theory]*100%

Conclusions